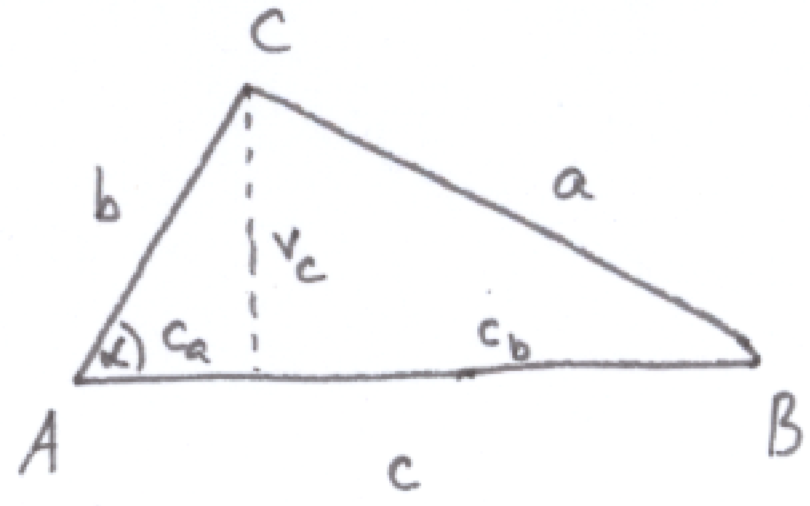
Héronův vzorec



(1) $S_\Delta = \frac{1}{2}\, c\cdot v_c$ kosinová věta $v_c = ?$

• $\sin\alpha = \frac{v_c}{b} \Rightarrow v_c = b\cdot\sin\alpha$

$v_c^2 = b^2\sin^2\alpha = b^2(1-\cos^2\alpha)$

$$S_\Delta = \frac{1}{2}\, c\cdot b\cdot\sin\alpha = \frac{1}{2}\, c\cdot b\sqrt{1-\cos^2\alpha} =$$

• $\cos\alpha$ vyjádříme z kosinové věty

$a^2 = b^2+c^2-2bc\cdot\cos\alpha$

$2bc\cos\alpha = b^2+c^2-a^2$

$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$

$$= \frac{1}{2}\, c\cdot b\cdot\sqrt{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2} =$$

$$= \frac{1}{4}\sqrt{4b^2c^2-(b^2+c^2-a^2)^2} =$$

$$= \frac{1}{4}\sqrt{[2bc+b^2+c^2-a^2]\cdot[2bc-b^2-c^2+a^2]} =$$

$$= \frac{1}{4}\sqrt{[(b+c)^2-a^2]\cdot[a^2-(b-c)^2]} =$$

$$= \frac{1}{4}\sqrt{(b+c+a)\underbrace{(b+c-a)}_{a+b+c-2a}\cdot\underbrace{(a+b-c)}_{a+b+c-2c}\underbrace{(a-b+c)}_{a+b+c-2b}} = \sqrt{\frac{\sigma}{2}\cdot\frac{\sigma-2a}{2}\cdot\frac{\sigma-2b}{2}\cdot\frac{\sigma-2c}{2}} =$$

$s := \frac{\sigma}{2}$

$$= \boxed{\sqrt{s\cdot(s-a)(s-b)\cdot(s-c)} = S_\Delta}$$

(2) řešení rovnice

$c = c_a + c_b \overset{\text{Pýthag.}}{\Rightarrow} c = \sqrt{b^2-v_c^2} + \sqrt{a^2-v_c^2}$

$c - \sqrt{a^2-v_c^2} = \sqrt{b^2-v_c^2} \quad /^2$

$c^2+a^2-\cancel{v_c^2}-2c\sqrt{a^2-v_c^2} = b^2-\cancel{v_c^2}$

$c^2+a^2-b^2 = 2c\sqrt{a^2-v_c^2} \quad /^2$

$(c^2+a^2-b^2)^2 = 4c^2(a^2-v_c^2)$

$(c^2+a^2-b^2)^2 = 4a^2c^2-4c^2v_c^2$

$S_\Delta = \frac{1}{2}\, c v_c$

$c\cdot v_c = 2S_\Delta$

$$(c^2+a^2-b^2)^2 - 4a^2c^2 = -4\cdot 4S_\Delta^2$$

$$16S_\Delta^2 = 4a^2c^2 - (c^2+a^2-b^2)^2$$

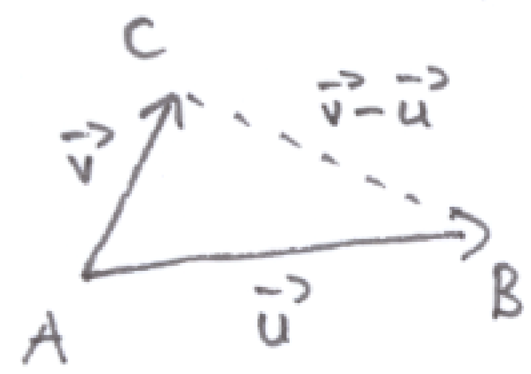
$$16S_\Delta^2 = [2ac + (c^2+a^2-b^2)]\cdot[2ac-(c^2+a^2-b^2)]$$

$$[(a+c)^2-b^2]\cdot[b^2-(a-c)^2]$$

$$16S_\Delta^2 = [(a+c+b)(a+c-b)]\cdot[(a-c+b)(b+c-a)]$$

$$S_\Delta^2 = s\cdot(s-a)(s-b)(s-c) \Rightarrow S_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

③ analyticky

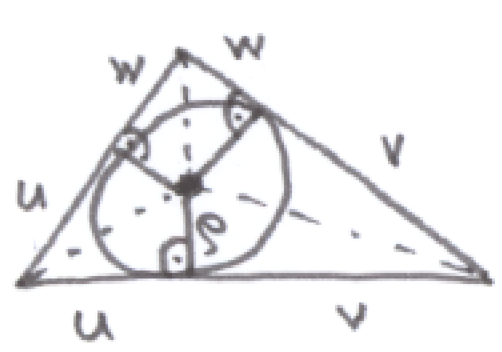


$$S_\Delta = \frac12\left|[\vec u,\vec v]\right| = \frac12\sqrt{G(\vec u,\vec v)} =$$

$$= \frac12\sqrt{\begin{vmatrix} \vec u\cdot\vec u & \vec u\cdot\vec v\\ \vec u\cdot\vec v & \vec v\cdot\vec v\end{vmatrix}} =$$

$$= \frac12\sqrt{\|\vec u\|^2\cdot\|\vec v\|^2 - (\vec u\cdot\vec v)^2} = \dots$$

④ poloměr kružnice vepsané



$$u+v+w = \frac{\sigma}{2} = s$$

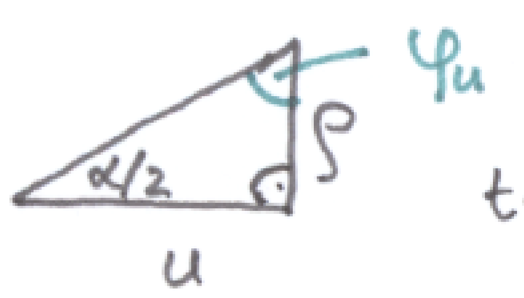
$$S_\Delta = \sum_{i=1}^{6} S_i = 2S_1+2S_3+2S_6 = 2\cdot\frac{u\cdot\rho}{2} + 2\cdot\frac{v\cdot\rho}{2} + 2\cdot\frac{w\cdot\rho}{2}$$

6 pravoúhlých Δ, vždy 2 shodné

$$= \rho(u+v+w) = \rho\cdot\frac{\sigma}{2} = \rho\cdot s$$

$$T_1\cong T_2 \quad T_3\cong T_4 \quad T_5\cong T_6$$

$$\boxed{S_\Delta = \rho\cdot s}$$



$$\mathrm{tg}\,\varphi_u = \frac{u}{\rho}$$

$$u = \mathrm{tg}\,\varphi_u\cdot\rho$$

$$\underbrace{u+v}_{a}+w = s \Rightarrow w = s-a = \rho\cdot\mathrm{tg}\,\varphi_w$$

$$u+\underbrace{v+w}_{c} = s \Rightarrow u = s-c = \rho\cdot\mathrm{tg}\,\varphi_u$$

$$\underbrace{u}_{}+v+\underbrace{w}_{b} = s \Rightarrow v = s-b = \rho\cdot\mathrm{tg}\,\varphi_v$$

$$S_\Delta = \rho\cdot s = \rho\cdot(u+v+w) = \rho\cdot(\rho\,\mathrm{tg}\,\varphi_u + \rho\,\mathrm{tg}\,\varphi_v + \rho\,\mathrm{tg}\,\varphi_w) = \rho^2(\mathrm{tg}\,\varphi_u+\mathrm{tg}\,\varphi_v+\mathrm{tg}\,\varphi_w)$$

$$= \rho^2\cdot(\mathrm{tg}\,\varphi_u\cdot\mathrm{tg}\,\varphi_v\cdot\mathrm{tg}\,\varphi_w) = \rho^2\cdot\frac{u}{\rho}\cdot\frac{v}{\rho}\cdot\frac{w}{\rho} = \frac{1}{\rho}uvw =$$

$$= \frac{1}{\rho}(s-a)(s-b)(s-c) \Rightarrow \rho^2 = \frac{1}{s}(s-a)(s-b)(s-c) \Rightarrow S_\Delta = s\cdot\sqrt{\rho^2}$$

$$\rho^2 = \frac{1}{s}(s-a)(s-b)(s-c)$$

$$S_\Delta = s \cdot \sqrt{\rho^2} = s \cdot \sqrt{\frac{1}{s}(s-a)(s-b)(s-c)} = \underline{\underline{\sqrt{s(s-a)(s-b)(s-c)}}}$$

L/ $\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma = \operatorname{tg}\alpha \cdot \operatorname{tg}\beta \cdot \operatorname{tg}\gamma$

pokud $\alpha + \beta + \gamma = 180^\circ$ a tg je pro α, β, γ i součty každých dvou definován a jsou def. i výrazy

$$\operatorname{tg}(\alpha+\beta+\gamma) = \underbrace{\operatorname{tg} 180^\circ}_{0}$$

$$\frac{\operatorname{tg}(\alpha+\beta) + \operatorname{tg}\gamma}{1 - \operatorname{tg}(\alpha+\beta) \cdot \operatorname{tg}\gamma} = 0 \quad \Rightarrow \quad \operatorname{tg}(\alpha+\beta) + \operatorname{tg}\gamma = 0$$

$$\frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta} + \operatorname{tg}\gamma = 0$$

$$\frac{\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma - \operatorname{tg}\alpha \operatorname{tg}\beta \operatorname{tg}\gamma}{1 - \operatorname{tg}\alpha \operatorname{tg}\beta} = 0$$

$$\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta \cdot \operatorname{tg}\gamma = 0$$

... cbd.