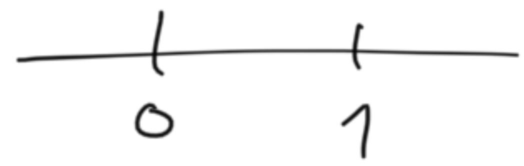
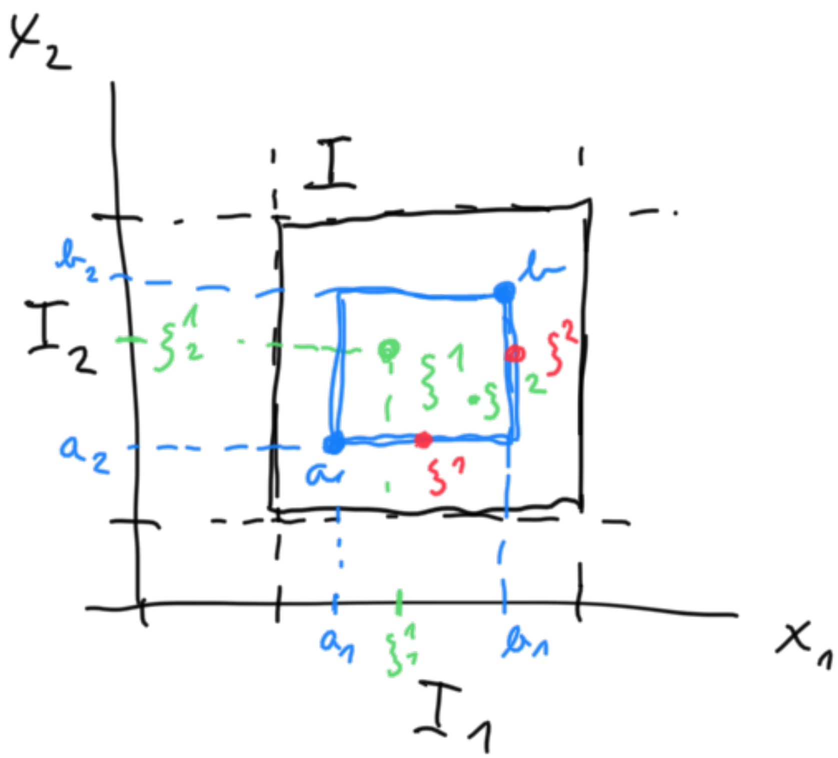
Úmluva: $a, b \in \mathbb{R}$

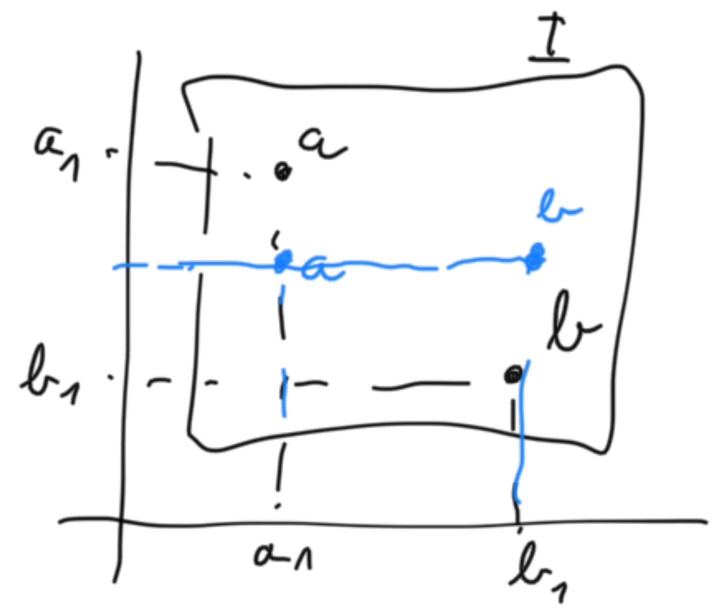
Symbolem $\langle a, b \rangle$ budeme rozumět uz. interval s kr. body a, b i v případě, že $a > b$.



$\langle 0,1 \rangle = \langle 1,0 \rangle$



$\xi^i_j \in \langle a_j, b_j \rangle$



Důkaz: krok 1 ($n=1$): Pokud $a = b$, zvolme $\xi^1 = a$.

Pokud $a < b$, je fce f na $\langle a, b \rangle \subset I$ spojitá (má v I vl. derivaci) a má v (a,b) derivaci $\underset{\text{Lagr. v.}}{\Rightarrow}$ $\exists\, \xi^1 \in (a,b)$: $\dfrac{f(b)-f(a)}{b-a} = f'(\xi^1)$, tj.

$$f(b) - f(a) = f'(\xi^1)(b-a) = \frac{\partial f}{\ldots}$$

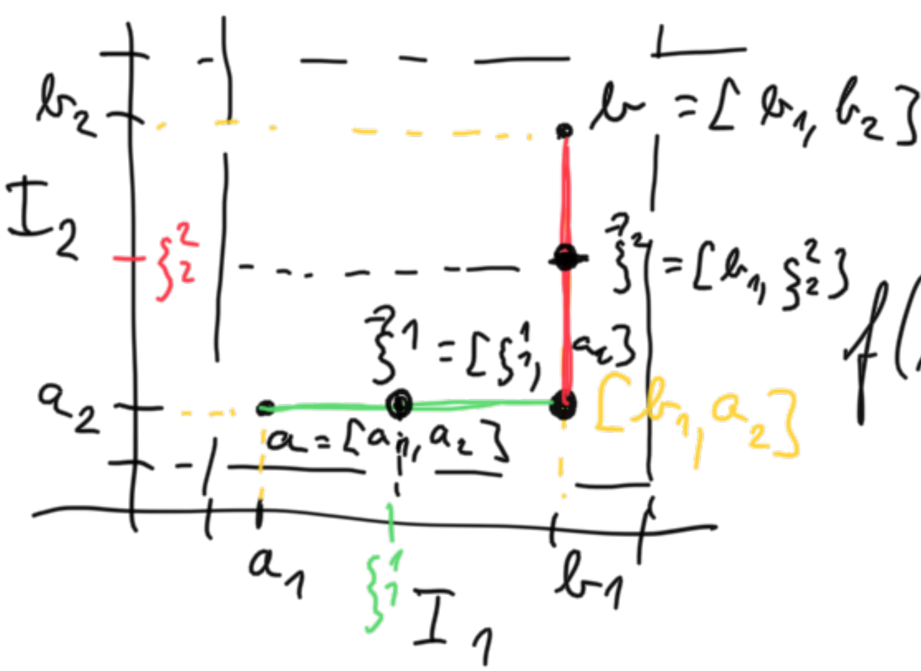
$\frac{\partial f}{\partial x_1}(\xi^1)(b-a)$.

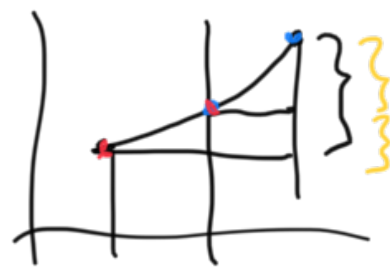
Pokud $a>b$, pak použijeme Lagr. větu na interval $\langle b,a\rangle$:

$$\exists\, \xi^1 \in (b,a):\quad f(a)-f(b) = f'(\xi^1)(a-b), \quad \Big/ \cdot(-1)$$

tedy
$$f(b)-f(a) = \frac{\partial f}{\partial x_1}(\xi^1)(b-a).$$

<u>krok 2</u> ($n=2$)





$$f(b)-f(a) = \overbrace{f(b_1,b_2)-f(b_1,a_2)}^{g_2(b_2)-g_2(a_2)} + \underbrace{f(b_1,a_2)-f(a_1,a_2)}_{g_1(b_1)-g_1(a_1)} \qquad (*)$$

Označme řezy $g_1(s)=f(s,a_2)$, $g_1: I_1\to\mathbb{R}$

$g_2(s)=f(b_1,s)$, $g_2: I_2\to\mathbb{R}$

$g_1\in C^1(I_1)$, $g_2\in C^1(I_2)$

podle kroku 1 existují čísla $\xi_1^1 \in \langle a_1, b_1 \rangle$ splňující
$\xi_2^2 \in \langle a_2, b_2 \rangle$

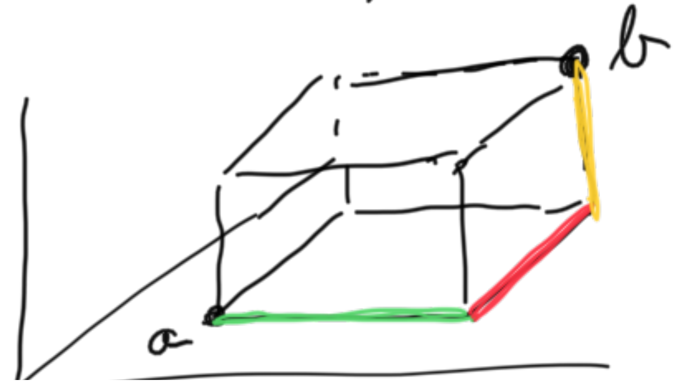
$$g_1(b_1) - g_1(a_1) = g_1'(\xi_1^1)(b_1 - a_1) = \frac{\partial f}{\partial x_1}\underbrace{(\xi_1^1, a_2)}_{\xi^1} \cdot (b_1 - a_1)$$
$$g_2(b_2) - g_2(a_2) = g_2'(\xi_2^2)(b_2 - a_2) = \frac{\partial f}{\partial x_2}\underbrace{(b_1, \xi_2^2)}_{\xi^2} \cdot (b_2 - a_2)$$

Položme $\vec{\xi}^1 = [\xi_1^1, a_2]$, $\vec{\xi}^2 = [b_1, \xi_2^2]$.

Po dosazení do (*) dostaneme, že

$$f(b) - f(a) = \frac{\partial f}{\partial x_1}(\vec{\xi}^1)(b_1 - a_1) + \frac{\partial f}{\partial x_2}(\vec{\xi}^2)(b_2 - a_2).$$

<u>krok 3</u> $(n > 2)$



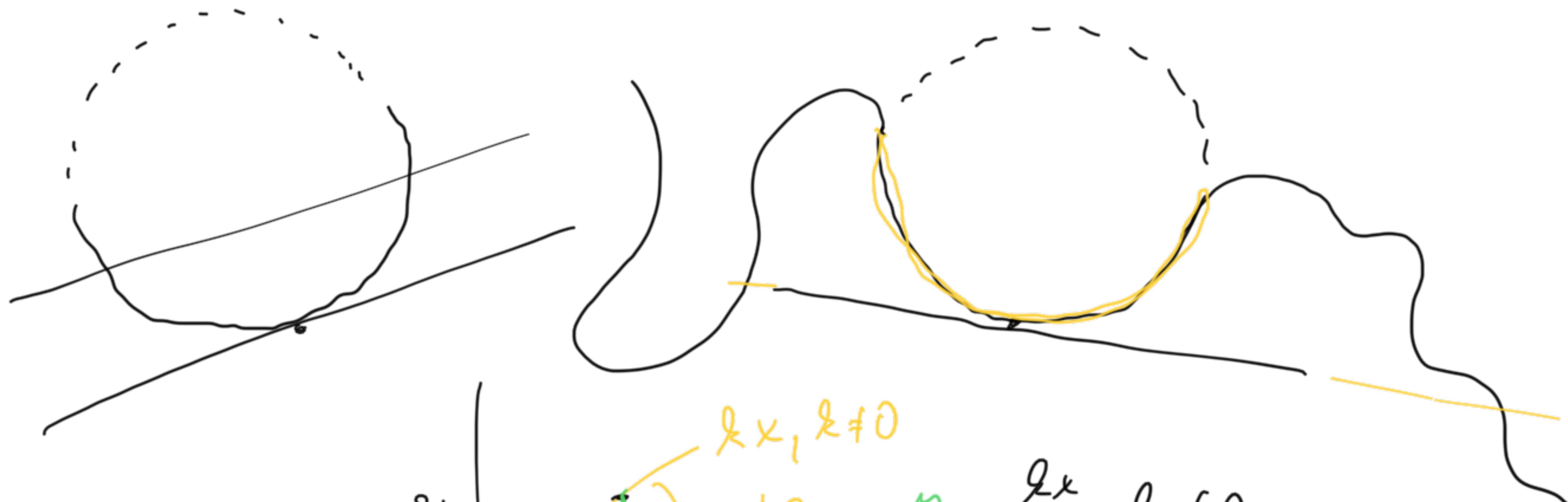
$$f(b)-f(a)=\sum_{i=1}^{n}\underbrace{\Big(f(b_1,\dots,b_{i-1},b_i,a_{i+1},\dots,a_n)-f(b_1,\dots,b_{i-1},a_i,a_{i+1},\dots,a_n)\Big)}_{\substack{\text{lze použít krok 1}\\ \text{tedy analogicky jako v kroku 2}}}$$

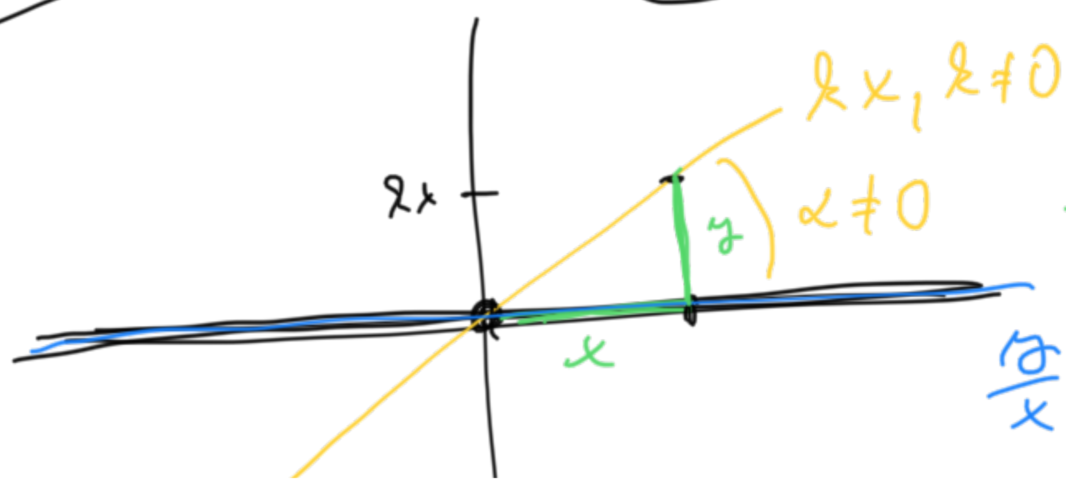
$\square$

Pozn: $n=1$ … tečna

$n=2$ … tečná rovina

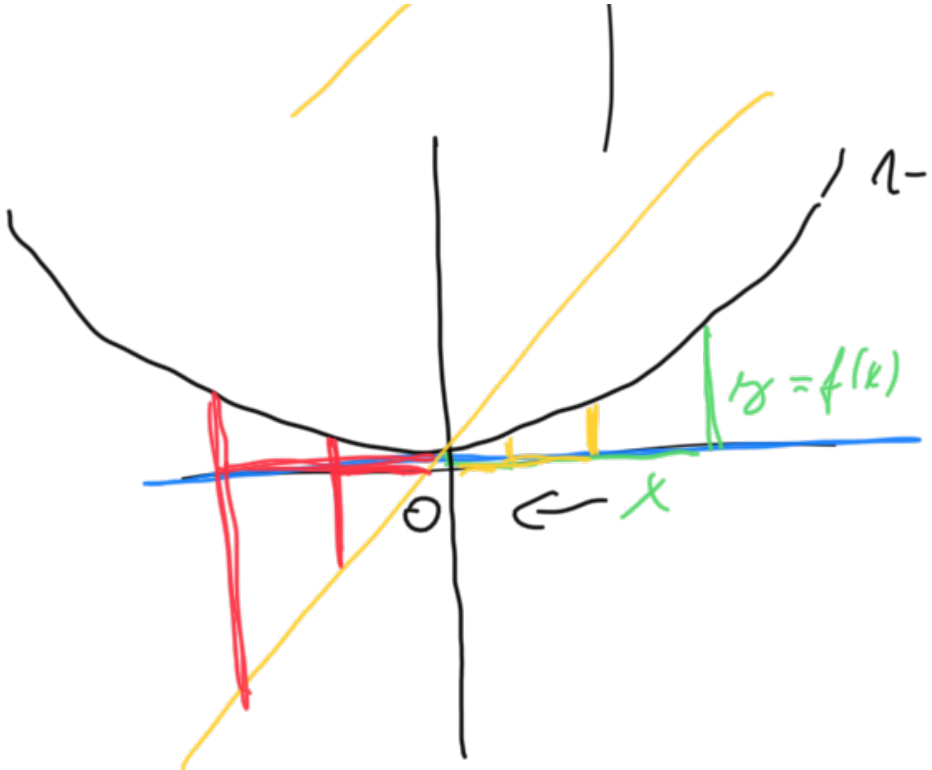
$$\overset{f(a)}{d}+\overset{\frac{\partial f}{\partial x_1}(a)}{c_1}(x_1-a_1)+\overset{\frac{\partial f}{\partial x_2}(a)}{c_2}(x_2-a_2)$$





$$\frac{y}{x}=\frac{kx}{x}=k\neq 0$$

$$\frac{y}{x}=0$$



$1-\sqrt{1-x^2} = f(x)$

$$\frac{y}{x} = \frac{f(x)}{x} = \frac{1-\sqrt{1-x^2}}{x}$$

$$\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x} = \lim \frac{1-(1-x^2)}{x\left(1+\sqrt{1-x^2}\right)} = \lim \frac{x^2}{\cancel{x}(\quad)} = 0$$

$\circ \ll \circ$

Def. V20: Zvolme $a \in G$ libovolně.

Nechť T je fce z def. tečné nadroviny ke f v bodě $[a, f(a)]$.

Pak $$\lim_{x\to a} f(x) = \lim_{x\to a}\left(\underbrace{\frac{f(x)-T(x)}{\rho(x,a)}}_{\to 0 \ \text{V19}}\cdot\underbrace{\rho(x,a)}_{\to 0}+\underbrace{T(x)}_{\to T(a) = f(a)}\right) = 0\cdot 0 + f(a) = f(a)$$

VOAL

$\Rightarrow f$ je spojitá v a

$\square$