2.3.6 Vyrovnání kubickými spliny

One of the successful and recently frequently used models is the model of *cubic splines*. Assuming $T_1 < T_2 < \cdots < T_N$, we consider functions g such that (i) g is a piecewise cubic function, i.e., g equals

$$g_n(t) := \alpha_n + \beta_n t + \gamma_n t^2 + \delta_n t^3 \quad \text{for } t \in [T_{n-1}, T_n], \quad n = 2, \dots, N, \tag{2.78}$$

(ii) g is twice continuously differentiable everywhere; this is (together with (i)) equivalent to

$$g_n(T_n) = g_{n+1}(T_n), \quad g_n'(T_n) = g_{n+1}'(T_n), \quad g_n''(T_n) = g_{n+1}''(T_n), \quad n = 2, \dots, N-1.$$

We then choose the function $\widehat{g}$ from this class that minimizes a combination of the residual sum of squares and the integrated squared 2nd derivative of g:

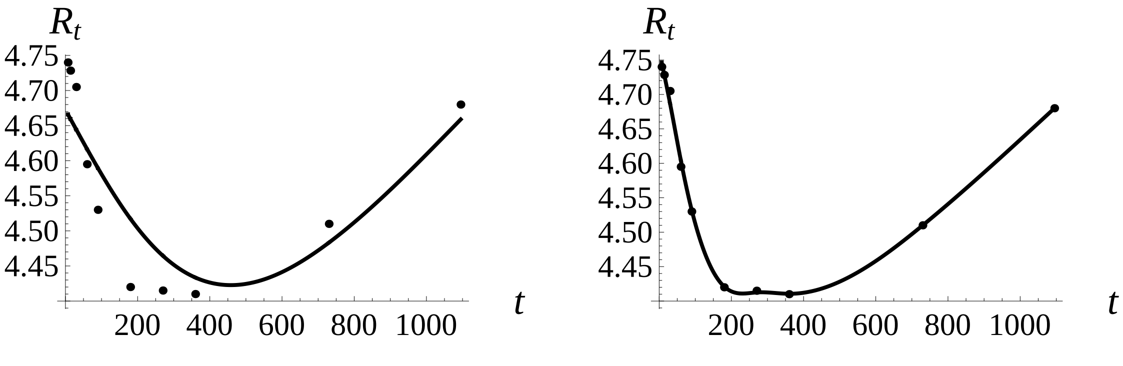
$$\widehat{g} = \operatorname*{argmin}_g \{ \sum_{n=1}^N (y_n - g(T_n))^2 + \lambda \int_{T_1}^{T_N} (g''(t))^2 dt \}$$

with a smoothing constant $\lambda > 0$. The resulting $\widehat{g}$ represents a compromise between fit of data and smoothness of the fitting curve. Values of the smoothing constant λ cover ordinary least squares fitting by a straight line ($\lambda \to \infty$) as one extreme, and pure numerical interpolation by a piecewise cubic functions ($\lambda = 0$) as the other one. Details of the method together with an algorithm can be found in [150].

$$\widehat{g} = \operatorname*{argmin}_g \{ \sum_{n=1}^N w_n (y_n - g(T_n))^2 + \int_{T_1}^{T_N} \lambda(t) (g''(t))^2 dt \}$$

w_i may take into account the market capitalization, e.g. (Risk J. 2(1), p. 25), $\lambda(\tau) = 0.1$, $0 \le \tau < 1$, $\lambda(\tau) = 100$, $1 \le \tau < 10$, $\lambda(\tau) = 100000$, $\tau \ge 10$,

2.3.16 Příklad. Viděli jsme, že „nejdivočejší“ chování úrokových měr z poznámky 2.3.14 je v nejkratším časovém horizontu. Proto uvedeme ilustraci pro prvních deset kotací $N = 10$, tj. poslední kotace pro $T_{10} = 1095$. Metoda je v tomto případě velmi citlivá na volbu konstanty λ. Na obr. 2.11 je vlevo vyrovnání pro $\lambda = 10^7$, vpravo pro $\lambda = 10000$, body znázorňují pozorované kotace.



Obr. 2.11. Vyrovnání kubickými spliny