ON NON-NEGATIVE INTEGER QUADRATIC FORMS

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The use of quadratic forms as a tool for characterizing classes of finite dimensional algebras and Lie algebras is well known and widely accepted.

We prove that each $\mathbb{Z}$-equivalence class of integer quadratic forms contains a disjoint union of multiplied unit Dynkin diagrams and some form related to radical, and each Gabrilov-equivalence class of integer quadratic forms contains a disjoint union of multiplied Dynkin diagrams and some form related to radical. For an integer quadratic form a non-negativity criterion is given.

A square matrix with integer coefficients $A$ is called a quasi-Cartan matrix if it is symmetrizable (there exists a diagonal matrix $D$ with positive diagonal entries such that $DA$ is symmetric) and $A_{ii} = 2$ for all $i$. A quasi-Cartan matrix is called Cartan matrix if it is positive definite and $A_{ij} \leq 0$ for all $i \neq j$. To any integer form we associate a Lie algebra in generators and relations in terms of the positive quasi-Cartan matrix. Quasi-Cartan matrix $A_q$ of quadratic form $q$ is Cartan matrix iff form $q$ is positive definite and classic.

Each Cartan matrix determines a unique semisimple complex Lie algebra via the Chevalley-Serre, sometimes called simply the Serre relations. We develop the classical Serre relations for quasi-Cartan matrix. Serre proved that if $q$ is positive definite and classic integer form then $g(q)$ is a semisimple (and finite dimensional) Lie algebra.

Two forms $q$ and $q'$ are called $G$-equivalent if one comes from another after a sequence of Gabrilov transformations, sign-inversions or a permutation of the variables. It is shown, that two connected, positive integer forms $q$ and $q'$ are $\mathbb{Z}$-equivalent and define identical sets of roots if and only if they are $G$-equivalent. We show that two connected, positive integer forms $q$ and $q'$ are $G$-equivalent and define identical sets of roots if and only if they are $G$-equivalent. Finally we show that if $q$ and $q'$ are $G$-equivalent, then $g(q)$ and $g(q')$ are isomorphic as graded Lie algebras. It is shown that corresponding isomorphism type of algebras is determined by the Gabrilov-equivalence class of integer quadratic form.

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