Some new classes of ideals in subtraction algebras

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International conference on algebras and lattices
June 21-25, Prague

June 24, 2010
Seminar List

1. Preliminaries and definitions
2. Primal ideals
3. Weakly prime and weakly primal ideals
4. 2-absorbing and weakly 2-absorbing ideals
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- the composition "\(\circ\)" of functions (and hence \((\Phi, \circ)\) is a function semigroup), and
- the set theoretic subtraction "\(\setminus\)" (and hence is a subtraction algebra in the sense of Abbott (1969)).
Preliminaries and definitions

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By a subtraction algebra we mean an algebra \((X; -)\) with a single binary operation "," that satisfies the following identities: for any \(x, y, z \in X\),
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\[(S1)\]
\[x - (y - x) = x;\]

\[(S2)\]
\[x - (x - y) = y - (y - x)\]

\[(S3)\]
\[(x - y) - z = (x - z) - y.\]

The last identity permits us to omit parentheses in expressions of the form \((x - y) - z\).
The subtraction determines an order relation on $X$:

$$a \leq b \text{ if and only if } a - b = 0$$

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- $a \wedge b = a - (a - b)$;
- The complement of an element $b \in [0, a]$ is $a - b$;
An order in a subtraction algebra

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- $a \wedge b = a - (a - b)$;
- The complement of an element $b \in [0, a]$ is $a - b$;
- If $b, c \in [0, a]$, then
  \[ b \vee c = (b' \wedge c')' = a - ((a - b) \wedge (a - c)) = a - ((a - b) - (((a - b) - (a - c)))). \]
Properties of a subtraction algebra

In a subtraction algebra $X$, the following are true:

- $(p1)$ $(x - y) - y = x - y$.
- $(p2)$ $x - 0 = x$ and $0 - x = 0$.
- $(p3)$ $(x - y) - x = 0$.
- $(p4)$ $x - (x - y) \leq y$.
- $(p5)$ $(x - y) - (y - x) = x - y$.
- $(p6)$ $x - (x - (x - y)) = x - y$.
- $(p7)$ $(x - y) - (z - y) \leq x - z$.
- $(p8)$ $x \leq y$ if and only if $x = y - w$ for some $w \in X$.
- $(p9)$ $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$.
- $(p10)$ $x, y \leq z$ implies that $x - y = x \land (z - y)$.
In a subtraction algebra $X$, the following are true:

(p1) $(x - y) - y = x - y$.
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(p9) $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$.
(p10) $x, y \leq z$ implies that $x - y = x \land (z - y)$.
Definition of an ideal

A nonempty subset $A$ of a subtraction algebra $X$ is called an ideal of $X$ if it satisfies:

(I1) $0 \in A$.

(I2) $y \in A$ and $x - y \in A$ imply $x \in A$ for all $x, y \in A$.

Definition of a prime ideal

Let $X$ be a subtraction algebra. A prime ideal of $X$ is defined to be a proper ideal $P$ of $X$ such that if $x \land y \in P$ then $x \in P$ or $y \in P$. 
**Definition and proposition**

Let $X$ be a subtraction algebra, $A$ an ideal of $X$ and $S$ a nonempty subset of $X$. Set

$$(A :_X S) = \{x \in X | x \land s \in A \text{ for every } s \in S\}$$
Definition and proposition

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Then

- If $S = \{s\}$, then we write $(A : X s)$ instead of $(A : X S)$.
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Then

- If $S = \{ s \}$, then we write $(A :_X s)$ instead of $(A :_X S)$.
- $(A :_X S)$ is an ideal of $X$ and is called the residual of $A$ by $S$. 
Definition and proposition

Let \( X \) be a subtraction algebra, \( A \) an ideal of \( X \) and \( S \) a nonempty subset of \( X \). Set

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\]

Then

- If \( S = \{s\} \), then we write \((A :_X s)\) instead of \((A :_X S)\).
- \((A :_X S)\) is an ideal of \( X \) and is called the residual of \( A \) by \( S \).
- The annihilator of \( S \) in \( X \) is the set \((0 :_X S)\) and we denote it by \( Ann(S) \).
**Definition**

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$. An element $a \in X$ is called prime to $A$ if

$$a \land b \in A \ (b \in X) \Rightarrow b \in A.$$ 

Denote by $S(A)$ the set of all elements of $X$ that are not prime to $A$, so

$$S(A) = \{a \in X | a \land b \in A \text{ for some } b \in X \setminus A\}.$$
Lemma

Let $X$ be a subtraction algebra, $A$ an ideal of $X$ and $S$ a nonempty subset of $X$. Then
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1. $A \subseteq (A :_X S)$. In particular $A \subseteq (A :_X x)$ for every $x \in X$. 

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Some new classes of ideals in subtraction algebras
Lemma

Let $X$ be a subtraction algebra, $A$ an ideal of $X$ and $S$ a nonempty subset of $X$. Then

(1) $A \subseteq (A :_X S)$. In particular $A \subseteq (A :_X x)$ for every $x \in X$.

(2) $x \in X$ is prime to $A$ if and only if $A = (A :_X x)$. 

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Some new classes of ideals in subtraction algebras
**Example 1**

Let $X = \{0, x, y, 1\}$ and define "−" on $X$ by

<table>
<thead>
<tr>
<th>−</th>
<th>0</th>
<th>x</th>
<th>y</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>0</td>
<td>x</td>
<td>0</td>
</tr>
<tr>
<td>y</td>
<td>y</td>
<td>y</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>y</td>
<td>x</td>
<td>0</td>
</tr>
</tbody>
</table>

It is easy to check that $(X; −)$ is a subtraction algebra. Then the operation $\land$ on $X$ is as follows:
Now set $I = \{0, x\}$. Then $I$ is an ideal of $X$. 

\[
\begin{array}{c|cccc}
\wedge & 0 & x & y & 1 \\
\hline
0 & 0 & 0 & 0 & 0 \\
x & 0 & x & 0 & x \\
y & 0 & 0 & y & y \\
1 & 0 & x & y & 1 \\
\end{array}
\]
Now set $I = \{0, x\}$. Then $I$ is an ideal of $X$.

- The element $x$ is not prime to $I$ since $y \in X \setminus I$ with $x \land y = 0 \in I$. 

\[
\begin{array}{c|cccccc}
\land & 0 & x & y & 1 \\
\hline
0 & 0 & 0 & 0 & 0 \\
x & 0 & x & 0 & x \\
y & 0 & 0 & y & y \\
1 & 0 & x & y & 1
\end{array}
\]
Now set $I = \{0, x\}$. Then $I$ is an ideal of $X$.

- The element $x$ is not prime to $I$ since $y \in X \setminus I$ with $x \land y = 0 \in I$.
- Also $y$ is prime to $I$, for if $z \in X$ is such that $y \land z \in I$, then $y \land z = 0$. Thus either $z = 0$ or $z = x$ both lie in $X$. 

\[
\begin{array}{c|cccc}
\land & 0 & x & y & 1 \\
0 & 0 & 0 & 0 & 0 \\
x & 0 & x & 0 & x \\
y & 0 & 0 & y & y \\
1 & 0 & 0 & y & 1 \\
\end{array}
\]
Lemma

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$. If $S(A)$ is a proper ideal of $X$, then $S(A)$ is a prime ideal of $X$. 
Definition of a primal ideal

Definition

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$.

- $A$ is said to be a primal ideal of $X$ provided that $S(A)$ forms an ideal of $X$. If $S(A)$ is a proper ideal of $X$, then it is a prime ideal of $X$, called the adjoint prime ideal $P$ of $A$. In this case we also say that $A$ is a $P$-primal ideal of $X$. 

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- $X$ is called a coprimal subtraction algebra provided that the zero ideal of $X$ is primal.
An example of a primal ideal

Example 2

Let \( X = \{0, 1, 2, 3, 4, 5\} \) and define "−" on \( X \) by

\[
\begin{array}{ccccccc}
\text{−} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 3 & 4 & 3 & 1 \\
2 & 2 & 5 & 0 & 2 & 5 & 4 \\
3 & 3 & 0 & 3 & 0 & 3 & 3 \\
4 & 4 & 0 & 0 & 4 & 0 & 4 \\
5 & 5 & 5 & 0 & 5 & 5 & 0 \\
\end{array}
\]
Then \((X; -)\) is a subtraction algebra. The operation \(\wedge\) on \(X\) is as follows:

\[
\begin{array}{cccccc}
\wedge & 0 & 1 & 2 & 3 & 4 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 4 & 3 & 4 & 0 \\
2 & 0 & 4 & 2 & 0 & 4 & 5 \\
3 & 0 & 3 & 0 & 3 & 0 & 0 \\
4 & 0 & 4 & 4 & 0 & 4 & 0 \\
5 & 0 & 0 & 5 & 0 & 0 & 5 \\
\end{array}
\]
Set $A = \{0, 4\}$. Then $A$ is an ideal of $X$ and:
- $S(A) = X$. So $A$ is a primal ideal of $X$. 
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- $S(A) = X$. So $A$ is a primal ideal of $X$.
- $A$ is not a prime ideal of $X$ since $3 \land 2 = 0 \in A$ but neither $3$ nor $2$ belong to $A$. Therefore a primal ideal of $X$ need not be primal. We will prove in a theorem that every prime ideal of $X$ is primal.
Set \( A = \{0, 4\} \). Then \( A \) is an ideal of \( X \) and:

- \( S(A) = X \). So \( A \) is a primal ideal of \( X \).
- \( A \) is not a prime ideal of \( X \) since \( 3 \land 2 = 0 \in A \) but neither 3 nor 2 belong to \( A \). Therefore a primal ideal of \( X \) need not be primal. We will prove in a theorem that every prime ideal of \( X \) is primal.
- By (1), for an ideal \( A \) of a subtraction algebra \( X \), \( S(A) \) need not be a proper ideal of \( X \).
Lemma

Let $X$ be a subtraction algebra and $A$ an ideal of $X$.
(1) If $A$ is proper, then $A \subseteq S(A)$.
(2) If $A$ is a $P$-primal ideal of $X$, then $A \subseteq P$. 
**Theorem**

Let $X$ be a subtraction algebra. Then every prime ideal of $X$ is primal.
Definition of a zero-divisor

**Definition**

Let $X$ be a subtraction algebra. An element $a \in X$ is called a zero-divisor of $X$ provided that $a \wedge b = 0$ for some nonzero element $b \in X$.

**Is $Z(X)$ an ideal of $X$?**

Let $X = \{0, x, y, 1\}$ and assume that ”$-$” is defined on $X$ as in Example 1. Then:

- $Z(X) = \{0, x, y\}$. 
Definition of a zero-divisor

**Definition**

Let $X$ be a subtraction algebra. An element $a \in X$ is called a zero-divisor of $X$ provided that $a \land b = 0$ for some nonzero element $b \in X$.

**Is $Z(X)$ an ideal of $X$?**

Let $X = \{0, x, y, 1\}$ and assume that " $-$ " is defined on $X$ as in Example 1. Then:

- $Z(X) = \{0, x, y\}$.
- Since $1 - x = y \in X$, $x \in X$ but $1 \notin X$, so $Z(X)$ is not an ideal of $X$. This example shows that, for a subtraction algebra $X$, $Z(X)$ need not necessarily be an ideal of $X$. 
Determining the coprimality via $Z(X)$

**Theorem**

Let $X$ be a subtraction algebra. Then $X$ is coprimal if and only if $Z(X)$ is an ideal of $X$. 
Weakly prime ideals

**Definition**

Let $X$ be a subtraction algebra. An ideal $P$ of $X$ is said to be a weakly prime ideal of $X$ if whenever $0 \neq x \land y \in P$ then either $x \in P$ or $y \in P$. 

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Some new classes of ideals in subtraction algebras
Prime $\Rightarrow$ weakly prime but not conversely

**Example**

Let $X$ be a subtraction algebra.
(1) Every prime ideal of $X$ is weakly prime.
(2) Let $X = \{0, a, b, c, d\}$ be a set with the following Cayley table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>a</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>b</td>
<td>0</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>b</td>
<td>a</td>
<td>0</td>
<td>c</td>
</tr>
<tr>
<td>d</td>
<td>d</td>
<td>d</td>
<td>d</td>
<td>d</td>
<td>0</td>
</tr>
</tbody>
</table>

Then $(X; -)$ is a subtraction algebra. The operation $\land$ on $X$ is as follows:
Set $P = \{0, b\}$. Then
Set $P = \{0, b\}$. Then

- $P$ is a weakly prime ideal of $X$ since if $0 \neq x \land y \in P$, then $x \land y = b$. One can check that in any cases either $x = b$ or $y = b$, that is either $x \in P$ or $y \in P$. 

\[
\begin{array}{cccccc}
- & 0 & a & b & c & d \\
0 & 0 & 0 & 0 & 0 & 0 \\
a & 0 & a & 0 & a & 0 \\
b & 0 & 0 & b & b & 0 \\
c & 0 & a & b & c & 0 \\
d & 0 & 0 & 0 & 0 & d \\
\end{array}
\]
Set $P = \{0, b\}$. Then

- $P$ is a weakly prime ideal of $X$ since if $0 \neq x \land y \in P$, then $x \land y = b$. One can check that in any cases either $x = b$ or $y = b$, that is either $x \in P$ or $y \in P$.

- $c \land d = 0 \in P$ while $c \notin P$ and $d \notin P$. Therefore $P$ is not a prime ideal of $X$.

This example shows that a weakly prime ideal of $X$ need not necessarily be prime.
A characterization for weakly prime ideals

Theorem

Let $P$ be a proper ideal of a subtraction algebra $X$. Then the following are equivalent:

(i) $P$ is weakly prime.

(ii) For every pair of ideals $A$ and $B$ of $X$, $0 \neq A \land B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. 
Definition

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$. An element $a \in X$ is called weakly prime to $A$ if $0 \neq a \land b \in A$ ($b \in X$) implies that $b \in A$. We denote by $w(A)$ the set of all elements of $X$ that are not weakly prime to $A$. 
Let $A$ be a proper ideal of a subtraction algebra $X$. 

Remark
**Remark**

Let $A$ be a proper ideal of a subtraction algebra $X$.

- $0$ is always weakly prime to $A$, so $0 \notin w(A)$. 

Remark

Let $A$ be a proper ideal of a subtraction algebra $X$.

- 0 is always weakly prime to $A$, so $0 \notin w(A)$.
- If $a \in X$ is prime to $A$, then $a$ is weakly prime to $A$. Consequently $w(A) \subseteq S(A)$. 


**Remark**

Let $A$ be a proper ideal of a subtraction algebra $X$.

- $0$ is always weakly prime to $A$, so $0 \notin w(A)$.
- If $a \in X$ is prime to $A$, then $a$ is weakly prime to $A$. Consequently $w(A) \subseteq S(A)$.
- $w(0) = \emptyset$ where $0$ is the zero ideal of $X$. 
Lemma

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$. If $P := w(A) \cup \{0\}$ is an ideal of $X$, then $P$ is a weakly prime ideal of $X$. 

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Some new classes of ideals in subtraction algebras
Definition of weakly primal ideals

**Definition**

Let $X$ be a subtraction algebra and let $A$ be an ideal of $X$. $A$ is said to be a weakly primal ideal of $X$ provided that $P := w(A) \cup \{0\}$ forms an ideal of $X$; this ideal is always a weakly prime ideal, called the weakly adjoint ideal $P$ of $A$. In this case we also say that $A$ is a $P$-weakly primal ideal of $X$. 
In this example we show that the concepts ”primal ideal” and ”weakly primal ideal” are different concepts. Indeed we show that neither imply the other. Let $X = \{0, 1, 2, 3, 4, 5\}$ and define ”–” on $X$ as in the Example2.
In this example we show that the concepts ”primal ideal” and ”weakly primal ideal” are different concepts. Indeed we show that neither imply the other. Let \( X = \{0, 1, 2, 3, 4, 5\} \) and define ” − ” on \( X \) as in the Example2.

**Example (Primal \( \not\Rightarrow \) weakly primal)**

Set \( A = \{0, 4\} \). Then, by Example2, \( A \) is a primal ideal of \( X \). It is easy to see that \( w(A) = \{1, 2, 4\} \). Set \( P = w(A) \cup \{0\} = \{0, 1, 2, 4\} \). Since \( 1 \in P \), \( 3 - 1 = 0 \in P \) and \( 3 \not\in P \), \( P \) is not an ideal of \( X \). So \( A \) is not a weakly primal ideal of \( X \). This example shows that a primal ideal need not be weakly primal.
**Example (Weakly primal $\nRightarrow$ primal)**

Now set $B = \{0, 3\}$. Then $B$ is an ideal of $X$. Also $S(B) = \{0, 1, 3, 4, 5\}$. Since $1 \in S(B)$, $2 - 1 = 5 \in S(B)$ and $2 \notin S(B)$, $S(B)$ is not an ideal of $X$. So $B$ is not a primal ideal of $X$. Moreover $w(B) = \{3\}$. Hence $w(B) \cup \{0\} = B$. So $B$ is a weakly primal ideal of $X$. This example shows that a weakly primal ideal of $X$ need not be primal.
Definition

A proper ideal $A$ of a subtraction algebra $X$ is said to be a 2-absorbing (resp. weakly 2-absorbing) ideal if whenever $a, b, c \in X$ with $a \land b \land c \in A$, (resp. $0 \neq a \land b \land c \in A$) then $a \land b \in A$ or $a \land c \in A$ or $b \land c \in A$. 
We can generalize the concept of 2-absorbing ideals in a subtraction algebra $X$ to the concept of $(n, m)$-absorbing ideals. Suppose that $m, n$ are two positive integers with $n > m$. We say that an ideal $A$ of $X$ is a $(n, m)$-absorbing ideal if whenever $a_1, a_2, ..., a_n \in X$ and $a_1 \land a_2 \land ... \land a_n \in A$, then there are $m$ of $a_i$’s whose meet lies in $X$. The concept of weakly $(m, n)$-absorbing ideals is defined in a similar way.
Proposition

Let $X$ be a subtraction algebra and assume that $A$ is an ideal of $X$. Then

- Every 2-absorbing ideal of $X$ is weakly 2-absorbing.
- Every prime ideal of $X$ is 2-absorbing.
- Every weakly prime ideal of $X$ is weakly 2-absorbing.
- $A$ is $\left(n, m\right)$-absorbing if and only if it is $\left(m + 1, m\right)$-absorbing.
- If $A$ is $\left(n, m\right)$-absorbing, then it is $\left(n, k\right)$-absorbing for every positive integer $k > n$.
- $A$ is a prime ideal if and only if it is a $\left(2, 1\right)$-absorbing ideal.
- $A$ is a 2-absorbing ideal if and only if it is a $\left(3, 2\right)$-absorbing ideal.
Proposition

Let $X$ be a subtraction algebra and assume that $A$ is an ideal of $X$. Then

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- Every 2-absorbing ideal of $X$ is weakly 2-absorbing.
- Every prime ideal of $X$ is 2-absorbing.
- Every weakly prime ideal of $X$ is weakly 2-absorbing.
- $A$ is $(n, m)$-absorbing if and only if it is $(m + 1, m)$-absorbing.
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- Every 2-absorbing ideal of $X$ is weakly 2-absorbing.
- Every prime ideal of $X$ is 2-absorbing.
- Every weakly prime ideal of $X$ is weakly 2-absorbing.
- $A$ is $(n, m)$-absorbing if and only if it is $(m + 1, m)$-absorbing.
- If $A$ is $(n, m)$-absorbing, then it is $(n, k)$-absorbing for every positive integer $k > n$. 

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**Proposition**

Let $X$ be a subtraction algebra and assume that $A$ is an ideal of $X$. Then

- Every 2-absorbing ideal of $X$ is weakly 2-absorbing.
- Every prime ideal of $X$ is 2-absorbing.
- Every weakly prime ideal of $X$ is weakly 2-absorbing.
- $A$ is $(n, m)$-absorbing if and only if it is $(m + 1, m)$-absorbing.
- If $A$ is $(n, m)$-absorbing, then it is $(n, k)$-absorbing for every positive integer $k > n$.
- $A$ is a prime ideal if and only if it is a $(2, 1)$-absorbing ideal.
Proposition

Let $X$ be a subtraction algebra and assume that $A$ is an ideal of $X$. Then

- Every $2$-absorbing ideal of $X$ is weakly $2$-absorbing.
- Every prime ideal of $X$ is $2$-absorbing.
- Every weakly prime ideal of $X$ is weakly $2$-absorbing.
- $A$ is $(n, m)$-absorbing if and only if it is $(m + 1, m)$-absorbing.
- If $A$ is $(n, m)$-absorbing, then it is $(n, k)$-absorbing for every positive integer $k > n$.
- $A$ is a prime ideal if and only if it is a $(2, 1)$-absorbing ideal.
- $A$ is a $2$-absorbing ideal if and only if it is a $(3, 2)$-absorbing ideal.
Examples of 2-absorbing and weakly 2-absorbing ideals

**Theorem**

Let $X$ be a subtraction algebra.

- If $P_1$ and $P_2$ are distinct prime ideals of $X$, then $P_1 \cap P_2$ is a 2-absorbing ideal of $X$. 

- If $P_1$ and $P_2$ are distinct weakly prime ideals of $X$, then $P_1 \cap P_2$ is a weakly 2-absorbing ideal of $X$. 

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A. Yousefian Darani

Some new classes of ideals in subtraction algebras
Examples of $2$-absorbing and weakly $2$-absorbing ideals

**Theorem**

Let $X$ be a subtraction algebra.

- If $P_1$ and $P_2$ are distinct prime ideals of $X$, then $P_1 \cap P_2$ is a $2$-absorbing ideal of $X$.

- If $P_1$ and $P_2$ are distinct weakly prime ideals of $X$, then $P_1 \cap P_2$ is a weakly $2$-absorbing ideal of $X$. 
Thank you for your attention.