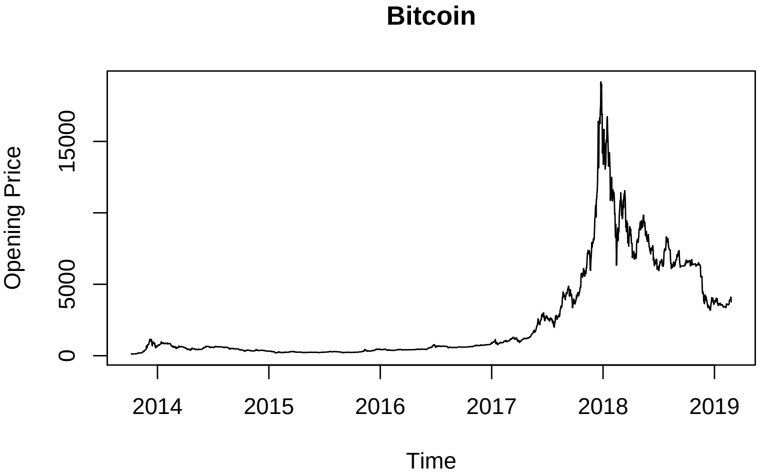
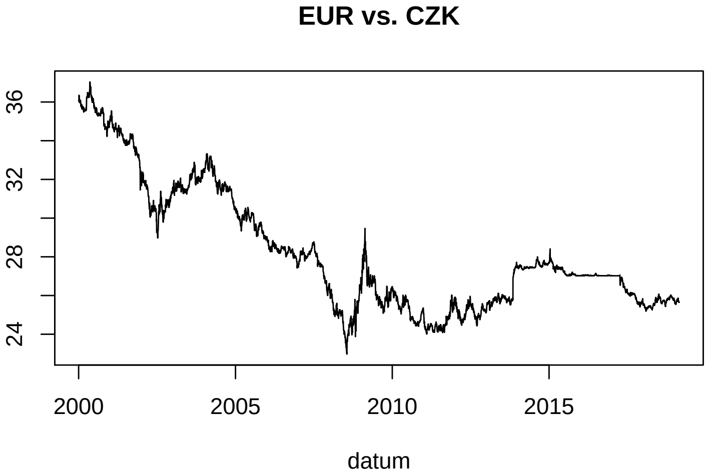
Week 1: Introduction

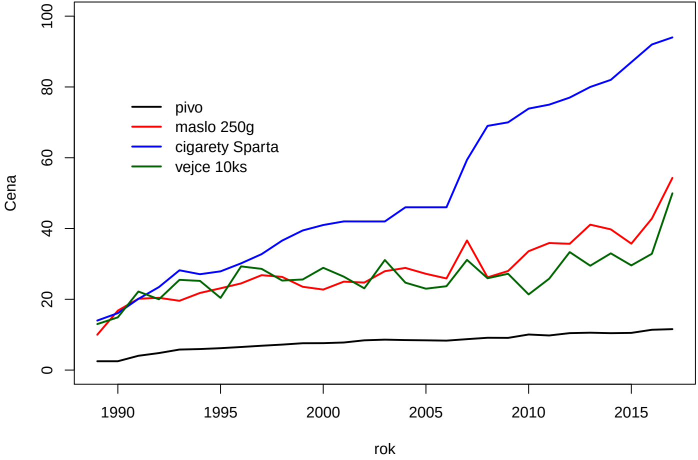
Time Series


Bitcoin
Opening Price
0
5000
15000
2014
2015
2016
2017
2018
2019
Time

Time Series

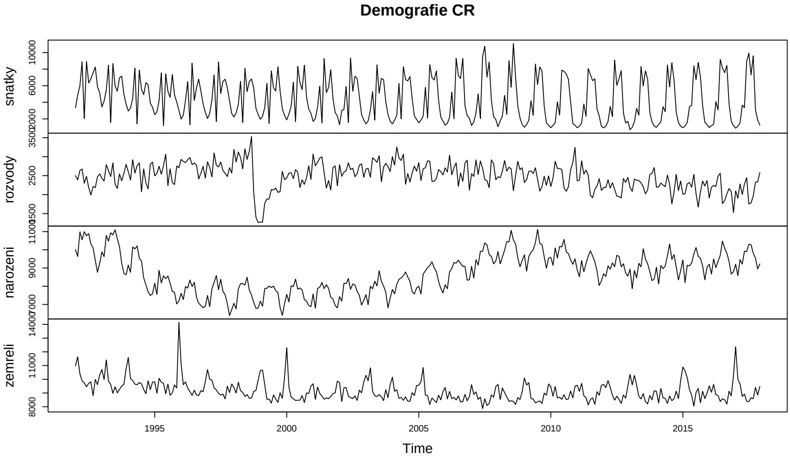


Time Series



pivo
maslo 250g
cigarety Sparta
vejce 10ks
Cena
0
20
40
60
80
100
1990
1995
2000
2005
2010
2015
rok

Time Series

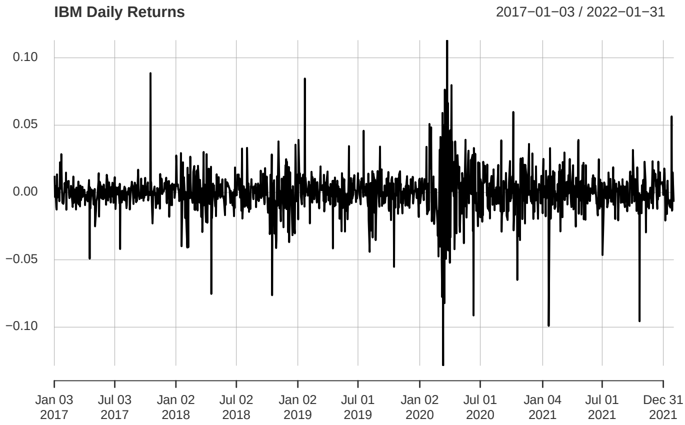


Demografie CR
snatky
rozvody
narozeni
zemreli
2000
6000
10000
1500
2500
3500
7000
9000
11000
8000
11000
14000
1995
2000
2005
2010
2015
Time

Time Series


IBM Closing
2017−01−03/2022−01−31
160
140
120
100
Jan 03 2017
Jul 03 2017
Jan 02 2018
Jul 02 2018
Jan 02 2019
Jul 01 2019
Jan 02 2020
Jul 01 2020
Jan 04 2021
Jul 01 2021
Jan 03 2022

Time Series



IBM Daily Returns
2017−01−03 / 2022−01−31
0.10
0.05
0.00
−0.05
−0.10
Jan 03 2017
Jul 03 2017
Jan 02 2018
Jul 02 2018
Jan 02 2019
Jul 01 2019
Jan 02 2020
Jul 01 2020
Jan 04 2021
Jul 01 2021
Dec 31 2021

Time Series

- sequence of random variables $\{Y_t\}$,
  - $\hookrightarrow$ t is time
  - $\hookrightarrow$ different time units
  - $\hookrightarrow$ equidistant times
- stochastic process $\rightsquigarrow$ we observe a piece of one trajectory $Y_1(\omega), \ldots, Y_n(\omega)$
- possible dependence ➞ needs to be taken into account in the analysis

Aims of a time series analysis

1. description of time behavior

2. statistical inference

3. prediction

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    - point and interval
    - various criteria for prediction evaluation → comparison of different methods
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Prediction is very difficult, especially if it is about the future.

Niels Bohr

Time series decomposition

Additive model

$$Y_t = Tr_t + S_t + E_t$$

Multiplicative model

$$Y_t = Tr_t \cdot S_t \cdot E_t$$

Time series decomposition

Additive model

$$Y_t = Tr_t + S_t + E_t$$

Multiplicative model

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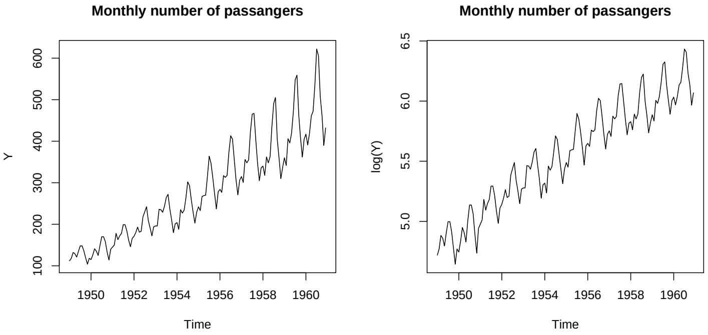
$\rightsquigarrow \log Y_t$ satisfies an additive model

Additive or multiplicative model?



Additive or multiplicative model?

No seasonality: See that

$$Y_t = Tr_t + E_t \quad \Longleftrightarrow Y_t = Tr_t \cdot \varepsilon_t, \quad \varepsilon_t = \left(1 + \frac{E_t}{Tr_t}\right),$$

If $\text{Var}\, E_t = const$, then

$$\text{Var}\, \varepsilon_t = \frac{\text{Var}\, E_t}{Tr_t^2} \neq const,$$

General approach

$$Y_t = Tr_t + S_t + E_t$$

1. estimate a deterministic trend Tr_t and seasonality S_t
2. possible approaches:
   - parametric
   - non-parametric
3. model for stationary series $\{E_t\}$, constructed using $\widehat{E}_t = Y_t - \widehat{Tr}_t - \widehat{S}_t$

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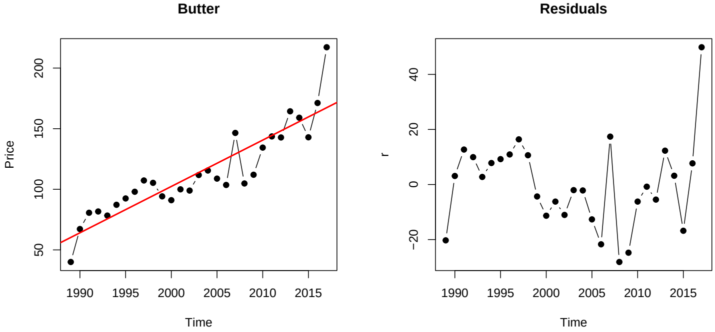


Parametric trend estimation

$$Y_t = Tr_t + E_t$$

- mathematical curves:

$$Tr_t = f(t, \theta)$$

- least squares estimation:

$$\widehat{\theta} = \min_{\theta} \sum_{t=1}^{n} (Y_t - f(t, \theta))^2$$

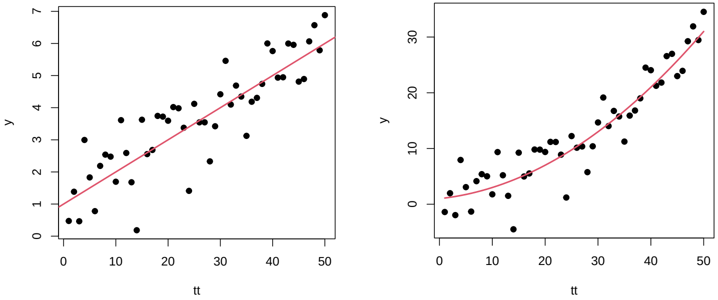
  - f polynomial $\rightsquigarrow$ ordinary least squares
  - f nonlinear $\rightsquigarrow$ non-linear least squares

- then

$$\widehat{Tr}_t = f(t, \widehat{\theta})$$

  is the fitted trend, $Y_t - f(t, \widehat{\theta})$ a detrended series

Linear and polynomial trend



Estimation via ordinary least squares

Also recall spline methods