Cvičení 7 - výsledky

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Příklad 1. $(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0} \frac{2\sin\frac{h}{2}\cdot\cos\frac{2x+h}{2}}{h} = \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}\cdot\cos\frac{2x+h}{2} = 1\cdot\cos\frac{2x}{2} = \cos x.$

Příklad 2.

$$f'(2) = \lim_{h\to 0}\frac{(2+h)^2\cdot\sin(2+h-2)-2^2\cdot\sin(2-2)}{h} = \lim_{h\to 0}(4+4h+h^2)\cdot\frac{\sin h}{h} = 1\cdot\lim_{h\to 0}(4+4h+h^2) = 4.$$

$$t: y = f(2)+f'(2)(x-2) = 0+4(x-2) = 4x-8,$$

$$n: y = f(2)-\frac{1}{f'(2)}(x-2) = 0-\frac{1}{4}(x-2) = -\frac{x}{4}+\frac{1}{2}.$$

Příklad 3. $f(4) = \frac{1+4-16}{1-4+16} = -\frac{11}{13}$,

$$f'(x) = \left(\frac{1+x-x^2}{1-x+x^2}\right)' = \frac{(1-2x)(1-x+x^2)-(-1+2x)(1+x-x^2)}{(1-x+x^2)^2} =$$

$$= \frac{(1-2x)(1+x-x^2+1-x+x^2)}{(1-x+x^2)^2} = \frac{2-4x}{(1-x+x^2)^2},$$

odtud $f'(4) = -\frac{14}{169}$.

$$t: y = -\frac{11}{13}-\frac{14}{169}(x-4) = -\frac{14}{169}x-\frac{87}{169},$$

$$n: y = -\frac{11}{13}+\frac{169}{14}(x-4) = \frac{169}{14}x-\frac{169\cdot 4\cdot 14+11\cdot 14}{182} = \frac{169}{14}x-\frac{8942}{182}.$$

Příklad 4. $f'(x) = \left(-2x^2+2x+\frac{1}{2}\right)' = -4x+2$ *Tečna v bodě* x_0 *je rovnoběžná s osou* x, *jestliže* $f'(x_0) = 0$. *Tedy* $f'(x_0) = -4x_0+2 = 0$ *a* $x_0 = \frac{1}{2}$. *Odtud* $y_0 = f(x_0) = -2\cdot(\frac{1}{2})^2+2\cdot\frac{1}{2}+\frac{1}{2} = 1.$

$$t: y = 1,$$

$$n: x = \frac{1}{2}.$$

Příklad 5. $f'(x) = \left(\frac{x+9}{x+5}\right)' = \frac{x+5-(x+9)}{(x+5)^2} = \frac{-4}{x^2+10x+25}$, *Hledaná tečna má tedy v počátku tvar*

$$0 = \frac{x_0+9}{x_0+5}-\frac{4}{x_0^2+10x_0+25}(0-x_0) = \frac{(x_0+9)\cdot(x_0+5)-4\cdot(-x_0)}{(x_0+5)^2} = \frac{x_0^2+18x_0+45}{(x_0+5)^2}.$$

Stačí tedy vyřešit rovnici $x_0^2+18x_0+45 = 0$. *Kořeny této rovnice jsou* $x_{01} = -3$ *a* $x_{02} = -15$.

$$t_{-3}: y = \frac{6}{2}-\frac{4}{4}(x+3) = -x,$$

$$n_{-3}: y = \frac{6}{2} + \frac{4}{4}(x+3) = x + 6,$$

$$t_{-15}: y = \frac{-6}{-10} - \frac{4}{100}(x+15) = -\frac{x}{25},$$

$$n_{-15}: y = \frac{-6}{-10} + \frac{100}{4}(x+15) = 25x + \frac{1878}{5}.$$

Příklad 6. $f'(x) = (\ln(x-1)^2)' = \frac{1}{(x-1)^2} \cdot 2 \cdot (x-1) = \frac{2}{x-1}$. *Přímka q kolmá na přímku $p: y = \frac{2}{5}x + 2$ je tvaru $y = -\frac{5}{2}x + c$.*

Odtud $\frac{2}{x_0 - 1} = -\frac{5}{2}$ a $x_0 = \frac{1}{5}$.

$$t: y = \ln(\frac{1}{5} - 1)^2 - \frac{5}{2}(x - \frac{1}{5}) = -\frac{5}{2}x + \frac{1}{2} + \ln\frac{16}{25},$$

$$n: y = \ln(\frac{1}{5} - 1)^2 + \frac{2}{5}(x - \frac{1}{5}) = \frac{2}{5}x - \frac{2}{15} + \ln\frac{16}{25}.$$

Příklad 7. *Najděte derivaci funkce f, jestliže*

a) $((\sin x)^{\cos x})' = (\exp(\cos x \cdot \ln(\sin x)))' = (\sin x)^{\cos x} \cdot \left(-\sin x \ln(\sin x) + \frac{\cos^2 x}{\sin x}\right)$,

b) $\left(\ln\left(\arccos\frac{1}{\sqrt{x}}\right)\right)' = \frac{1}{\arccos\frac{1}{\sqrt{x}}} \cdot \frac{1}{2\cdot\sqrt{x^3}\cdot\sqrt{1-\frac{1}{|x|}}}$,

c) $(x^2 + \operatorname{arctg}(x^3 + x))' = 2x + \frac{3x+1}{x^6+2x^4+x^2+1}$,

d) $\left(\frac{\cos(2x+5)}{\sin^2 x}\right)' = \frac{2\sin(2x+5)\cdot\sin x - \cos(2x+5)\cdot\cos x}{\sin^3 x} = -2\frac{\cos(3x+5)}{\sin^3 x}$,

e) $\left(4\sqrt[3]{\operatorname{cotg}^2 x} + \sqrt[3]{cotg^8 x}\right)' = -\frac{8}{3\sin^2 x}\left(\frac{1}{\sqrt[3]{\operatorname{cotg} x}} + \sqrt[3]{\operatorname{cotg}^5 x}\right)$,

f) $\left(e^{-x^2}\cdot(x^2 - 2x + 2)\right)' = e^{-x^2}\cdot(-x^3 + 4x^2 - 2x - 2)$,

g) $\left(\log^3 x^2\right)' = \frac{6\ln^2 x^2}{x}$,

h) $(x^{-3}\cdot \operatorname{tg} x)' = \frac{x - 3\sin x\cdot\cos x}{x^4\cdot\cos^2 x} = \frac{x - \frac{3}{2}\sin 2x}{x^4\cdot\cos^2 x}$,

i) $\left(\sqrt{x + \sqrt{x + \sqrt{x}}}\right)' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\cdot(1 + \frac{1}{2\sqrt{x}})\right)$,

j) $\left(\frac{\sqrt[4]{x^3+x}}{1+x}\right)' = \frac{-4x^3+3x^2+1}{4\cdot(x^2+2x+1)\cdot\sqrt[4]{(x^3+x)^3}}$,

k) $\left(\operatorname{arctg}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)\right)' = 1$,

l) $(x(\sin(\ln x) + \cos(\ln x)))' = 2\cos(\ln x)$.