Numerical solution of IVP (ODE)

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Quiz # 5

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Question #2

Let us consider the 4-stage Runge-Kutta method of order 4 ($s = p = 4$).
We can estimate the local error by the following techniques:

(i) half-step size method,

(ii) Runge-Kutta-Fehlberg method ($s = 6$).

What is the ratio between the amount of computational time of these two techniques (i), (ii) necessary for the performing of one time step?

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- half-step size method $4 + 2 \cdot 4 = 12$
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Which size of h^{opt} gives us the the estimate of the local error equal to TOL?

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