THE ELEMENTARY THEORY OF THE LATTICE OF EQUATIONAL THEORIES

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The lattice $\mathcal{L}_\Delta$ of all equational theories of signature $\Delta$ has an undecidable elementary theory, according to a theorem of Burris and Sankappanavar from 1975, provided $\Delta$ is large in the sense of providing at least one operation symbol of rank at least two or at least two operation symbols of rank one. On the other hand, Burris also noted in 1971 that the equational theory of $\mathcal{L}_\Delta$ is decidable. We use the work of Jaroslav Ježek in a effort to find the point along the spectrum from the equational theory to the elementary theory where undecidability enters. We provide three additional proofs that $\mathcal{L}_\Delta$ has an undecidable elementary theory. Our sharpest result is that the $\forall^*\exists^*\forall^*$ theory of $\mathcal{L}_\Delta$ is hereditarily undecidable.

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