The character and the pseudo-character in endomorphism rings

Horea Abrudan

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Let A be an Abelian group and End(A) the ring of endomorphisms viewed as a topological ring with the topology of pointwise convergence (finite topology). We estimate the character of End(A) for a torsion group. A characterization of torsion group A for which the pseudo-character is $\leq m$, where m is a cardinal number, is given.

Quasialgebras, Lattices and Matrices

Helena Albuquerque

coauthored by Rolf Soeren Kraushnar

Departamento de Matemtica- Universidade de Coimbra- Portugal

In this talk we will present the notion of quasialgebra and we will give some examples like the octonions. We will study deformed group algebras as Cayley algebras or Clifford algebras and in this context the structure of a lattice defined in the space is induced naturally.

On the term functions and fundamental relation of a fuzzy hyperalgebra

Reza Ameri

coauthored by Tahere Nozari

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In this paper, we start from this idea that the set of nonzero fuzzy subsets of a fuzzy hyperalgebra can be organized naturally as a universal algebra, and constructing the term functions over this algebra we can deduce some results on fuzzy hyperalgebras from some of the very known properties of the term functions of a universal algebra. Also, we present the form of the generated subfuzzy hyperalgebra in two theorem, which in the particular cases of the universal algebras and multialgebras are already known. In the last section we will present the form of the fundamental relation of a fuzzy hyperalgebra, i.e. the smallest equivalence relation for which the quotient set, considered as a hyperalgebra, is a universal algebra.

Enveloping algebra for simple Malcev algebras

Elisabete Barreiro

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In this talk we will present some results on the enveloping algebra for simple Malcev algebras. We present the classical theory of enveloping algebra for Lie algebras, generalizing this theory for the Malcev case. In this communication we discuss again the known question of how one can embed a Malcev algebra in an alternative algebra, so that the product defined in the first algebra is naturally induced by the commutator defined in the second. We will make reference to some published results that address this question.

Counting CSPs: Exact, Weighted, Approximate

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We give a brief survey of recent developments in the study of the counting CSP. Then we discuss how counting CSPs that allow different degrees of approximation are different from those allowing exact counting, and investigate possibilities of an algebraic theory for CSPs allowing approximation.

Proper Classes associated to Grothendieck Categories

Fernando Cornejo-Montaño

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We study some relations between prerradicals and proper classes, and we analyze those proper classes injectively or coinjectively generated.

On Stone Duality for Skew Boolean Algebras

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The Stone duality states that the categories of [generalized] Boolean algebras with morphisms, and [locally-compact] Boolean spaces with proper continuous maps are equivalent. Skew Boolean algebras present a non-commutative generalization of Boolean algebras. In the talk we shall describe a generalization of Stone's duality to a category of skew Boolean algebras satisfying certain additional constraints.

The Jordan-Hölder theorem with uniqueness for groups and semimodular lattices

Gábor Czédli

coauthored by E. Tamás Schmidt

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For subnormal subgroups $A \triangleleft B$ and $C \triangleleft D$ of a given group G, the factor B/A will be called *subnormally down-and-up projective* to D/C, if there are subnormal subgroups $X \triangleleft Y$ such that $AY = B$, $A \cap Y = X$, $CY = D$ and $C \cap Y = X$. Clearly, $B/A \cong D/C$ in this case. As G. Grätzer and J. B. Nation have just pointed out, the standard proof of the classical Jordan-Hölder theorem yields somewhat more than widely known; namely, the factors of any two given composition series are the same up to subnormal down-and-up projectivity and a permutation. We prove the *uniqueness* of this permutation.

The main result is the analogous statement for *semimodular lattices*. Most of the proof belongs to pure lattice theory; the group theoretical part is only a simple reference to a classical theorem of H. Wielandt.

New classes of ideals in subtraction algebras

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In [4] B. M. Schein considered systems of the form $(\Phi; o, \backslash)$,where Φ is a set of functions closed under the composition "o" of functions (and hence (Φ, o) is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B. Zelinka [5] discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. A particular attention was paid to the ideals of a subtraction algebra (cf. [2, 3] and papers cited there).

Prime ideals play a central role in the theory of subtraction algebras. Of course, a prime ideal P of a subtraction algebra X is a proper ideal of X with the property that for $a, b \in X$, $a \wedge b \in P$ implies $a \in P$ or $b \in P$. In this paper we give some generalizations of prime ideals: *weakly prime ideals and* 2*-absorbing ideals*. We introduce also new classes of ideals of X: *Primal ideals and weakly primal ideals*. Then we give some basic results about these classes of ideals.

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Amalgamation for Quasi-Stone Algebras

Sara-Kaja Fischer

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Quasi-Stone Algebras were introduced by N. and H. Sankappanavar in 1993 as a generalisation of Stone algebras. They were further investigated by H. Gaitán in 2000 by considering them as special cases of Q-distributive lattices and establishing a duality based on Priestley duality for distributive lattices. By means of this duality, we can show that the variety of Quasi-Stone algebras and most of its subvarieties do not have the amalgamation property and classify the amalgamation bases within these subvarieties.

Algorithms in Universal Algebra and the UACalculator

Ralph Freese

University of Hawaii

This talk will be about the algorithms employed by the latest version of the UACalculator (http://uacalc.org/).

A Class of Rings for which the Lattice of Preradicals is not a Set

Silvia Gavito

coauthored by Henry Chimal-Dzul and Rogelio Fernández-Alonso

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In this paper we define Z-coinitial rings, where Z is an integral domain. We define radical modules and radical rings, and we prove that every countable Z-coinitial and left hereditary ring is a left radical ring. Finally, we prove that the lattice of preradicals of every left radical ring is not a set.

On lattices with a compact top congruence

Pierre Gillibert

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We denote by $\mathrm{Con_c} L$ the $(\vee, 0)$-semilattice of all *compact* (finitely generated) congruences of a lattice L. Given a variety of lattices $\mathcal{V}$ we denote $\mathcal{V}^{\mathrm{b}}$ the class of all lattices $L \in \mathcal{V}$ such that $\mathrm{Con_c} L$ is bounded. We denote $\mathcal{V}^{0,1}$ the class of all bounded lattices in $\mathcal{V}$.

The containment $\mathcal{V}^{0,1} \subseteq \mathcal{V}^{\mathrm{b}}$ holds, but it is not an equality in general, for example $\mathcal{M}_3^{0,1} \subsetneq \mathcal{M}_3^{\mathrm{b}}$. We study the following two assertions.

(Q1) For all $K \in \mathcal{V}^{\mathrm{b}}$ there is $L \in \mathcal{V}^{0,1}$ such that $\mathrm{Con_c} K \cong \mathrm{Con_c} L$.

(Q2) Every $K \in \mathcal{V}^{\mathrm{b}}$ has a congruence-preserving extension in $\mathcal{V}^{0,1}$.

Given a finitely generated variety of lattices $\mathcal{V}$, we prove that (Q2) holds only in the trivial case, that is $\mathcal{V}^{0,1} = \mathcal{V}^{\mathrm{b}}$. However (Q1) holds for the variety $\mathcal{M}_3$ and the construction can be made functorial.

Algebraic properties of the Beltrami equation

Dmitry Goldstein

coauthored by Eduard Yakubov, Uri Srebro

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The Beltrami equation has a long and a dramatic history. This equation plays a crucial role in many fields of analysis, geometry, complex dynamics, string theory, mechanics, elasticity and robotics. We study algebraic properties of solutions of the Beltrami equations. Our main result asserts that the set of open and discrete solutions of the Beltrami equations forms an algebra and, in some cases, a lattice. We find the dependence between the algebraic properties of the solutions and the Beltrami coefficient.

The lattice of strongly permutative semigroups

Mariusz Grech

University of Wrocaw

Strongly permutative semigroups are semigroups satisfying a permutation identity

$$x_1 \cdots x_n = x_{\sigma(1)} \cdots x_{\sigma(n)}$$

with $\sigma(1) > 1$ and $\sigma(n) < n$.

We show that each equational theory of strongly permutative semigroups is described by five objects: an order filter, an equivalence relation, and three integers. This extend Kisielewicz's description of the lattice of equational theory of commutative semigroups.

In addition, we show, how this description can be used to prove that every equational theory of strongly permutative semigroups is first-order definable, up to duality.

On the equational theory of projection lattices of finite von-Neumann factors

Christian Herrmann

TU Darmstadt

For a finite von-Neumann algebra factor $\mathbf{M}$, the projections form a modular ortholattice $L(\mathbf{M})$. We show that the equational theory of $L(\mathbf{M})$ coincides with that of some resp. all $L(\mathbb{C}^{n\times n})$ and is decidable. In contrast, the uniform word problem for the variety generated by all $L(\mathbb{C}^{n\times n})$ is shown to be undecidable. This is a report on joint work with Luca Giudici. A few remarks, based on joint work with Martin Ziegler, on the complexity of the decidable problem will be added.

The complexity of the equivalence problem for commutative rings

Gábor Horváth

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The equivalence problem over finite rings asks whether or not a polynomial over a given finite ring is identically 0, i.e. whether or not the polynomial attains value 0 for every substitutions from the ring. We investigate the complexity of this problem.

The equivalence problem has two versions for finite rings depending on the form of the input polynomial. In the original version the input polynomial can have any form, and in the *sigma* version the input polynomial is written as a sum of monomials. Lawrence and Willard conjectured that for a finite ring R with Jacobson radical J the complexity of the sigma equivalence problem is in P if R/J is commutative, and coNP-complete otherwise. We confirm this conjecture for commutative rings.

Krull dimension of power series rings

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We show that the Krull dimension of the power series ring V[[X]] over a rank-one nondiscrete valuation domain V is uncountable, in fact, continuum.

Serre's problem and p-Schreier varieties of semimodules

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coauthored by S. N. Yl'in

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As is well known, *Schreier varieties* of algebras are varieties in which any subalgebra of any free algebra is free. In the same time, in homological algebra and algebraic geometry, *projective algebras* — algebras which are retracts of free algebras — play a very special and important role. Combining these concepts, one naturally comes up with the concept of a *p-Schreier variety* — a variety whose all projective algebras are free. Perhaps the most well-known results concerning p-Schreier varieties are the following: the category of modules over a local ring is a p-Schreier variety (Kaplansky); and a confirmation of Serre's famous conjecture that the category of modules over a polynomial ring $R[x_1, x_2, , x_n]$ over a field R is a p-Schreier variety, independently proved by D. Quillen and A. Suslin in 1976.

In this talk, we consider categories of all semimodules over semirings which are p-Schreier varieties; and, among other results, we first single out the following important for further research ones of [1].

Theorem 1. *Over division semirings, the categories of semimodules are Schreier varieties iff the division semirings are rings.*

As usual, a commutative monoid $(S, +, 0)$ is called π-*regular* (or an *epigroup*) if every its element has a power in some subgroup of S; and a *proper* semiring is a semiring that is not a ring. The next result, being of interest in its own rights as well as for tropical algebraic geometry, also solves Problem 1 of [2].

Theorem 2. *Over additively π-regular proper semirings, the categories of semimodules are not p-Schreier varieties.*

In contrast to the previous theorem, we have the following promising result.

Theorem 3. *Over additively cancellative division semirings, the categories of semimodules constitute p-Schreier varieties.*

Finally, we conclude the talk by discussing some consequences of these results for the Serre's problem in the context of polynomial semirings and presenting some open problems and directions for further investigations.

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Maltsev digraphs have a majority polymorphism

Alexandr Kazda

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We prove that when a digraph G has a Maltsev polymorphism, then G also has a majority polymorphism. We consider the consequences of this result for the structure of Maltsev digraphs and the complexity of the Constraint Satisfaction Problem.

Nilpotent semigroups of partial automorphisms of rooted trees

Eugenia Kochubinska

Kiev National Taras Shevchenko University

A semigroup S is called nilpotent if for some integer k and elements $a_1, a_2, \ldots, a_k$ $a_1 \cdot a_2 \cdot \ldots a_k = 0$, where 0 is a zero element of a semigroup S. However, if T is a nilpotent subsemigroup of a semigroup S it means that T contains a zero element, say e, but it does not mean that this element e coincides with a zero element of S. We will call such a subsemigroup a proper nilpotent subsemigroup.

We consider a semigroup of partial automorphisms of rooted trees. For this semigroup we give a description of its maximal nilpotent subsemigroups, its proper nilpotent subsemigroups and also get some combinatorial results concerning these types of subsemigroups.

Additively divisible commutative semirings

Miroslav Korbelář

coauthored by Tomáš Kepka

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A (commutative) *semiring* is an algebraic structure with two commutative and associative binary operations (an addition and a multiplication) such that the multiplication distributes over the addition.

We present a conjecture concerning finitely generated additively divisible commutative semirings, show the connection to the related problems and provide results for this hypothesis in particular cases.

Categories of algebras based on distributive (0,1)-lattices

V. Koubek

coauthored by J. Sichler

Charles University in Prague

A category $\mathcal{K}$ is $a$lg-$u$niversal if any category of algebras has a full embedding into $\mathcal{K}$. This is equivalent to the existence of a full embedding $\Phi : \mathbb{G}@ >>> \mathcal{K}$ of the category $\mathbb{G}$ of undirected graphs and their compatible mappings into $\mathcal{K}$. If $\mathcal{K}$ is a concrete category and the $\mathcal{K}$-object ΦG has finite underlying set for every finite graph G, we say that $\mathcal{K}$ is $f$inite-to-finite alg-universal. There are weaker forms of universality. For instance, a concrete category $\mathcal{K}$ need not be universal, but augmenting each $\mathcal{K}$-object by $k \geq 1$ constants and requiring that each of these constants be preserved gives rise to a proper subcategory of $\mathcal{K}$ called the *k-expansion* and denoted as $k\mathcal{K}$ that may well be alg-universal. A stronger version of weak universality is based on the concept of an ideal $\mathcal{J}$ of a category $\mathcal{K}$. A class $\mathcal{J}$ of $\mathcal{K}$-morphisms is an *ideal* of $\mathcal{K}$ if $f \circ g \in \mathcal{J}$ whenever $f \in \mathcal{J}$ or $g \in \mathcal{J}$. And $\mathcal{K}$ is *$\mathcal{J}$-relatively alg-universal* if there is a faithful functor $\Phi : \mathbb{G}@ >>> \mathcal{K}$ such that $\Phi m \notin \mathcal{J}$ for every $\mathbb{G}$-morphism m and every $\mathcal{K}$-morphism $k : \Phi G@ >>> \Phi G'$ outside of $\mathcal{J}$ has the form $k = \Phi m$ for some $m \in \mathbb{G}(G, G')$. The variety $\mathbb{D}$ of distributive $(0,1)$-lattices and the category $\mathbb{P}$ of all Priestley spaces dual to $\mathbb{D}$ are far from alg-universal, while their respective expansions $2\mathbb{D}$ and $2\mathbb{P}$ are. The main result used here is that

(R): the expansions $1\mathbb{D}$ and $1\mathbb{P}$ are $\mathcal{J}$-relatively alg-universal for the ideal $\mathcal{J}$ formed by all morphisms whose image is finite.

When combined with earlier results, (R) implies this

Theorem: For every non-regular variety $\mathbb{V}$ of distributive double p-algebras there are integers $m \leq 5$ and $n \leq 6$ such that its m-expansion $m\mathbb{V}$ is alg-universal and its n-expansion $n\mathbb{V}$ is finite-to-finite alg-universal. If $\mathbb{V}$ is a finitely generated regular variety of distributive double p-algebras then for no cardinal α the α-expansion $\alpha\mathbb{V}$ is alg-universal.

A more complete categorical classification of such varieties will be discussed in the talk.

Second look at cyclic terms

Marcin Kozik

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A term t is *cyclic* if it satisfies the following identity

$$t(x_1, x_2, \ldots, x_n) \approx t(x_2, \ldots, x_n, x_1).$$

During the talk I will present the improved (and TCT-free) proof of the cyclic theorem: A finite algebra generates a variety with a Taylor term, if and only if, it has a cyclic term.

NP vs. coNP and algebraic proof systems

Jan Krajíček

Charles University in Prague, Mathematics and Physics Faculty

The computational complexity class NP is closed under complementation iff there exist a proof system for propositional logic in which every tautology admits a polynomial size proof. It is conjectured that no such proof system exists. To establish this conjecture is one of fundamental problems of mathematical logic and complexity theory.

One may shift the whole set up to various algebraic settings. This lead to topics like the efficiency of proof systems for ideal membership in polynomial rings over finite fields or for integer linear inequalities, to the growth of Dehn function for some finitely presented groups, or to the existence of an abstract Euler characteristic on a first-order structure.

I shall outline some of these topics, and describe some known results and open problems.

On non-negative integer quadratic forms

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The use of quadratic forms as a tool for characterizing classes of finite dimensional algebras and Lie algebras is well known and widely accepted.

We prove that each $\mathbb{Z}$-equivalence class of integer quadratic forms contains a disjoint union of multiplied unit Dynkin diagrams and some form related to radical, and each Gabrielov-equivalence class of integer quadratic forms contains a disjoint union of multiplied Dynkin diagrams and some form related to radical. For an integer quadratic form a non-negativity criterion is given.

A square matrix with integer coefficients A is called a *quasi-Cartan matrix* if it is symmetrizable (there exists a diagonal matrix D with positive diagonal entries such that DA is symmetric) and $A_{ii} = 2$ for all i. A quasi-Cartan matrix is called *Cartan matrix* if it is positive definite and $A_{ij} \leq 0$ for all $i \neq j$. To any integer form we associate a Lie algebra in generators and relations in terms of the positive quasi-Cartan matrix. Quasi-Cartan matrix A_q of quadratic form q is Cartan matrix iff form q is positive definite and classic.

Each Cartan matrix determines a unique semisimple complex Lie algebra via the Chevalley-Serre, sometimes called simply the Serre relations. We develop the classical Serre relations for quasi-Cartan matrix. Serre proved that if q is positive definite and classic integer form then $\mathfrak{g}(q)$ is a semisimple (and finite dimensional) Lie algebra.

Two forms q and q' are called *G-equivalent* if one comes from another after a sequence of Gabrielov transformations, sign-inversions or a permutation of the variables. It is shown, that two connected, positive integer forms q and q' are $\mathbb{Z}$-equivalent and define identical sets of roots if and only if they are G-equivalent. We show that two connected, positive integer forms q and q' are $\mathbb{Z}$-equivalent and define identical sets of roots if and only if they are G-equivalent. Finally we show that if q and q' are G-equivalent, then $\mathfrak{g}(q)$ and $\mathfrak{g}(q')$ are isomorphic as graded Lie algebras. It is shown that corresponding isomorphism type of algebras is determined by the Gabrielov-equivalence class of integer quadratic form.

Monoids of languages, monoids of reflexive relations and ordered monoids

Ganna Kudryavtseva

We discuss the connection between the classes of semigroups of positive languages, upper-triangular reflexive relations over posets and semigroups of order-preserving and extensive transformations of sets. From our constructions we derive some interesting consequences for the finite case. For example, we prove that the class of finite semigroups of languages coincides with the class of semigroups of reflexive upper-triangular binary relations on finite sets.

Self-commuting lattice polynomial functions

Erkko Lehtonen
coauthored by Miguel Couceiro
University of Luxembourg

We provide sufficient conditions for a lattice polynomial function to be self-commuting. We explicitly describe self-commuting polynomial functions over chains.

Classifying 2-nilpotent Lie algebras and graphs: Equivalent wild problems

R. Lipyanski
coauthored by N. Vanetik
Department of Mathematics, Ben Gurion University of the Negev

Reduction of graph isomorphism problem to the problem of isomorphism of various algebraic structures is an important sub-area of combinatorics. Some reductions are well-known, such as reducing graphs to associative algebras or reducing graphs to (possibly infinite) groups in works of Makar-Limanov, Kim, Neggers and Roush, K. Droms and commutative rings (work of Kayal and Saxena).

We take the next logical step and reduce the graph isomorphism problem to the problem of isomorphism of finite 2-nilpotent p-groups. This reduction is not straightforward and is constructed using 2-nilpotent Lie algebras over the ring Z_{p^3} and anticommutative rings. Moreover, a simple reduction from groups back to graphs shows that the isomorphism problems for graphs, finite 2-nilpotent groups and 2-nilpotent Lie algebras are polynomially equivalent.

A wild classification problem contains (in some sense) a problem of classification of pairs of matrices up to simultaneous similarity. There exist different embeddings the W-problem into the problem classification up to isomorphism of various algebraic structures. We use a well-known result of V. Sergeichuk regarding the wildness of classification of 2-nilpotent p-groups to show that classification problems for graphs and 2-nilpotent Lie algebras are wild.

Invertibility of multiplication modules

Majid M. Ali

Department of Mathematics, Sultan Qaboos University

Invertibility of multiplication modules All rings are commutative with 1 and all modules are unital. Let R be a ring and M an R-module. M is called multiplication if for each submodule N of M, N=IM for some ideal I of R. Multiplication modules have recently received considerable attention during the last twenty years. In this talk we give the definition of invertible submodules as a natural generalization of invertible ideals, then we introduce the concept of Dedekind modules and Prufer modules. An R-module M is Dedekind (resp. Prufer) if every non-zero (resp. non-zero finitely generated) submodule of M is invertible. We introduce and investigate the concepts of generalized multiplication Dedekind modules and almost multiplication Dedekind modules. We also give some properties of nonfinitely generated submodules of faithful multiplication valuation modules and finally we characterize faithful multiplication modules via m-canonical submodules.

A personal history of tournaments represented as groupoids

Petar Marković

coauthored by Jaroslav Ježek, Miklós Maróti and Ralph McKenzie

Department of Mathematics and Informatics, University of Novi Sad

This talk will present the problems the authors collaborated on for a period of four years around the turn of the century and give the current state of affairs in the topic.

Tournaments are directed graphs where for any pair of distinct vertices exactly one possible directed edge is in the graph (edge goes from the 'winner' of the 'match' between the two into the 'loser'). They have an obvious representation as idempotent groupoids, namely that the operation is always equal to the winner. The universal class of groupoids defined so generates a variety which has fairly nice properties, but one where proofs of these properties happen to be unexpectedly difficult. Several results and an open problem will be presented.

Minimal quasivarieties of semilattices with a group of automorphisms

Miklós Maróti

coauthored by Ildikó Nagy

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An **F**-semilattice is a semilattice expanded by a group **F** of automorphisms acting as new unary basic operations. For a fixed group **F**, the class of **F**-semilattices forms a variety. The subdirectly irreducible and simple members of this variety, as well as its minimal subvarieties have been described for various groups by Jaroslav Ježek and others. We investigate the minimal quasivarieties of this variety and give complete characterizations in certain cases.

Weakly Oligomorphic Clones

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In their 2007 paper (Oligomorphic Clones, Algebra Universalis 57(1): 109–125, 2007), M. Bodirsky and H. Chen introduced and investigated clones on countable sets having the property that permutations contained in the clone form an oligomorphic permutation group. In this talk we introduce and propose the study of a wider class of clones that we refer to as weakly oligomorphic clones. Following (D. Mašulović, M. Pech: Oligomorphic Transformation Monoids and Homomorphism-Homogeneous Structures, accepted for publication in Fundamenta Mathematicae), we say that a clone C on a countable set is weakly oligomorphic if the transformation monoid $C^{(1)}$ is weakly oligomorphic.

Algebras with few subpowers are finitely related

Peter Mayr

coauthored by Erhard Aichinger, Ralph McKenzie

CAUL, Lisbon, Portugal

We show that if a clone of functions on a finite set A contains a Malcev operation, then it is finitely related (that is, equal to the clone of all operations preserving some finitary relation R over A). As a consequence the number of all such Malcev clones on A is countable. More generally, we obtain that every finite algebra with few subpowers has a finitely related clone of term operations. Hence modulo term equivalence and renaming the elements, there are only countably many finite algebras with few subpowers. This is joint work with Erhard Aichinger (JKU Linz) and Ralph McKenzie (Vanderbilt, Nashville).

Inherently finitely related algebras and sparse relational clones

Ralph McKenzie

Vanderbilt University

We say that an algebra $\mathbf{A}$ has *sparse relational clone* (or equivalently, has *few subpowers*) if there is a positive constant k so that $|\mathrm{Sub}(\mathbf{A}^n)| \leq 2^{n^k}$ for all integers $n > 1$. It is known that a finite algebra $\mathbf{A}$ has sparse relational clone iff $\mathbf{A}$ has one (or equivalently, all) of the following: a cube term, an edge term, a parallelogram term. Recently, Aichinger, Mayr and McKenzie have shown that every finite algebra $\mathbf{F}$ with sparse relational clone is finitely related; that is, there is a finitary relation ρ over F so that the finitary operations that respect ρ are precisely the term operations of $\mathbf{F}$.

In this talk, we describe, for any finite set F, finitely many special idempotent clones such that an idempotent clone over F fails to have sparse relational clone iff the clone is a subset of one of these special clones. The special nature of these clones yields, for fixed

F, a polynomial time algorithm to input a finite list of idempotent operations over F and determine if these operations generate a cube-term, and to determine if they generate a near-unanimity operation. Applying the theorem of Aichinger-Mayr-McKenzie, our results yield that a finite idempotent algebra $\mathbf{F}$ has sparse relational clone if and only if $\mathbf{F}$ is inherently finitely related; i.e., every algebra obtained by enriching the set of operations of $\mathbf{F}$ is finitely related. The special clones in our list turn out to be precisely the maximal non-finitely related idempotent clones over F.

The elementary theory of the lattice of equational theories

George F. McNulty

University of South Carolina

The lattice $\mathcal{L}_\Delta$ of all equational theories of signature Δ has an undecidable elementary theory, according to a theorem of Burris and Sankappanavar from 1975, provided Δ is large in the sense of providing at least one operation symbol of rank at least two or at least two operation symbols of rank one. On the other hand, Burris also noted in 1971 that the equational theory of $\mathcal{L}_\Delta$ is decidable. We use the work of Jaroslav Jezek in a effort ot find the point along the spectrum from the equational theory to the elementary theory where undecidability enters. We provide three additional proofs that $\mathcal{L}_\Delta$ has an undecidable elementary theory. Our sharpest result is that the $\forall^*\exists^*\forall^*$ theory of $\mathcal{L}_\Delta$ is hereditarily undecidable.

Ring Closures

Frank Patrick Murphy-Hernandez

coauthored by Francisco Raggi

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We study the closures of a ring defined as idempotent superfuntors of the identity funtor in $RNG1$, we analyze certain examples as matrix closure and differential closure. We also compare the closure with superfuntors induced by free algebras.

All that restricted dualities

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The finite dualities seem to be well understood in the context of CSP, logic and structural combinatorics. For restricted dualities the characterization problem was solved only recently. We present several characterizations of classes with all restricted dualities. Interestingly this relates to a classical Erdos-Hajnal problem.

Some classes of modules defined by certain finiteness conditions

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We study certain fniteness conditions on modules defned in terms of their sets of direct summands. We focus on the study of some types of direct decompositions of those modules that admit these conditions.

On the distance of a polynomial near-ring code

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For a polynomial $f(x) \in \mathbb{Z}_2[x]$ it is natural to consider the near-ring code generated by the polynomials $f \circ x, f \circ x^2, \ldots, f \circ x^k$ as a vectorspace. It is a 16 year old conjecture of Günter Pilz that for the polynomial $f(x) = x^n + x^{n-1} + \cdots + x$ the distance of this code is n.

The conjecture is equivalent to the following purely number theoretical problem. Let $\underline{m} = \{1, 2, \ldots, m\}$ and $A \subset \mathbb{N}$ be an arbitrary finite subset of $\mathbb{N}$. Show that the number of products that occur odd many times in $\underline{n} \cdot A$ is at least n. Pilz also formulated the conjecture for the special case when $A = \underline{k}$. We show that for $A = \underline{k}$ the conjecture holds and that the distance of the code is at least $n/(\log n)^{0.223}$.

While proving the case $A = \underline{k}$ we use different number theoretical methods depending on the size of k (respect to n). Furthermore, we apply several estimates on the distribution of primes.

Power representation of modals

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In a natural way we can "lift" any operation defined on a set A to an operation on the set $\wp(A)$ of all (non-empty) subsets of A and obtain from an algebra (A, F) its *power algebra* $(\wp(A), F)$ of subsets.

The set $\wp(A)$ carries also the join semilattice structure under the set-theoretical union $\cup$. By adding $\cup$ to the set of basic operations we obtain *the extended power algebra* $(\wp(A), F, \cup)$.

On the other hand, a *modal* is an algebra $(M, \Omega, +)$ such that (M, Ω) is a mode (i.e. algebra which is idempotent in the sense that each singleton is a subalgebra, and entropic, i.e. any two of its operation commute), $(M, +)$ is a (join) semilattice and the operations $\omega \in \Omega$ distribute over $+$.

In this talk we show that each modal may be represented as a subalgebra of a quotient of some extended power algebra.

Clones on homogeneous structures and Schaefer's theorem for graphs

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My talk will be an opera in a prologue, three acts, and an epilogue.

Prologue. In a café in Paris, young mathematician Manuel Bodirsky tells his friend Michael Pinsker that he is being haunted by the *graph satisfiability problem*, a generalization of the Boolean satisfiability problem. For a fixed set Ψ of allowed formulas in the language of graphs, an instance of the computational problem Graph-SAT(Ψ) consists of a set W of vertices and statements $\psi_1, \ldots, \psi_n$ about these vertices, where each statement is a formula from Ψ. The task is to decide whether there exists a graph on W such that all statements become true. The friends discuss how to translate the problem into a constraint satisfaction problem for a template with a definition in the countably infinite random graph, and conjecture that the problem is either NP-complete or in P, for all Ψ. They set out to study structures with a first-order definition in (="reducts of") the random graph.

Act I. In a small village in Britain, combinatorialist Peter Cameron is happy because he just proved that the dense linear order has, up to first-order equivalence, 5 reducts. Model-theorist Simon Thomas enters the stage, and sings an air in which he states that the same is true for the random graph. The other villagers count the number of reducts of some other structures, and the act ends with Simon Thomas claiming that the number of reducts of any homogeneous structure in a finite language is finite.

Act II. Manuel and Michael have received a magic Ramsey-theoretic potion produced by Jaroslav Nešetřil and Vojtech Rödl that allows them to find regular patterns in the behavior of functions on reducts of structures with the *Ramsey property*, in particular the random graph. They hear about Thomas' conjecture and want to apply their elixir to arbitrary homogeneous structures with the Ramsey property, but a technical obstacle prevents their success.

Act III. The two friends have decided to consult the three wise men Alexander Kechris, Vladimir Pestov, and Stevo Todorcevic to translate the obstacle into a strange language called topological dynamics. Being approached in this language, young tenor Todor Tsankov does not hesitate one second to resolve the problem. Relieved, the protagonists sing some theorems about minimal reducts of homogeneous structures with the Ramsey property, and dream of proving Thomas'conjecture for such structures in the future.

Epilogue. Back in Paris, and with the new tools at hand, Manuel and Michael prove the complexity dichotomy for Graph-SAT(Ψ).

The Coset Structure of Categorical Skew Lattices

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Pascual Jordan, motivated by questions in quantum logic, brought interest on noncommutative lattices. The most studied of these algebras have been the skew lattices. With the lack of commutativity on these algebras, its coset structure reveals to be of fundamental interest. Introduced in 1993 by J. Leech, a categorical skew lattice is a skew lattice for which composites of coset bijections are coset bijections. These algebras constitute a variety and, together with the parallelogram laws studied by K. Cvetko-Vah in 2005, unveil other aspects of the coset structure of a skew lattice. In this talk we will present a characterization for this algebras and some results that lead us to a better understanding of skew lattices in general.

On finite distributive congruence lattices

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Let $\mathcal{V}$ be a finitely generated congruence-distributive variety of algebras, and let Con$\mathcal{V}$ be the class of all congruence lattices of algebras in $\mathcal{V}$. We consider the problem of characterizing the finite members of Con$\mathcal{V}$. Such a description is not difficult under the additional condition that the congruence lattices of all subdirectly irreducible members of $\mathcal{V}$ are chains. However, without this assumption, the situation is much more difficult and we can only present some partial results and ideas.

Polynomial operators on classes of languages and monoids

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For regular languages $L_0, \ldots, L_k$ over a finite alphabet A and $a_1, \ldots, a_k \in A$, we consider languages of the form $L_0 a_1 L_1 a_2 \ldots a_k L_k \quad (*)$. Usually $L_0, \ldots, L_k$ are from a given Boolean (positive, disjunctive) variety of languages and we form finite unions (positive Boolean combinations, Boolean combinations) of the languages of the form $(*)$.

Here we rather bound the number k by a fixed n. We recall the basic results concerning such hierarchies.

If $L_0, \ldots, L_k$ are recognized by finite monoids (ordered monoids, idempotent semirings) $S_0, \ldots, S_k$ we are looking for a construction of a monoid (...) recognizing the language $(*)$. Such structures should not recognize too much languages, the so-called Schützenberger products do the job.

Affine complete G-sets

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Affine complete algebras were first investigated by G. Grätzer. An algebra is affine complete if no other functions than polynomials are compatible with every congruence. Unlike many other classes of algebras (Boolean algebras, semilattices, etc.) affine complete group actions are not characterized. We investigate affine complete regular G-sets motivated by a recent result of J. Snow. In particular, finite nonabelian simple groups and Frobenius groups yield affine complete regular G-sets. We introduce two characteristic subgroups and their connection with affine completeness.

Semicoprime Preradicals

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Semicoprime preradicals are those that satisfy the condition $\sigma \leq (\tau : \tau)$ implies $\sigma \leq \tau$ for every prerradical σ.

We study this kind of preradicals from several points of view, for instance, that of being the dual of semiprime, or as a generalization of coprime preradicals

On congruence lattices of nil-semigroups

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It is proved in [P. Růžička, J. Tůma and F. Wehrung, *Distributive congruence lattices of congruence-permutable algebras*, Journal of Algebra **311** (2007), 96–116] that every distributive algebraic lattice with the set of compact elements of cardinality at most $\aleph_1$ is isomorphic to the lattice of normal subgroups of some group (note that the lattice of normal subgroups of a group G is the congruence lattice of G considered as a semigroup). We are interested in congruence lattices of semigroups which are far in a sense from groups. For an integer n, a semigroup is called a nil-semigroup of index n if it has a zero and satisfies the identity $x^n = 0$. Our talk is devoted to representing lattices by congruence lattices of such semigroups.

Dyadic polygons

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Dyadic rationals are rationals whose denominator is a power of 2. *Dyadic triangles* and *dyadic polygons* are respectively defined as the intersections with the dyadic plane of a triangle or polygon in the real plane whose vertices lie in the dyadic plane. The one-dimensional analogues are *dyadic intervals*. Algebraically, dyadic polygons carry the structure of a commutative, entropic and idempotent algebra under the binary operation of arithmetic mean. In this talk, dyadic intervals and triangles will be classified to within affine or algebraic isomorphism, and dyadic polygons will be shown to be finitely generated as algebras.

Higman embeddings

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I will discuss several constructions of embeddings of recursively presented groups into finitely presented groups. The constructions preserve and improve algorithmic properties of groups and allow to construct an example of a finitely presented non-amenable group without free subgroups.

Minimal and minimal compact left distributive groupoids

Denis I. Saveliev

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A groupoid is minimal if it includes no proper subgroupoids, and minimal compact if it is compact and includes no proper compact subgroupoids (provided that it carries a topology). Often both types of minimality display a similarity; e.g. any groupoid satisfying certain weak versions of associativity consists of a unique element whenever is either minimal or minimal compact with continuous left translations.

Our talk concerns minimal and minimal compact groupoids satisfying left distributivity. We give a complete description of all minimal left distributive groupoids. In particular, we show that all they are finite and for any finite n there exists exactly one (up to isomorphism) such groupoid of cardinality n. Then we show that there exists a minimal compact topological left distributive groupoid of cardinality $2^{2^{\aleph_0}}$. The proof uses algebra of ultrafilters.

Congruence lattices of finite intransitive G-sets

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Let $\mathbf{X}$ be a finite **G-set**; that is, $\mathbf{X} = (X, G)$ is a finite unary algebra whose operations (G) form a group. When G acts transitively on X, then it is well known that the congruence lattice of $\mathbf{X}$, $Con(\mathbf{X})$, is isomorphic to an interval of the subgroup lattice of G. But what if G does not act transitively? Can anything of interest be said about the congruence lattices of finite intransitive G-sets?

It turns out that if L is a finite graded lattice and is isomorphic to the congruence lattice of a finite intransitive G-set having $n > 1$ components, then L has very special structure– it is isomorphic to a Π-product lattice, a certain sublattice of $K_1 \times \ldots K_n \times \Pi(n)$, where $\Pi(n)$ is the lattice of partitions of an n-element set, and for $i = 1, \ldots, n$, K_i is isomorphic to the congruence lattice of the i-th component of the G-set. In fact, the finite graded lattices that can occur as congruence lattices of finite intransitive G-sets are completely classified in the talk, at least up the classification of the congruence lattices of transitive G-sets (a project made important by the celebrated 1980 paper of Palfy and Pudlak).

In summary, the talk will describe results concerning the interplay between transitive and intransitive finite unary algebras, in particular their congruence lattices.

Special elements of the lattice of commutative semigroup varieties

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An element x of a lattice L is called *distributive* if

$$\forall y, z \in L : \quad x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z),$$

standard if

$$\forall y, z \in L : \quad y \wedge (x \vee z) = (y \wedge x) \vee (y \wedge z),$$

and *neutral* if, for any $y, z \in L$, the elements x, y and z generate a distributive sublattice of L. It is known that any neutral element is standard and any standard element is distributive.

Neutral elements of the lattice **SEM** of all semigroup varieties have been classified in [1]; distributive elements of this lattice have been completely described in [2]. Results of [2] readily imply also that an element of **SEM** is distributive if and only if it is standard. Here we describe elements of these three types in the lattice **Com** of all commutative semigroup varieties. By $\mathcal{T}$, $\mathcal{SL}$ and $\mathcal{COM}$ we denote the trivial variety, the variety of all semilattices and the variety of all commutative semigroups, respectively. We replace the identities $ux = xu = u$, where the letter x does not occur in the word u, by the symbolic identity $u = 0$. Such identities are called 0-*reduced*.

Theorem 1. *For any commutative semigroup variety $\mathcal{V}$, the following are equivalent:*

a) *$\mathcal{V}$ is a distributive element of the lattice* **Com***;*

b) *$\mathcal{V}$ is a standard element of the lattice* **Com***;*

c) *either $\mathcal{V} = \mathcal{COM}$ or $\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ coincides with one of the varieties $\mathcal{T}$ or $\mathcal{SL}$ while $\mathcal{N}$ satisfies the following restrictions:* (i) *$\mathcal{N}$ is defined in $\mathcal{COM}$ by* 0*-reduced identities only;* (ii) *$\mathcal{N}$ satisfies the identities $x^3yz = x^2y^2z = 0$;* (iii) *if $\mathcal{N}$ satisfies one of the identities $x^3y = 0$ and $x^2y^2 = 0$ then it satisfies the other one.*

Theorem 2. *For any commutative semigroup variety $\mathcal{V}$ the following are equivalent:*

a) *$\mathcal{V}$ is a neutral element of the lattice* **Com***;*

b) *$\mathcal{V}$ is both a distributive and a codistributive element of the lattice* **Com***;*

c) *either $\mathcal{V} = \mathcal{COM}$ or $\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ coincides with one of the varieties $\mathcal{T}$ or $\mathcal{SL}$ while $\mathcal{N}$ satisfies the identity $x^2y = 0$.*

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Congruence modularity of algebras with constants

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We recall the definition of 0-modularity and we prove that 0-modular varieties can be characterized by a Mal'cev condition.

Categorically-algebraic topology

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This talk aims at presenting a new way of approaching topological structures, induced by recent developments in the field of lattice-valued topology, and deemed to incorporate in itself both crisp and many-valued settings. Based on category theory and universal algebra, the new framework is called *categorically-algebraic* (*catalg*) [9-12], to underline its motivating theories, on one hand, and to distinguish it from the currently dominating among the respective researchers *point-set lattice-theoretic* (*poslat*) *topology* of S. E. Rodabaugh [7], on the other.

At the bottom of the new approach lies a generalization of several aspects of the standard topological framework. Taking the case of classical topological spaces as an example, there are three main cornerstones in their theory.

1. The starting *backward powerset operator*, which can be represented as a functor $\mathbf{Set} \xrightarrow{(-)^{\leftarrow}} \mathbf{CBAlg}^{op}$ from the category of sets to the dual of the category of complete Boolean algebras, assigning to every set X its powerset 2^X, and to every map $X \xrightarrow{f} Y$ its extension $2^Y \xrightarrow{f^{\leftarrow}} 2^X$, $f^{\leftarrow}(\alpha) = \alpha \circ f$.

2. The induced *topological theory*, which describes the underlying algebraic structure of topological spaces, and which essentially is the composition of the powerset operator and the forgetful functor $\mathbf{CBAlg} \xrightarrow{\|-\|} \mathbf{Frm}$ to the category of frames.

3. The resulting category **Top** of *topological spaces*, which has pairs (X, τ), with τ a subframe of $\|2^X\|$, as objects, and maps $(X, \tau) \xrightarrow{f} (Y, \sigma)$, with $(f^{\leftarrow})^{\rightarrow}(\sigma) \subseteq \tau$, as morphisms (forward powerset operator $(-)^{\rightarrow}$ can be easily avoided).

The proposed catalg framework chooses the starting backward powerset operator to be a functor $\mathbf{X} \xrightarrow{P} \mathbf{LoA}$ to the dual category of some variety of algebras $\mathbf{A}$ (in an extended form, including, e.g., the above-mentioned case of frames). The induced topological theory $\mathbf{X} \xrightarrow{T} \mathbf{LoB}$ is the composition of P and some reduct (in the obvious algebraic sense, but considered as a forgetful functor) $\mathbf{A} \xrightarrow{\|-\|} \mathbf{B}$ of $\mathbf{A}$. The resulting category $\mathbf{Top}(T)$ of topological structures (T-*spaces*) has then pairs (X, τ), with τ (T-*topology*) a subalgebra of $T(X)$, as objects, and $\mathbf{X}$-morphisms $(X, \tau) \xrightarrow{f} (Y, \sigma)$, with $((Tf)^{op})^{\rightarrow}(\sigma) \subseteq \tau$ (T-*continuity*), as morphisms.

The case of the powerset theory P having the form of $\mathbf{Set} \times \mathbf{C} \xrightarrow{(-)^{\leftarrow}} \mathbf{LoA}$ ($\mathbf{C}$ is a subcategory of $\mathbf{LoA}$), $((X_1, A_1) \xrightarrow{(f,\varphi)} (X_2, A_2))^{\leftarrow} = A_1^{X_1} \xrightarrow{((f,\varphi)^{\leftarrow})^{op}} A_2^{X_2}$, $(f, \varphi)^{\leftarrow}(\alpha) = \varphi^{op} \circ \alpha \circ f$ provides a rich source of examples, which cover almost all approaches to many-valued topology (taking in each case the appropriate variety $\mathbf{A}$). Moreover, one easily incorporates the non-standard example of *closure spaces* of D. Aerts *et al.* [1]. The still missing settings can be included through extending the machinery in two ways.

1. Replacing a single powerset theory P by a set-indexed family $(\mathbf{X} \xrightarrow{P_i} \mathbf{LoA}_i)_{i \in I}$ of theories, with the respective set $(\| - \|, \mathbf{B}_i)_{i \in I}$ of reducts, provides a *composite* topological theory $\mathbf{X} \xrightarrow{T_I} \prod_{i \in I} \mathbf{LoB}_i$, resulting in the category $\mathbf{CTop}(T_I)$ of *composite* topological structures, with objects pairs $(X, (\tau_i)_{i \in I})$, where τ_i is a subalgebra of $T_i(X)$ for every $i \in I$, and morphisms defined accordingly. The framework incorporates *bitopological spaces* of J. C. Kelly [3] as well as their lattice-valued analogues of T. Kubiak [4] and S. E. Rodabaugh [8].

2. Introducing the category $\mathbf{C}$-$\mathbf{A}$ of *lattice-valued* $\mathbf{A}$-*algebras* over some extension $\mathbf{C}$ of the variety $\mathbf{CSLat}(\bigvee)$ of $\bigvee$-semilattices, gives the category $\mathbb{L}\mathbf{Top}(T)$ of *lattice-valued* topological structures, which incorporates (L, M)-*fuzzy topological spaces* of T. Kubiak and A. Šostak [5] as well as their generalization of C. Guido [2].

The resulting category $\mathbb{L}_I\mathbf{CTop}(T_I)$ has the simple property of subbasic continuity for its morphisms and, therefore, is topological over its ground category $\mathbf{X} \times \prod_{i \in I} \mathbf{LoC}_i$. Moreover, it gives rise to the category $\mathbb{L}_I\mathbf{CTopSys}(T_I)$ of *topological systems* in the sense of S. Vickers [13] (introduced as a convenient tool for doing *pointless topology* with), which, in case of completely distributive underlying $\bigvee$-semilattices of $\mathbb{L}_I$, includes the category $\mathbb{L}_I\mathbf{CTop}(T_I)$ as a full coreflective subcategory. The generalization of systems starts a completely new area of research called *soft topology*, which is induced by the concept of *soft set* of D. Molodtsov [6] (presented as a promising tool to deal with uncertainty) extended, for the occasion, to the notion of *soft algebra*. Strikingly enough, it appears that soft topology is a proper extension of lattice-valued topology, thereby stimulating the study on its properties. On the other hand, the theories of both lattice-valued and soft algebras (the latter incorporating the former) also open a challenging area for exploration.

By the opinion of the author, the proposed catalg framework has a reasonable balance between universality and fruitfulness of the obtained theory and thus, can provide a good substitute for the above-mentioned poslat approach to topology.

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Quasi-equational axiomatizations for graphs of semigroups, monoids and groups

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The *graph* of an algebra $\mathbf{A} = (A, \Omega)$ is the relational structure

$$\mathsf{G}(\mathbf{A}) = (A, \{R_\omega\}_{\omega\in\Omega}),$$

where each R_ω is the graph of an operation ω. This means that if ω is an n-ary operation, then R_ω is the $(n+1)$-ary relation consisting of those tuples $(a_1, \ldots, a_{n+1})$ which satisfies $\omega(a_1, \ldots, a_n) = a_{n+1}$. For a class $\mathcal{C}$ of algebras by $\mathsf{G}(\mathcal{C})$ we denote the class of all graphs of algebras from $\mathcal{C}$.

A *quasivariety* is a class defined by *quasi-identities*, i.e. by sentences of the form

$$(\forall \bar{x})\ [\varphi_1(\bar{x}) \wedge \cdots \wedge \varphi_n(\bar{x}) \rightarrow \varphi(\bar{x})],$$

where n is a natural number and $\varphi_1, \ldots, \varphi_n, \varphi$ are atomic formulas. For a class $\mathcal{K}$ by $\mathsf{Q}(\mathcal{K})$ we denote the smallest quasivariety containing $\mathcal{K}$. It is the class defined by the quasi-identities true in $\mathcal{K}$.

O. M. Gornostaev proved that $\mathsf{QG}(\mathbb{Z}_2, +)$ and $\mathsf{QG}(\mathbb{Z}_2, \vee)$ are not finitely axiomatizable. This sharply contrasts with the facts that all varieties and quasivarieties generated by two element algebras are finitely axiomatizable. Our motivation was to check whether Gornostaev's result is just a curious exception or there is a deeper reason for the lack of a finite quasi-equational basis. We obtained the following fact.

Theorem. *Let $\mathcal{C}$ be a class of semigroups possessing a nontrivial member with a neutral element. Then $\mathsf{QG}(\mathcal{C})$ is not finitely axiomatizable.*

Small modifications in the proof yield the following result.

Corollary. *Let $\mathcal{C}$ be a class of monoids or groups possessing a nontrivial member. Then $\mathsf{QG}(\mathcal{C})$ is not finitely axiomatizable.*

Elementary problems in number theory

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An easy pigeonhole-principle argument shows that if you have $(\mathbf{n}-\mathbf{1})^{\mathbf{2}}+\mathbf{1}$ many numbers, then you can choose n of them such that their sum is divisible by n. A slightly easy Ramsey argument shows that if you have a polynomial $f(\bar{x}) \in R[x_1, x_2, \ldots, x_n]$, where R is a nilpotent ring of size r and nilpotency class k, then for every $\bar{a} \in R^n$ there is a $\bar{b} \in R^n$ such that $b_i = 0$ or $b_i = a_i$ and $b_i = a_i$ for only $\mathbf{r}^{\mathbf{r}^{\cdots^{\mathbf{r}^{\mathbf{k}}}}}$ many i-s (there are k-many r-s in the tower).

These bounds can be reduced to $2n-1$ and $k(p-1)+1$ using the following generalization of Chevalley's theorem, which is a simple consequence of Alon's Combinatorial Nullstellensatz.

Lemma. Let $A_1, \dots, A_n$ be subsets of F_p, the p-element field, and $f \in F_p[x_1, \dots, x_n]$ such that

$$\sum_{i=1}^{n}(|A_i|-1) > (p-1)\deg f.$$

If the set $\{a \in A_1 \times \dots \times A_n | f(a) = 0\}$ is not empty, then it has at least two different elements.

On avtotopies and antiavtotopies quasigroups

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The report is devoted to an avtotopies and antiavtotopies of a quasigroups. A connection between a sets of antiavtotopies isotopic quasigroups is established. This connection is analogue of Belousov's theorem about isomorphism of a groups of avtotopies isotopic quasigroups [1]; a structure of antiavtotopies of arbitrary a group is found, an avtotopies and antiavtotopies of linear and alinear quasigroups are described.

A triplet of permutations $T = (\alpha, \beta, \gamma)$ of a set Q such that $\gamma(x \cdot y) = \alpha x \cdot \beta y$ $(\gamma(x \cdot y) = \alpha y \cdot \beta x)$ is called *an avtotopy (antiavtotopy)* of a quasigroup $(Q, \cdot)$.

Theorem 1 (V.D. Belousov,[1]). *If a quasigroups $(Q, \cdot)$ and $(Q, \circ)$ are isotopic then their a groups of avtotopies are isomorphic, namely*

$$A\nu t(Q, \cdot) = T^{-1} A\nu t(Q, \circ) T$$

where $Avt(Q, \cdot)$ and $Avt(Q, \circ)$ are a groups of avtotopies of a quasigroups $(Q, \cdot)$ and $(Q, \circ)$ respectively, where $T = (\alpha, \beta, \gamma)$ is an isotopy.

Denote by $\bar{A}vt(Q, \cdot)$ and $\bar{A}vt(Q, \circ)$ a sets of antiavtotopies of a quasigroups $(Q, \cdot)$ and $(Q, \circ)$ respectively. The following theorem is an analogue of the theorem 1.

Theorem 2. *If a quasigroups $(Q, \cdot)$ and $(Q, \circ)$ are isotopic, $\gamma(x \circ y) = \alpha x \cdot \beta y$, then*

$$\bar{A}\nu t(Q, \cdot) = T^{-1} \bar{A}\nu t(Q, \circ) T_1$$

where $T = (\alpha, \beta, \gamma)$, $T_1 = (\beta, \alpha, \gamma)$.

In the following theorem a structure of antiavtotopies of arbitrary a group is found:

Theorem 3. *An any antiavtotopy of a group $(Q, +)$ has the following form:*

$$T = (\tilde{L}_a, \tilde{R}_b, \tilde{L}_a \tilde{R}_b)\bar{\theta},$$

where $\bar{\theta}$ is an antiautomorphism of the group $(Q, +)$, a, b are fixed elements of Q, $\tilde{R}_s x = x + s$, $\tilde{L}_s x = s + x$, $s, x \in Q$.

Using the theorems 1-3 a structure of an avtotopies and antiavtotopies of linear and alinear quasigroups are found.

A quasigroup $(Q,\cdot)$ is called *linear (alinear)* over a group $(Q,+)$ if it has the form: $xy=\varphi x+c+\psi y$, where $\varphi,\psi\in Aut(Q,+)$, (φ,ψ is an antiautomorphism of the group $(Q,+)$), c is fixed element of Q.

Theorem 4. *An any avtotopy of a linear (an alinear) quasigroup $(Q,\cdot)$, $xy=\varphi x+c+\psi y$ $(xy=\bar\varphi x+c+\bar\psi y)$ has the following form:*

$$A=\left(\tilde R_c L_{\varphi a}\varphi\theta\varphi^{-1}\tilde R_{-c},\tilde L_{\psi b}\bar\psi\theta\bar\psi^{-1},\tilde L_a\tilde R_b\theta\right),$$

$$\left(\overline{A}=\left(\tilde R_d\bar\varphi\theta\bar\varphi^{-1}\tilde R_{-c},\tilde L_{\psi b}\bar\psi\theta\bar\psi^{-1},\tilde L_a\tilde R_b\theta\right)\right).$$

where $\varphi,\psi,\theta\in Aut(Q,+)$, a,b,c are fixed elements of Q.

Theorem 5. *An any antiavtotopy of a linear quasigroup $(Q,\cdot)$, $xy=\varphi x+c+\psi y$ has the following form:*

$$\mathrm{P}=(\varphi^{-1}\tilde R_{-c}\tilde L_a\bar\theta\psi,\psi^{-1}\tilde R_b\bar\theta\tilde R_c\varphi,\tilde L_a\tilde R_b\bar\theta),$$

where $\varphi,\psi\in Aut(Q,+)$, $\bar\theta$ is antiautomorphism of a group $(Q,+)$, a,b,c are fixed elements of Q.

Corollary. An any automorphism γ of a linear quasigroup $(Q,\cdot)$ $xy=\varphi x+c+\psi y$ can be submit in the form:

$$\gamma=\tilde R_c\tilde L_{\varphi a}\varphi\theta\varphi^{-1}\tilde R_{-c}=\tilde R_{\psi b}\psi\theta\psi^{-1}=\tilde L_a\tilde R_b\theta,$$

An analogical results for a quasigroups mixed type of linearity, T-quasigroups and a medial quasigroups are given.

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Compressible Modules and Compressible Dimensions

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We define the compressibility respect to the density and purity, we compare them and look for relations between the dense compressible and pure compressible submodules. This comparation is developed first with a fixed hereditary torsion theory and then, we investigate how the order of R-tors has influence on the two concepts of compressibility.

Functorial algebras and coalgebras

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Old period

I. *Structural problems:* basic properties of F-algebras where $F : \mathcal{K} \to K$ is an endofunctor of a category $\mathcal{K}$ and their use in Arbib-Manes functorial machines. Iterative constructions and their convergence.

II. *Representations*: Following the idea of J. R. Isbell, the representation of every group as the automorphism group of a distributive lattice (Birkhoff) or of a topological space (de Groot), were generalized to the investigation of the full embedings of categories. A category $\mathcal{K}$ is called *algebraically universal* if every category of universal algebras can be fully embedded in it. Then every monoid can be represented as the endomorphism monoid of its object. Algebraic universality of the categories of F-algebras was examined by V. Koubek in 1983.

THE TWENTY-YEARS-LONG BREAK

New period

III. *Structural problems:* Refreshing the field of problems, computer scientists use initial F-algebras and terminal F-coalgebras (J. Rutten: Universal coalgebra: a system theory, Theoret. Comp. Sci., 249, 2000)
Selected parts of the two recent papers

- J. Adámek, V. T.: Initial algebras and terminal coalgebras in many-sorted sets, sent for publication
- J. Adámek, V. T.: Relatively terminal coalgebras, in preparation

will be presented.

IV. *Representations:* Selected parts of the two recent papers

- J. Sichler and V. T.: On universal categories of coalgebras, accepted in AU
- V. Koubek, J. Sichler and V. T.: Universality of categories of coalgebras, accepted in Applied Cat. Structures

will be presented. Let us mention here at least that a category is *coalgebraically universal* (i.e. every category of universal coalgebras can be fully embeded in it) if and only if it is algebraically universal.
For the ultrafilter functor $\beta : Set \to Set$, though Coalg β contains rigid proper class, a monoid on 13 elements cannot be represented in it.

Normal Subalgebras in Universal Algebras

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We extend the notions of (ideals, clots and) normal subalgebras of a (universal) algebra, from the most common and best understood case of a single constant to the case of any number of constants. The natural idea for defining when a subalgebra is normal is to involve the inverse image under a morphism of the minimal subalgebra of the target, i.e. the subalgebra generated by constants in the target; this goes without further ado in case there is just one constant whose singleton is a subalgebra. Ideals (and clots) are defined instead by closure under derived operations which have the right behavior on constants. We give several characterizations of these notions; much care is taken to envisage the possibility of a categorical generalization. We characterize: the varieties in which ideals coincide with normal subalgebras, the varieties in which different congruences produce different normal subalgebras, the varieties in which ideals are normal for exactly one congruence, and the special class of protomodular varieties. As useful tools in these regards, we deal with (extended) notions of subtractivity, coherence and regularity (at the constants) of congruences. Some applications are discussed: boolean algebras, Heyting algebras, unitary rings, difference rings, and differential rings.

Maltsev Conditions for Omitting Types

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Tame congruence theory identifies six Maltsev conditions associated with locally finite varieties omitting certain subsets of types of local behaviour. Extending a result of Siggers, we show that of these six Maltsev conditions, only two of them are equivalent to strong Maltsev conditions for locally finite varieties.

This is joint work with Marcin Kozik, Andrei Krokhin, Miklos Maroti, and Ross Willard.

Codistributive elements of the lattice of semigroup varieties

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An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called *codistributive* if

$$\forall y, z \in L: \quad x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z),$$

and *costandard* if

$$\forall y, z \in L: \quad (x \wedge y) \vee z = (x \vee z) \wedge (y \vee z).$$

Distributive and *standard* elements are defined dually to codistributive and costandard ones respectively. An element $x \in L$ is called *neutral* if, for all $y, z \in L$, the elements x, y and z generate a distributive sublattice in L. It is known that a [co]standard element is [co]distributive.

Let **SEM** be the lattice of all semigroup varieties. By $\mathcal{T}$, $\mathcal{SEM}$, $\mathcal{SL}$, and $\mathcal{ZM}$ we denote the trivial variety, the variety of all semigroups, the variety of all semilattices, and the variety of all semigroups with zero multiplication respectively. Distributive elements of the lattice **SEM** are completely determined in [1]. Results of [1] readily implies also that an element of **SEM** is standard if and only if it is distributive. Neutral elements of **SEM** are completely determined in [2, Proposition 4.1].

Theorem 1. *If a semigroup variety $\mathcal{V}$ is a codistributive element of the lattice* **SEM** *then either $\mathcal{V} = \mathcal{SEM}$ or $\mathcal{V}$ satisfies an identity of the form $xy = (xy)^{n+1}$ for some natural number n.*

This theorem together with [2, Proposition 4.1] and [3, Proposition 1.6] readily imply

Corollary. *For a semigroup variety $\mathcal{V}$, the following are equivalent: a) $\mathcal{V}$ is a costandard element of* **SEM***; b) $\mathcal{V}$ is a neutral element of* **SEM***; c) $\mathcal{V}$ coincides with one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$, $\mathcal{SL} \vee \mathcal{ZM}$ or $\mathcal{SEM}$.*

Theorem 2. *Let $\mathcal{V}$ be a semigroup variety satisfying an identity of the form*

$$x_1 x_2 \cdots x_n = x_{1\alpha} x_{2\alpha} \cdots x_{n\alpha}$$

where α is a permutation on the set $\{1, 2, \ldots, n\}$ with $1\alpha \neq 1$ and $n\alpha \neq n$. The variety $\mathcal{V}$ is a codistributive element of the lattice **SEM** *if and only if $\mathcal{V} = \mathcal{G} \vee \mathcal{X}$ where $\mathcal{G}$ is an abelian periodic group variety, while $\mathcal{X}$ is one of the varieties $\mathcal{T}$, $\mathcal{SL}$, $\mathcal{ZM}$ or $\mathcal{SL} \vee \mathcal{ZM}$.*

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Lattice universal semigroup varieties

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A variety of semigroups is called *lattice universal* if its subvariety lattice contains an interval dual to the lattice of partitions of a countable set. In 1976 Ježek proved that the variety defined by the identity $x^2 = 0$ is lattice universal. Extending Ježek's techniques, we strengthen this result as follows. Let $\{Z_n\}_{n=1,2,\ldots}$ stand for the sequence of Zimin's words defined by $Z_1 = x_1$ and $Z_{n+1} = Z_n x_{n+1} Z_n$.

Theorem. Suppose that a semigroup variety $\mathcal{V}$ is defined by identities depending on at most n variables and satisfies no non-trivial identity of the form $Z_{n+1} = w$. Then $\mathcal{V}$ is lattice universal.

It is interesting that under a relatively week restriction, the converse is also true. Namely, we have the following result.

Theorem. Suppose that a lattice universal semigroup variety $\mathcal{V}$ is defined by identities depending on at most n variables and all periodic groups in $\mathcal{V}$ are locally finite. Then $\mathcal{V}$ satisfies no non-trivial identity of the form $Z_{n+1} = w$.

In other words, we have obtained a complete characterization of lattice universal semigroup varieties in the class of varieties defined by identities in finitely many variables and containing no infinite periodic groups with finitely many generators.

The relational clone membership problem is hard

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Given a finite set R of relations on a finite set D, and another relation s on D, the relational clone membership problem asks whether s belongs to the relational clone generated by R – equivalently, whether s is invariant under all the polymorphisms of R. We prove that the space required to store a witness for a positive answer (a primitive positive definition) or a negative answer (a non-invariant polymorphism) may be required to be exponentially large in comparison to the size of the input. We also show that the problem is co-NExpTime-complete.

The Project of Complexity Classification of Quantified Temporal Constraints

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Let τ be a finite relational signature and Γ a τ-structure over some domain D. A quantified constraint satisfaction problem QCSP(Γ) is a question whether a τ-sentence of the form:

$$Q_1 v_1 \dots Q_k v_k \bigwedge_{i=1}^{n} R_i(x_{i1}, \dots, x_{in_i}) \tag{1}$$

where each Q_i is either universal or existential and each $R_i \in \tau$ is true in Γ.

A structure $(R_1, \dots, R_k, Q)$, where Q is the set of rational numbers, is called a temporal constraint language if for each i the relation R_i is first order definable over $(Q, <)$.

The purpose of the titled project is to classify all temporal constraints languages due to the computational complexity of their quantified constraint satisfaction problems.

In the talk I will present partial results obtained so far and the methods which were successfully used for quantified constraint satisfaction problems for relations on finite domains. These methods are based on the so-called algebraic approach to constraint satisfaction problems, which means that they link the complexity of QCSP(Γ) with the set of operations (polymorphisms) preserving Γ. We believe that similar methods will be useful in the case of temporal languages.

Algebra of the solutions to the Beltrami equations

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The Beltrami equation has a long and a dramatic history. This equation plays a crucial role in many fields of analysis, geometry, complex dynamics, string theory, mechanics, elasticity and robotics.

We study algebraic properties of solutions to the Beltrami equations. Our main result asserts that the set of open and discrete solutions to the Beltrami equations forms an algebra and, in some cases, a lattice. We find the dependence between the algebraic properties of the solutions and the Beltrami coefficient.