Cvičení 5 - výsledky

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Příklad 1. $\lim_{x\to\infty} \frac{1}{x} = 0.$

Příklad 2.

$a)$ $\lim_{x\to 5} \frac{x+3}{7} = \frac{5+3}{7} = \frac{8}{7},$
$b)$ $\lim_{x\to 1} (2^x - 3^x) = 2^1 - 3^1 = -1,$
$c)$ $\lim_{x\to 0} \frac{\cos 2x + \sin 2x}{x+1} = \frac{\cos 0 + \sin 0}{0+1} = 1,$
$d)$ $\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \lim_{x\to 1} \frac{(x-1)\cdot(x+1)}{(x-1)\cdot(x^2+x+1)} = \lim_{x\to 1} \frac{x+1}{x^2+x+1} = \frac{1+1}{1+1+1} = \frac{2}{3},$
$e)$ $\lim_{x\to 0} \sqrt{x+1} - 2x + 6 = \sqrt{0+1} - 2\cdot 0 + 6 = 1 + 6 = 7,$
$f)$ $\lim_{x\to 3} \frac{x^2-6x+9}{81-x^4} = \lim_{x\to 3} \frac{(3-x)^2}{(9+x^2)\cdot(3-x)\cdot(3+x)} = \lim_{x\to 3} \frac{3-x}{(9+x^2)\cdot(3+x)} = \frac{3-3}{(9+9)\cdot(3+3)} = 0,$
$g)$ $\lim_{x\to 0} \operatorname{sign} x$ *nemá limitu,* $\lim_{x\to 0+} \operatorname{sign} x = 1, \lim_{x\to 0-} \operatorname{sign} x = -1,$
$h)$ $\lim_{x\to 1} 3x^2 + 6x - 5 = 3\cdot 1^2 + 6\cdot 1 - 5 = 4,$
$i)$ $\lim_{x\to 2} \frac{2x^2}{x^2+4} = \frac{2\cdot 2^2}{2^2+4} = \frac{8}{8} = 1.$

Příklad 3.

$a)$ $\lim_{x\to -\infty} \frac{2x^4-x^3+4}{5x^4+x^3+2} = \lim_{x\to -\infty} \frac{2\frac{x^4}{x^4} - \frac{x^3}{x^4} + \frac{4}{x^4}}{5\frac{x^4}{x^4} + \frac{x^3}{x^4} + \frac{2}{x^4}} = \frac{2-0+0}{5+0+0} = \frac{2}{5},$

$b)$ $\lim_{x\to\infty} \frac{2^{x+3}+4}{2^{x-1}+1} = \lim_{x\to\infty} \frac{\frac{2^{x+3}}{2^{x-1}} + \frac{4}{2^{x-1}}}{\frac{2^{x-1}}{2^{x-1}} + \frac{1}{2^{x-1}}} = \frac{2^4+0}{1+0} = 16,$

$c)$ $\lim_{x\to\infty} \frac{x^3+x^2}{x^2-1} = \lim_{x\to\infty} x \cdot \lim_{x\to\infty} \frac{\frac{x^2}{x^2} + \frac{x}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \lim_{x\to\infty} x \cdot \frac{1+0}{1-0} = \infty,$

$d)$ $\lim_{x\to\infty} \frac{5^x - \cdot 6^x}{6^x+1+2^x} = \lim_{x\to\infty} \frac{(\frac{5}{6})^x - (\frac{6}{6})^x}{(\frac{6}{6})^x + (\frac{1}{6})^x + (\frac{2}{6})^x} = \frac{0-1}{1+0+0} = -1,$

$e)$ $\lim_{x\to\infty} \frac{\sqrt{x^2-x+3}}{x} = \lim_{x\to\infty} \sqrt{\frac{x^2}{x^2} - \frac{1}{x} + \frac{3}{x^2}} = \sqrt{1-0+0} = 1,$

$f)$ $\lim_{x\to\infty} \sqrt{\frac{2x+3}{x-1}} = \lim_{x\to\infty} \sqrt{2 + \frac{5}{x-1}} = \sqrt{2+0} = \sqrt{2},$

$g)$ $\lim_{x\to\infty} (2x - \sqrt{4x^2+3x}) = \lim_{x\to\infty} (2x - \sqrt{4x^2+3x}) \cdot \frac{2x+\sqrt{4x^2+3x}}{2x+\sqrt{4x^2+3x}} = \lim_{x\to\infty} \frac{4x^2-(4x^2+3x)}{2x+\sqrt{4x^2+3x}} =$
$= \lim_{x\to\infty} \frac{-3x}{2x+\sqrt{4x^2+3x}} = \lim_{x\to\infty} \frac{-3\frac{x}{x}}{2\frac{x}{x} + \sqrt{4\frac{x^2}{x^2} + \frac{3}{x}}} = \frac{-3}{2+\sqrt{4+0}} = -\frac{3}{4},$

$h)$ $\lim_{x\to\infty} \frac{\sqrt{x+2}+3\sqrt{x^2-6}}{2x+1} = \lim_{x\to\infty} \frac{\sqrt{x+2}}{2x+1} + 3\cdot \lim_{x\to\infty} \frac{\sqrt{x^2-6}}{2x+1} =$
$= \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x^2} + \frac{2}{x^2}}}{2\frac{x}{x} + \frac{1}{x}} + 3\cdot \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{2}{x^2}}}{2\frac{x}{x} + \frac{1}{x}} = \frac{\sqrt{0+0}}{2+0} + 3\cdot \frac{\sqrt{1+0}}{2+0} = \frac{3}{2},$

$i)$ $\lim_{x\to\infty} \frac{-(x+1)^2}{(x+2)\cdot(x+3)} = \lim_{x\to\infty} \frac{-x^2-2x-1}{x^2+5x+6} = -1,$

$j)$ $\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \lim_{x\to\infty} \frac{\sqrt{\frac{x}{x}}}{\sqrt{\frac{x}{x} + \sqrt{\frac{x}{x^2} + \sqrt{\frac{x}{x^4}}}}} = \frac{\sqrt{1}}{\sqrt{1+\sqrt{0+\sqrt{0}}}} = 1,$

$k)$ $\lim_{x\to -\infty} \frac{\sqrt{x^2+14}+x}{\sqrt{x^2-2}+x} = \lim_{x\to -\infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{14}{x^2}} + \frac{x}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{2}{x^2}} + \frac{x}{x}} = \frac{\sqrt{1}+1}{\sqrt{1}+1} = 1,$

$l)$ $\lim_{x\to\infty} \frac{\sqrt[3]{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\sqrt[3]{\frac{x^2}{x^3} + \frac{1}{x^3}}}{\frac{x}{x} + \frac{1}{x}} = \frac{\sqrt{0+0}}{1+0} = 0.$

Příklad 4.

$a)$ $\lim_{x\to -2-} \frac{5x-6}{x^2+x-2} = -\infty,$ $\lim_{x\to -2+} \frac{5x-6}{x^2+x-2} = \infty,$

$b)$ $\lim_{x\to 1-} \frac{2-x^2}{x^2-1} = -\infty,$ $\lim_{x\to 1+} \frac{2-x^2}{x^2-1} = \infty,$

$c)$ $\lim_{x\to 4-} \frac{5x-2}{4-x} = \infty,$ $\lim_{x\to 4+} \frac{5x-2}{4-x} = -\infty.$