EE 261 The Fourier Transform and its Applications
Fall 2007
Problem Set Five   Due Wednesday, October 31

1. (30 points) *Evaluating integrals with the help of Fourier transforms* Evaluate the following integrals using Parseval's Theorem and one other method. (Yes, we expect you to evaluate the integral twice, and if you do it right you should get the same answer for both approaches (obviously)):

   (a)
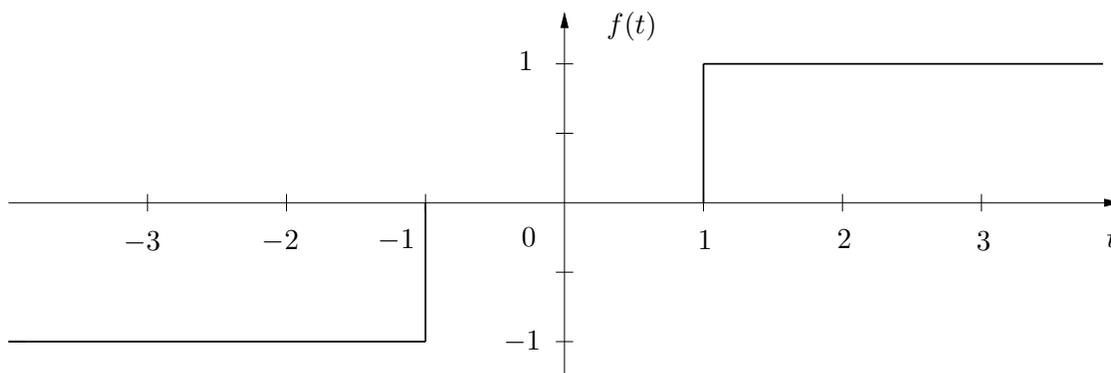 $$\int_{-\infty}^{\infty} \text{sinc}^4(t)\, dt$$

   (b)
   $$\int_{-\infty}^{\infty} \frac{2}{1+(2\pi t)^2}\, \text{sinc}(2t)\, dt$$
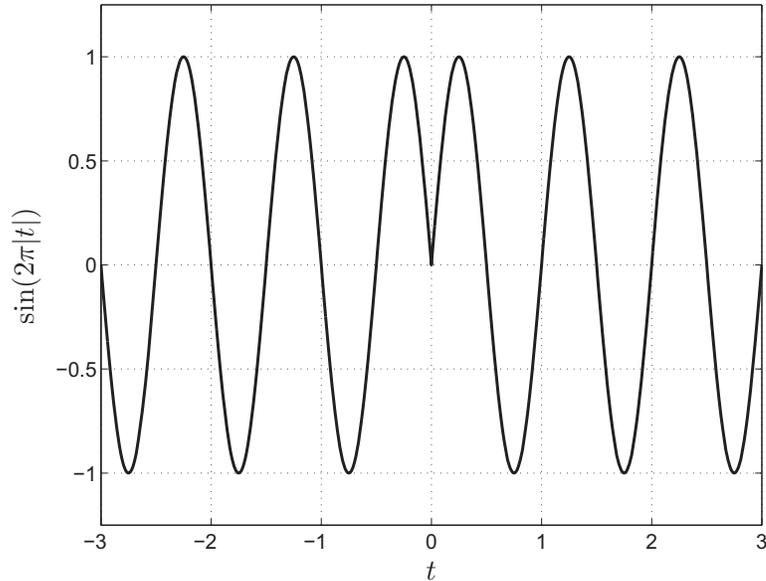
   (c)
   $$\int_{-\infty}^{\infty} t^2 \text{sinc}^4(t)\, dt$$

2. (20 points) *Generalized Fourier transforms*

   (a) Find the Fourier transform of the signal shown below.

   (b) Find the Fourier transform of the function $f(t) = \sin(2\pi|t|)$ plotted below. (Simplify your expression as *much* as possible.)

3. (20 points) *System identification*

   (a) Consider a filter with *real and even* transfer function $H(s)$. If the input to the filter is $cos(2\pi\nu t)$, show that the output is $H(\nu)cos(2\pi\nu t)$.

   (b) We shall use the result in part (a) to identify an unknown filter. Download **identme.m** and **transferfcn.m** from the class web page

   http://see.stanford.edu/see/materials/lsoftaee261/assignments.aspx

   The file **identme.m** is the filter which takes an input signal and produces an output. The file **transferfcn.m** stores the transfer function and is called by **identme.m**. Let the input be a sinusoid with some frequency ν Estimate $H(\nu)$ from the amplitude of the output. Sketch $H(s)$ by varying ν. What kind of filter is this? What is/are the cutoff frequency(ies)?
   [*Hint: (1) Try frequencies in the range 0 - 20 Hz. (2) Ignore the fact that the output signal is not a perfect sinusoid. This is because the signal is not infinite duration.*]

   (c) Plot $H(s)$ from **transferfcn.m**. Compare with part (b).

4. (10 points) Consider the signal

$$f(t) = 1 + \Lambda(3t) * III_{1/3}(t).$$

   Sketch the graph of $f(t)$, find the Fourier transform $\mathcal{F}f(s)$, and sketch its graph. Comment on what you see.

5. (10 points) *Sampling Images*

   Thomas and Lykomidis are chatting after class.
   Thomas: What's up Lykomidis? You look too skeptical today. Is everything ok?
   Lykomidis: Well . . . I was watching this movie yesterday and I am still thinking about it. . .
   Thomas: It must have been a great movie!
   Lykomidis: Actually, it was rather boring. However, I observed something which I cannot really explain. In one of the scenes, there was a fan in the background. It was a hot summer

day, so at some point, the guy in the movie turned the fan on. My problem is that the fan did not seem to rotate even after that point. That guy commented on how much nicer it was, after the fan had been turned on. I mean.. it was not the greatest movie ever, but I am pretty sure they would not have used a fan that was not functioning..

Thomas: You know, it is funny that you mention that. I had observed something similar a few years ago.

Brad, who was standing right behind them all this time, joins the conversation.

Brad: Well, maybe this can help you: What happens when you sample a periodic signal with a sampling period equal to the period of the signal? For what other sampling periods do you observe the same phenomenon?

Can you figure out what is happening? Why does a rotating fan appear stopped in the movie? You are further told that the particular fan has a three-blade propeller. How does that impact your answer?