Numerical solution of IVP (ODE)

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Quiz # 2

Question #1

Let us consider the following **initial value problem** given by the ODE: find $y : (a, b) \to \mathbb{R}^m$ such that

- $y'(x) = f(x, y(x))$,
- $y(a) = \eta$,

where $f : [a, b] \times \mathbb{R}^m \to \mathbb{R}^m$ and $\eta \in \mathbb{R}^m$ are given.

We say that this problem is stable if

(A) $\frac{\partial f(x,y)}{\partial x} < 0$,

(B) all eigenvalues of $\{\frac{\partial f_i}{\partial y_j}\}_{i,j=1}^N$ are negative,

(C) all eigenvalues of $\{\frac{\partial f_i}{\partial y_j}\}_{i,j=1}^N$ have negative real part,

(D) all eigenvalues of $\{\frac{\partial f_i}{\partial y_j}\}_{i,j=1}^N$ have positive real part,

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Question #2

Let us consider the ODE: find $y : (0, 1) \to \mathbb{R}$ such that

- $y'(x) = -2y + 3x$,
- $y(0) = 1$,

where $f : [a, b] \times \mathbb{R}^m \to \mathbb{R}^m$ and $\eta \in \mathbb{R}^m$ are given.

this problem is

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Jacobian $= \frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial y}[-2y + 3x] = -2 < 0 \qquad \Rightarrow$ stable

Question #3

Let us consider the following the ODE: find $y : (a, b) \to \mathbb{R}^3$ such that

- $y'(x) = f(x, y(x))$.

Let the eigenvalues of the Jacobian $\{\frac{\partial f_i}{\partial y_j}\}_{i,j=1}^3$ are $\lambda_1 = -1$, $\lambda_2 = -1000 + 20i$ and $\lambda_3 = -1000 - 20i$.

Which sentences are correct?

(A) This problem *is stiff* since all eigenvalues have negative real parts with very different magnitudes.

(B) This problem *is not stiff* since the system is stable.

(C) This problem *is stiff* since at least one eigenvalue has the imaginary part.

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