Numerical solution of IVP (ODE)

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Quiz # 4

Question #1

Let us consider the **initial value problem**

- $y'(x) = f(x, y(x)), \qquad y(a) = \eta.$ (P)

Implicit Euler method: $y_{k+1} = y_k + h_k f(x_{k+1}, y_{k+1})$

The implicit Euler method is unconditionally stable which means:

(A) the stability of the method is guaranteed for any $h_k > 0$ and any problem (P)

(B) the stability of the method is guaranteed for any $h_k > 0$ and any stable problem (P)

(C) the stability of the method is guaranteed for any $h_k > 0$ and any stable linear problem (P)

The unconditional stability was derived for linear problems.

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Let us consider the **initial value problem**

- $y'(x) = f(x, y(x)), \qquad y(a) = \eta,$

which is stiff.

It is advantageous to solve the stiff problem (P) by the implicit Euler method (in comparison to the explicit one) since

(A) it is more accurate

(B) since it is possible to choose larger time steps

(C) it is less sensitive to the rounding errors due to machine arithmetic

(D) it is more efficient

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larger stability $\Rightarrow$ larger time steps $\Rightarrow$ small number of time steps on (a, b) $\Rightarrow$ save computational time

Question #3

Let us consider the following the ODE: find $y : (a, b) \to \mathbb{R}^2$ such that

- $y'(x) = f(x, y(x))$,

the eigenvalues of the Jacobian $\{\frac{\partial f_i}{\partial y_j}\}_{i,j=1}^2$ are $\lambda_1 = -100$, $\lambda_2 = -120$.

It is advantageous to solve this problem (P) by

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problem is not stiff $\Rightarrow$ explicit method is more efficient