Písemka Sample test 2017

Příklad 1 (8 points)**.** There are 12 white balls and 8 black balls in the urn. Calculate:

(a) The probability that in the third round the black ball is drawn if we draw **without replacement**?

(b) What is the mean waiting time for black ball (expected round of first black ball drawn) if we draw taháme-li **without replacement**

(c) The probability that white ball was drawn in the first round given (conditionad by) the fact that black ball was drawn in the second round. Consider both drawith: **with replacement** and **without replacement**.

(d)* Consider drawing **with replacement.** The game ends when black ball is drawn immediately after white ball. What is the nxpected number of white balls drawn before the end of the game? (Is the game possibly endless?)

Příklad 2 (3 points)**.** Formulate and proof the inclusion/exclusion principle.

Příklad 3 (5 points)**.** X foolows exponential distribution with density $f(x) = \lambda e^{-\lambda x}\chi[x > 0]$. Define $Y := [X]$ (i.e. Y is the integer part of X). Find $P(X = k)$ for $k = 0, 1, \ldots$. What is the distribution of Y (including the parameter)?

Příklad 4 (8 points)**.** Anička likes gardening. She planted 100 seeds. She knows that the probability of germination is only 70 %. After succesful germination the small plants must be individually separated to pots (one plant to one pot).

(a) How many pots should Anička buy to have pot for each plant with probability at least 0.95?

(b) The plant survives two weeks with probability 0.8. What is the expected number of surviving plants (from 100 seeds)?

(c) The surviving plants must be watered by 50 ml of water daily. What is the lower estimate of water need for all plants during one week with probability 0.95?

(Use limit theorems and the fact that $\Phi(1, 65) = 0.95$.)

Příklad 5 (6 points)**.** Define random vector, explain and calculate (it is enough for bivariate r.v.).

(a) Cummulative distribution function of bivariate random vector and its properties.

(b) Marginal distribution and its relation to joint distribution. Characterisation of independence.

(c) Covariance and its properties.

(d)* Consider two independent **tetrahedral, i.e. with four faces** dice throws. Find the correlation between the result of first throw and the sum of the results of both throws.

Příklad 6 (6 points)**.** Formulate Slutsky theorem. Consider two random samples $X_1, X_2, \ldots, X_n$ a $Y_1, Y_2, \ldots, Y_n$ from unknown distributions. Use Centrla limit theorem and Slutsky theorem to find (asymptotic) interval estimate of $\mathrm{E}X - \mathrm{E}Y$, the difference of mean values. What are the assumptions of used theorems?

Remarks: Each problem is appointed by given number of points, 36 in total. The minimum number of points needed for succesful exam is 19. Questions with star mean two additional points.

The correct result need not to be simple value or function, it may be infinite series or integral. Try to get as compact and explicite form as possible. Infinite series which you cannot sum up may be correct result.

Each question and subquestion should be clearly indicated. Try to write legibly. **Please sign all submitted sheets of paper and write their total number**