An easy pigeonhole-principle argument shows that if you have \((n - 1)^2 + 1\) many numbers, then you can choose \(n\) of them such that their sum is divisible by \(n\). A slightly easy Ramsey argument shows that if you have a polynomial \(f(x) \in R[x_1, x_2, \ldots, x_n]\), where \(R\) is a nilpotent ring of size \(r\) and nilpotency class \(k\), then for every \(\bar{a} \in R^n\) there is a \(\bar{b} \in R^n\) such that \(b_i = 0\) or \(b_i = a_i\) and \(b_i = a_i\) for only \(r^{n-k}\) many \(i\)-s (there are \(k\)-many \(r\)-s in the tower).

These bounds can be reduced to \(2n - 1\) and \(k(p - 1) + 1\) using the following generalization of Chevalley's theorem, which is a simple consequence of Alon's Combinatorial Nullstellensatz.

**Lemma.** Let \(A_1, \ldots, A_n\) be subsets of \(F_p\), the \(p\)-element field, and \(f \in F_p[x_1, \ldots, x_n]\) such that

\[
\sum_{i=1}^{n} (|A_i| - 1) > (p - 1) \deg f.
\]

If the set \(\{a \in A_1 \times \cdots \times A_n \mid f(a) = 0\}\) is not empty, then it has at least two different elements.

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