

PRŮNIKY

Najděte průnik vektorových prostorů $U, W \subseteq V$.

1. $V = \mathbb{Z}_3^4$

Zadání.

(1)

$$U = \langle (2, 1, 1, 1), (1, 0, 0, 1), (1, 0, 0, 2), (1, 1, 1, 2) \rangle,$$

$$W = \langle (1, 0, 0, 2), (1, 0, 0, 0), (1, 2, 1, 0), (2, 2, 1, 1) \rangle.$$

(2)

$$U = \langle (2, 1, 2, 2), (0, 1, 0, 2), (1, 1, 2, 0), (0, 0, 1, 1) \rangle,$$

$$W = \langle (0, 2, 2, 2), (1, 0, 0, 2), (0, 1, 1, 1), (1, 0, 2, 1) \rangle.$$

(3)

$$U = \langle (0, 0, 0, 2), (2, 0, 0, 2), (1, 2, 2, 1), (0, 0, 0, 2) \rangle,$$

$$W = \langle (1, 2, 1, 2), (1, 2, 1, 0), (0, 1, 0, 2), (0, 2, 0, 1) \rangle.$$

(4)

$$U = \langle (2, 1, 0, 2), (1, 2, 0, 1), (1, 2, 2, 0), (0, 2, 2, 2) \rangle,$$

$$W = \langle (2, 2, 0, 2), (0, 2, 2, 1), (2, 2, 1, 1), (2, 1, 0, 2) \rangle.$$

(5)

$$U = \langle (1, 2, 1, 2), (2, 0, 0, 1), (1, 2, 0, 1), (2, 1, 2, 1) \rangle,$$

$$W = \langle (0, 0, 0, 0), (1, 0, 1, 0), (0, 0, 0, 1), (2, 0, 2, 0) \rangle.$$

(6)

$$U = \langle (1, 2, 2, 0), (1, 0, 1, 1), (2, 2, 0, 1), (1, 0, 1, 0) \rangle,$$

$$W = \langle (1, 0, 1, 1), (2, 2, 1, 2), (2, 2, 2, 0), (1, 2, 2, 0) \rangle.$$

(7)

$$U = \langle (1, 2, 1, 2), (0, 1, 2, 0), (2, 2, 2, 1), (0, 1, 1, 0) \rangle,$$

$$W = \langle (2, 2, 0, 1), (2, 2, 1, 0), (1, 2, 2, 2), (0, 1, 2, 0) \rangle.$$

(8)

$$U = \langle (1, 2, 2, 0), (2, 2, 0, 2), (1, 1, 2, 2), (1, 2, 2, 0) \rangle,$$

$$W = \langle (0, 0, 2, 1), (0, 1, 0, 2), (0, 1, 0, 1), (0, 2, 0, 1) \rangle.$$

(9)

$$U = \langle (1, 0, 1, 0), (0, 1, 1, 0), (2, 2, 0, 2), (0, 2, 2, 0) \rangle,$$

$$W = \langle (0, 1, 0, 2), (0, 1, 0, 2), (2, 2, 1, 2), (1, 2, 0, 0) \rangle.$$

(10)

$$U = \langle (0, 1, 1, 0), (2, 2, 2, 1), (0, 2, 2, 2), (2, 2, 2, 0) \rangle,$$

$$W = \langle (2, 1, 0, 2), (2, 0, 2, 1), (0, 2, 2, 0), (2, 1, 0, 0) \rangle.$$

(11)

$$U = \langle (0, 2, 0, 0), (2, 1, 1, 0), (1, 0, 2, 1), (1, 1, 2, 2) \rangle ,$$

$$W = \langle (2, 0, 0, 2), (1, 1, 1, 0), (1, 1, 2, 1), (0, 2, 0, 2) \rangle .$$

(12)

$$U = \langle (2, 0, 2, 0), (1, 2, 1, 0), (2, 0, 1, 2), (0, 2, 0, 0) \rangle ,$$

$$W = \langle (1, 0, 1, 2), (2, 0, 0, 2), (1, 0, 1, 2), (1, 0, 0, 1) \rangle .$$

(13)

$$U = \langle (0, 1, 0, 2), (1, 2, 1, 0), (1, 0, 1, 2), (0, 1, 1, 2) \rangle ,$$

$$W = \langle (2, 0, 1, 1), (0, 2, 0, 2), (2, 1, 2, 1), (2, 1, 0, 0) \rangle .$$

(14)

$$U = \langle (0, 0, 1, 2), (2, 0, 1, 2), (1, 0, 0, 2), (1, 0, 0, 0) \rangle ,$$

$$W = \langle (2, 2, 0, 0), (0, 0, 0, 2), (2, 1, 0, 2), (1, 0, 0, 0) \rangle .$$

(15)

$$U = \langle (2, 0, 1, 2), (2, 1, 0, 1), (0, 0, 2, 2), (0, 0, 0, 0) \rangle ,$$

$$W = \langle (1, 0, 0, 2), (1, 1, 1, 0), (1, 0, 0, 2), (2, 1, 1, 2) \rangle .$$

(16)

$$U = \langle (1, 2, 1, 1), (2, 1, 1, 2), (1, 0, 1, 2), (1, 0, 0, 2) \rangle ,$$

$$W = \langle (2, 1, 2, 0), (0, 2, 0, 2), (2, 2, 2, 1), (1, 0, 1, 1) \rangle .$$

Řešení.

(1)

$$\langle (1, 0, 0, 0), (0, 0, 0, 1) \rangle .$$

(2)

$$\langle (1, 0, 0, 2), (0, 0, 1, 1) \rangle .$$

(3)

$$\langle (1, 1, 1, 0), (0, 0, 0, 1) \rangle .$$

(4)

$$\langle (1, 0, 0, 1), (0, 1, 0, 0) \rangle .$$

(5)

$$\langle (1, 0, 1, 0) \rangle .$$

(6)

$$\langle (1, 0, 1, 1), (0, 1, 2, 1) \rangle .$$

(7)

$$\langle (1, 0, 1, 2), (0, 1, 2, 0) \rangle .$$

(8)

$$\langle (0, 1, 0, 1), (0, 0, 1, 2) \rangle .$$

$$(9) \quad \langle (1, 0, 0, 2), (0, 1, 0, 2) \rangle.$$

$$(10) \quad \langle (0, 1, 1, 0), (0, 0, 0, 1) \rangle.$$

$$(11) \quad \langle (1, 0, 2, 0), (0, 1, 0, 1) \rangle.$$

$$(12) \quad \langle (0, 0, 1, 1) \rangle.$$

$$(13) \quad \langle (1, 0, 2, 2), (0, 1, 2, 2) \rangle.$$

$$(14) \quad \langle (1, 0, 0, 0), (0, 0, 0, 1) \rangle.$$

$$(15) \quad \langle (1, 0, 0, 2), (0, 1, 1, 1) \rangle.$$

$$(16) \quad \langle (0, 1, 0, 1) \rangle.$$

$$2. V = \mathbb{Z}_5^4$$

Zadání.

$$(1) \quad U = \langle (0, 1, 2, 3), (2, 1, 4, 0), (1, 3, 4, 3), (2, 0, 3, 1) \rangle, \\ W = \langle (0, 4, 4, 3), (0, 1, 4, 3), (4, 3, 0, 3), (3, 0, 1, 3) \rangle.$$

$$(2) \quad U = \langle (2, 4, 1, 0), (1, 3, 0, 0), (1, 3, 4, 0), (0, 2, 4, 0) \rangle, \\ W = \langle (2, 1, 4, 4), (3, 0, 2, 1), (4, 3, 0, 2), (1, 4, 1, 4) \rangle.$$

$$(3) \quad U = \langle (0, 2, 0, 4), (0, 3, 1, 2), (0, 4, 4, 2), (1, 0, 0, 4) \rangle, \\ W = \langle (4, 4, 1, 1), (3, 0, 2, 0), (4, 1, 4, 2), (0, 0, 0, 0) \rangle.$$

$$(4) \quad U = \langle (1, 4, 2, 0), (3, 2, 2, 1), (3, 4, 4, 1), (1, 1, 1, 2) \rangle, \\ W = \langle (1, 3, 0, 0), (1, 2, 0, 4), (2, 2, 2, 4), (2, 3, 2, 0) \rangle.$$

$$(5) \quad U = \langle (1, 2, 1, 1), (1, 2, 1, 0), (2, 0, 2, 4), (1, 3, 1, 1) \rangle, \\ W = \langle (4, 2, 2, 4), (4, 3, 1, 2), (0, 0, 3, 2), (2, 0, 0, 1) \rangle.$$

(6)

$$U = \langle (0, 2, 2, 0), (3, 2, 4, 2), (2, 1, 0, 3), (2, 1, 1, 3) \rangle ,$$

$$W = \langle (3, 1, 2, 3), (1, 2, 0, 1), (4, 3, 1, 4), (2, 3, 2, 1) \rangle .$$

(7)

$$U = \langle (1, 3, 0, 3), (1, 4, 3, 2), (1, 0, 1, 1), (1, 0, 0, 1) \rangle ,$$

$$W = \langle (1, 4, 0, 3), (2, 4, 2, 4), (4, 3, 2, 1), (1, 0, 0, 4) \rangle .$$

(8)

$$U = \langle (2, 3, 1, 4), (0, 2, 4, 2), (4, 4, 3, 1), (1, 2, 4, 3) \rangle ,$$

$$W = \langle (1, 1, 1, 2), (1, 2, 2, 1), (4, 3, 3, 4), (4, 1, 1, 2) \rangle .$$

(9)

$$U = \langle (2, 0, 2, 2), (0, 3, 4, 3), (0, 2, 4, 4), (4, 3, 3, 2) \rangle ,$$

$$W = \langle (3, 0, 2, 4), (1, 1, 3, 4), (1, 3, 4, 3), (0, 0, 4, 1) \rangle .$$

(10)

$$U = \langle (4, 2, 0, 0), (1, 0, 3, 3), (3, 3, 3, 2), (1, 3, 3, 4) \rangle ,$$

$$W = \langle (3, 3, 1, 0), (0, 2, 3, 1), (3, 3, 0, 2), (3, 1, 0, 0) \rangle .$$

(11)

$$U = \langle (4, 0, 2, 4), (2, 4, 1, 3), (0, 1, 0, 4), (4, 1, 4, 3) \rangle ,$$

$$W = \langle (0, 4, 3, 2), (2, 2, 3, 4), (3, 3, 0, 1), (1, 0, 4, 4) \rangle .$$

(12)

$$U = \langle (1, 3, 0, 2), (1, 0, 2, 3), (4, 1, 1, 0), (2, 3, 3, 0) \rangle ,$$

$$W = \langle (0, 4, 1, 2), (1, 4, 1, 2), (3, 3, 4, 0), (4, 4, 4, 1) \rangle .$$

(13)

$$U = \langle (4, 2, 1, 1), (4, 3, 3, 4), (2, 0, 1, 0), (1, 4, 2, 0) \rangle ,$$

$$W = \langle (4, 4, 2, 0), (1, 2, 3, 4), (4, 3, 4, 2), (0, 1, 0, 4) \rangle .$$

(14)

$$U = \langle (2, 1, 0, 0), (0, 3, 2, 0), (1, 1, 1, 4), (1, 3, 4, 4) \rangle ,$$

$$W = \langle (3, 4, 4, 2), (3, 1, 1, 1), (2, 0, 4, 1), (1, 1, 4, 0) \rangle .$$

(15)

$$U = \langle (3, 3, 3, 1), (1, 1, 1, 0), (2, 3, 3, 1), (1, 2, 2, 3) \rangle ,$$

$$W = \langle (3, 4, 3, 3), (2, 1, 2, 2), (4, 4, 2, 4), (1, 0, 3, 0) \rangle .$$

(16)

$$U = \langle (0, 0, 1, 3), (2, 2, 1, 0), (3, 1, 1, 4), (2, 2, 2, 3) \rangle ,$$

$$W = \langle (0, 0, 4, 0), (3, 2, 2, 0), (2, 4, 3, 2), (0, 0, 0, 0) \rangle .$$

Řešení.

(1)

$$\langle (1, 0, 0, 2), (0, 1, 0, 0) \rangle .$$

(2)

$$\langle (1, 0, 1, 0), (0, 1, 1, 0) \rangle .$$

(3)

$$\langle (1, 4, 0, 2), (0, 0, 1, 1) \rangle .$$

(4)

$$\langle (1, 0, 0, 2), (0, 1, 1, 0) \rangle .$$

(5)

$$\langle (1, 0, 1, 2), (0, 1, 0, 2) \rangle .$$

(6)

$$\langle (1, 0, 0, 4), (0, 0, 1, 0) \rangle .$$

(7)

$$\langle (1, 0, 2, 1), (0, 1, 3, 4) \rangle .$$

(8)

$$\langle (1, 0, 0, 0), (0, 0, 0, 1) \rangle .$$

(9)

$$\langle (1, 0, 0, 2), (0, 0, 1, 4) \rangle .$$

(10)

$$\langle (1, 4, 0, 2), (0, 0, 1, 3) \rangle .$$

(11)

$$\langle (1, 3, 0, 3), (0, 0, 1, 0) \rangle .$$

(12)

$$\langle (1, 0, 1, 3), (0, 1, 4, 3) \rangle .$$

(13)

$$\langle (1, 2, 0, 0), (0, 0, 1, 3) \rangle .$$

(14)

$$\langle (1, 0, 1, 3), (0, 1, 1, 2) \rangle .$$

(15)

$$\langle (1, 0, 0, 2), (0, 1, 1, 2) \rangle .$$

(16)

$$\langle (1, 0, 4, 2), (0, 1, 2, 2) \rangle .$$

3. $V = \mathbb{Z}_7^5$ **Zadání.**

(1)

$$U = \langle (3, 3, 5, 0, 2), (6, 1, 1, 4, 5), (4, 1, 4, 4, 2), (0, 4, 3, 1, 2) \rangle,$$

$$W = \langle (4, 4, 0, 5, 6), (4, 3, 6, 2, 2), (1, 4, 3, 3, 6), (0, 3, 3, 1, 3) \rangle.$$

(2)

$$U = \langle (2, 6, 1, 0, 3), (5, 3, 4, 1, 4), (4, 5, 5, 3, 6), (6, 5, 2, 3, 1) \rangle,$$

$$W = \langle (6, 1, 0, 0, 2), (6, 0, 0, 0, 4), (0, 5, 3, 3, 3), (2, 3, 5, 5, 3) \rangle.$$

(3)

$$U = \langle (1, 2, 1, 3, 0), (5, 4, 6, 6, 5), (6, 2, 3, 2, 1), (1, 2, 1, 5, 3) \rangle,$$

$$W = \langle (2, 1, 2, 2, 5), (5, 2, 5, 5, 3), (6, 5, 0, 0, 1), (0, 6, 2, 0, 3) \rangle.$$

(4)

$$U = \langle (5, 3, 5, 5, 1), (0, 5, 2, 2, 4), (4, 2, 3, 0, 2), (5, 3, 3, 0, 3) \rangle,$$

$$W = \langle (4, 5, 6, 5, 1), (4, 2, 1, 4, 1), (3, 6, 3, 3, 2), (0, 1, 6, 5, 2) \rangle.$$

(5)

$$U = \langle (6, 1, 6, 3, 0), (2, 0, 2, 4, 5), (1, 6, 1, 0, 1), (1, 3, 5, 2, 4) \rangle,$$

$$W = \langle (6, 5, 3, 2, 2), (6, 0, 5, 3, 2), (1, 3, 5, 5, 5), (0, 1, 1, 3, 0) \rangle.$$

(6)

$$U = \langle (0, 5, 4, 0, 5), (5, 2, 4, 2, 5), (3, 4, 3, 3, 2), (2, 6, 5, 0, 0) \rangle,$$

$$W = \langle (1, 0, 6, 5, 6), (4, 4, 4, 1, 2), (1, 3, 1, 1, 6), (4, 2, 0, 6, 6) \rangle.$$

(7)

$$U = \langle (4, 4, 5, 5, 4), (2, 1, 0, 1, 2), (0, 5, 4, 4, 2), (5, 2, 3, 0, 3) \rangle,$$

$$W = \langle (5, 2, 5, 0, 1), (2, 0, 5, 3, 1), (2, 1, 3, 1, 0), (3, 0, 1, 1, 2) \rangle.$$

(8)

$$U = \langle (6, 6, 1, 2, 2), (0, 2, 3, 1, 4), (5, 1, 4, 6, 2), (1, 3, 1, 2, 4) \rangle,$$

$$W = \langle (1, 1, 4, 6, 4), (5, 0, 5, 3, 5), (3, 5, 4, 0, 2), (2, 2, 2, 6, 5) \rangle.$$

(9)

$$U = \langle (5, 5, 6, 4, 3), (6, 3, 6, 2, 5), (1, 6, 4, 2, 1), (4, 6, 3, 2, 6) \rangle,$$

$$W = \langle (3, 6, 2, 4, 2), (4, 5, 3, 1, 1), (5, 1, 6, 0, 4), (6, 1, 2, 6, 0) \rangle.$$

(10)

$$U = \langle (5, 3, 2, 1, 1), (4, 2, 3, 0, 2), (5, 2, 4, 3, 4), (6, 5, 1, 4, 4) \rangle,$$

$$W = \langle (6, 5, 3, 0, 4), (0, 0, 2, 6, 6), (3, 6, 6, 5, 6), (0, 1, 3, 5, 6) \rangle.$$

(11)

$$U = \langle (0, 0, 2, 0, 2), (2, 0, 6, 5, 0), (4, 4, 6, 1, 0), (1, 1, 4, 6, 4) \rangle,$$

$$W = \langle (2, 4, 6, 3, 4), (4, 3, 3, 4, 6), (1, 4, 1, 3, 0), (3, 6, 2, 1, 6) \rangle.$$

(12)

$$U = \langle (5, 6, 0, 0, 2), (1, 0, 3, 4, 1), (5, 5, 4, 5, 1), (6, 5, 6, 1, 4) \rangle, \\ W = \langle (0, 2, 2, 0, 3), (0, 6, 4, 3, 2), (3, 1, 5, 2, 1), (3, 0, 5, 5, 5) \rangle.$$

(13)

$$U = \langle (5, 6, 4, 1, 3), (2, 6, 4, 1, 5), (5, 1, 3, 6, 0), (5, 5, 6, 3, 4) \rangle, \\ W = \langle (0, 4, 6, 6, 6), (2, 1, 2, 1, 5), (5, 5, 3, 2, 5), (1, 0, 2, 2, 5) \rangle.$$

(14)

$$U = \langle (2, 1, 2, 4, 4), (4, 5, 5, 6, 0), (5, 2, 4, 1, 1), (1, 6, 4, 4, 6) \rangle, \\ W = \langle (5, 2, 1, 0, 6), (0, 5, 3, 4, 4), (1, 2, 1, 1, 2), (6, 0, 0, 2, 6) \rangle.$$

(15)

$$U = \langle (3, 5, 2, 2, 5), (6, 3, 5, 3, 3), (5, 4, 3, 6, 5), (1, 1, 6, 0, 6) \rangle, \\ W = \langle (5, 4, 1, 3, 6), (1, 3, 0, 4, 4), (1, 5, 2, 2, 0), (6, 0, 0, 0, 6) \rangle.$$

(16)

$$U = \langle (4, 2, 2, 2, 3), (6, 4, 6, 2, 0), (2, 0, 3, 4, 6), (0, 1, 6, 0, 5) \rangle, \\ W = \langle (6, 5, 3, 1, 6), (4, 2, 2, 4, 2), (1, 1, 5, 4, 2), (4, 2, 3, 3, 1) \rangle.$$

Řešení.

(1)

$$\langle (1, 5, 4, 3, 4) \rangle.$$

(2)

$$\langle (1, 0, 0, 0, 3), (0, 1, 2, 2, 2) \rangle.$$

(3)

$$\langle (1, 0, 6, 6, 6), (0, 1, 1, 3, 2) \rangle.$$

(4)

$$\langle (1, 0, 4, 0, 3), (0, 1, 2, 0, 1), (0, 0, 0, 1, 5) \rangle.$$

(5)

$$\langle (1, 0, 2, 5, 5), (0, 1, 1, 0, 0) \rangle.$$

(6)

$$\langle (1, 0, 0, 0, 1), (0, 1, 0, 2, 1), (0, 0, 1, 0, 2) \rangle.$$

(7)

$$\langle (1, 0, 0, 5, 0), (0, 1, 0, 5, 0), (0, 0, 1, 0, 0), (0, 0, 0, 0, 1) \rangle.$$

(8)

$$\langle (1, 0, 0, 4, 1), (0, 1, 0, 5, 1), (0, 0, 1, 4, 1) \rangle.$$

(9)

$$\langle (1, 0, 4, 0, 5), (0, 1, 5, 5, 3) \rangle.$$

(10)

$$\langle (1, 0, 1, 3, 2), (0, 1, 3, 1, 4) \rangle .$$

(11)

$$\langle (0, 1, 6, 6, 6) \rangle .$$

(12)

$$\langle (1, 0, 0, 0, 5), (0, 1, 0, 0, 2), (0, 0, 1, 0, 3), (0, 0, 0, 1, 2) \rangle .$$

(13)

$$\langle (1, 0, 0, 0, 5), (0, 1, 0, 4, 3), (0, 0, 1, 3, 6) \rangle .$$

(14)

$$\langle (1, 0, 1, 5, 4), (0, 1, 3, 5, 1) \rangle .$$

(15)

$$\langle (1, 5, 4, 2, 1) \rangle .$$

(16)

$$\langle (1, 0, 1, 0, 1), (0, 1, 4, 4, 1) \rangle .$$