Agafonov, Serguei: Integrable four-component systems of conservation laws and linear congruences in P^5

Abstract:

We propose a differential-geometric classification of the four-component hyperbolic systems of conservation laws which satisfy the following properties: (a) they do not possess Riemann invariants; (b) they are linearly degenerate; (c) their rarefaction curves are rectilinear; (d) the cross-ratio of the four characteristic speeds is harmonic. This turned out to provide a classification of projective congruences in P^5 whose developable surfaces are planar pencils of lines, each of these lines cutting the focal variety at points forming an harmonic quadruplet. Symmetry properties and the connection of these congruences to Cartan's isoparametric hypersurfaces are discussed.

Agricola, Ilka: Metric connections with torsion and string theory

Abstract:

(see the abstract of Thomas Friedrich for general motivation and references)

In my talk, I will report on our recent work on metric connections with skew-symmetric torsion. In particular, I will sketch the foundations for a holonomy theory for such connections, describe integral formulas of Weitzenboeck type and establish the relation to Kostant's cubic Dirac operator. Explicit examples of homogeneous manifolds with spinors that are parallel with respect to these connections will also be described.

Akivis, Maks & Goldberg, Vladislav: Dually degenerate varieties and the generalization of a theorem of Griffiths-Harris

Abstract:

The dual variety X^* for a smooth n-dimensional variety X of the projective space $\mathbb{P}^N$ is a set of the tangent hyperplanes to X. Then in the general case, the variety $X*$ is a hypersurface in the dual space $(\mathbb{P}^N)^*$. If $\dim X^* < N-1$, then the variety X is called dually degenerate. The authors refine these definitions for a variety $X \subset \mathbb{P}^N$ with a degenerate Gauss map of rank r. For such a variety, in the general case, the dimension of its dual variety X^* is $N-l-1$, where $l = n-r$, and X is dually degenerate if $\dim X^* < N-l-1$. In 1979 Griffiths and Harris proved that a smooth variety $X \subset \mathbb{P}^N$ is dually degenerate if and only if all its second fundamental forms are degenerate. The authors generalize this theorem for a variety $X \subset \mathbb{P}^N$ with a degenerate Gauss map of rank r.

Aldea, Nicoleta: Complex Finsler spaces of constant holomorphic curvature

Abstract:

This paper is about complex Finsler metrics of constant holomorphic curvature. For a complex Finsler space (M,F), the holomorphic flag curvature is defined with respect to the Chern complex linear connection on the pull-back holomorphic tangent bundle. A complex Finsler metric, which satisfies some regularity conditions, is called generalized Einstein. By means of such connection, a special approach is devoted to obtain the equivalence conditions that a complex Finsler space be generalized Einstein. A Schur-type theorem for a generalized Einstein complex Finsler space, weakly Kähler, and other characterizations of the holomorphic curvature of this space are given in section 3. We deal with the conformal generalized Einstein complex

Finsler metrics. The final section applies our results to some examples, better illustrating the interest for this work.

Alekseevsky, Dmitri: Pseudo-Riemannian symmetric spaces of quaternionic type

Abstract:

A pseudo-Riemannian n-dimensional manifold M with holonomy group H is called quaternionic Kaehler (respectively, para-quaternionic Kaehler) if $\mathbb{H} \subset \mathrm{Sp}_1 \cdot \mathrm{Sp}_{p,q}$, $p + q = n$ (respectively, $\mathbb{H} \subset \mathrm{Sp}_1(\mathbb{R}) \cdot Sp_n(\mathbb{R})$). If $\mathbb{H} \subset \mathrm{Sp}_{p,q}$ (respectivel, $\mathbb{H} \subset \mathrm{Sp}_n(\mathbb{R})$) then the manifold M is called hyperKaehler (respectively, para-hyperKaehler). We prove that any symmetric quaternionic Kaehler and para-quaternionic Kaehler manifold with non zero Ricci curvature has a simple isometry group, generated by transvections and give a classification of such manifolds. A description of hyperKaehler and para-hyperKaehler symmetric spaces is also given. They have solvable group generated by transvections. The talk is based on joint work with V. Cortes.

Alexander, Stephanie & Bishop, Richard: Generalized Cone Constructions in Riemannian, Alexandrov, and Lorentz spaces.

Abstract:

We give a new global criterion for warped products of metric spaces to have a given curvature bound above or below. This construction extends the known criterion for linear cones, by replacing the radial coordinate with elements of a rich class of generalized convex functions on metric spaces. The proof considers propagation of curvature bounds, and the geometry of vertex sets (vanishing points of the warping function). In addition to generalizing coning from 1-dimensional to arbitrary base, the construction generalizes standard gluing theorems from 0-dimensional to arbitrary fiber. Metric space convergence is used in proving the criterion's necessity. Another element of the proof fills in a missing duality, pairing linear cones over spaces of curvature bound 1 with Lorentz cones over spaces of curvature bound -1.

Altay, Sezgin: On Ricci tensor of weakly concircular symmetric spaces

Abstract:

In this paper, we study on Ricci tensor of a weakly concircular symmetric space $(WZS)_n$. In the first part, we give some details about weakly concircular symmetric spaces and in the other part, Ricci tensor of $(WZS)_n$ is examined and some properties of it are obtained. After then, we define the cyclic Ricci tensor of this space and we prove some theorems about it.

Andrada, Adrian: Complex product structures on manifolds

Abstract:

A complex product structure on a manifold is given by a complex structure and a product structure which anticommute. The existence of such a structure implies that the manifold admits a pair of complementary foliations whose leaves carry affine structures. This is due to the existence of a unique torsion-free connection on the manifold which preserves both the complex and the product structure; this connection however is not necesarily flat. We establish some similarities and differences between complex product structures and hypercomplex structures, and we also study the relationship between complex product structures and hypersymplectic metrics.

Andrica, Dorin & Cicortas, Gratiela: Perturbation result on Morse theory for G-Hilbert-Riemann manifolds

Abstract:

We extend a perturbation method introduced in classical Morse theory by A. Marino and G. Prodi. The main result is contained in the following theorem:

Theorem. Let M be a C^2-G-Hilbert Riemann manifold and let $\Phi, \Psi \in C^2(M, \mathbf{R})$ be two functionals G-invariant. Suppose that Φ and Ψ satisfy the Palais-Smale condition (PS). Assume that exist $c \in R$ and $\varepsilon > 0$ such that:

(i) c is the only critical value of Φ in $[c-\varepsilon, c+\varepsilon]$;

(ii) $\Phi_{c-\varepsilon}$ is not a G-equivariant deformation retract of $\Phi_{c+\varepsilon}$;

(iii) $\sup\limits_{x \in M} \mid \Phi(x) - \Psi(x) \mid \leq \frac{\varepsilon}{3}$.

Then there exists a critical value of Ψ in $[c-\varepsilon, c+\varepsilon]$.

Particular cases when the second assumption is replaced by algebraic conditions are analyzed. Examples are given.

Antic, Miroslava: Totally real totally geodesic submanifolds of the six sphere

Abstract:

A six dimensional sphere S^6 has an almost complex structure J induced by Cayley algebra. A submanifold M of the S^6 is totally real if the almost complex structure J carries each tangent space of Minto its corresponding normal space.We prove that compact 3-dimensional totally real submanifolds of S^6 is totally geodesic if and only if the Ricci curvature $Ricc(M)$ satisfies $Ricc(M) \geq 3/4$.

This is a joint research with M. Djorić and L. Vrancken.

Arias-Marco, Teresa: About the classification of six-dimensional naturally reductive spaces.

Abstract:

Naturally reductive homogeneous spaces have been studied by a number of authors as a natural generalization of Riemannian symmetric spaces. A general theory with many examples was well-developed by D'Atri and Ziller. D'Atri and Nickerson proved that all naturally reductive spaces are spaces with volume-preserving local geodesic symmetries.

Others authors drew their attention to the relationship between the naturally reductive spaces and the commutative Riemannian spaces (in the sense of I.M. Gelfand) which are known to generalize symmetric spaces, as well. For the purpose of testing various conjectures, we started with the classification of naturally reductive spaces in small dimensions. The three-dimensional naturally reductive spaces have been classified by F. Tricerri and L. Vanhecke in [T-V]. In addition, O. Kowalski in [K] found the same classification in a different context, and he also proved that the naturally reductive spaces and the commutative spaces form the same class in dimension three. For other hand, O. Kowalski and L. Vanhecke gave the complete classification for the naturally reductive spaces as well as for the commutative spaces in dimension four. Once again, both classes were shown to be the same.

In [K-V] the same authors gave an explicit (local) classification of naturally reductive spaces in dimension five. They also proved that all five-dimensional naturally reductive spaces are again commutative spaces in the sense of I.M. Gelfand. This gives an additional piece of evidence for a general conjecture "all naturally reductive spaces may be Gelfand spaces." (The general converse is surely not true because we have a six-dimensional counter-example-a generalized Heisenberg group).

Now, we are intending to obtain the classification of six- dimensional naturally reductive spaces, using for that many of the developed techniques in the lesser dimensions classifications. In this sense, we have obtained some results. In addition to, obtained this classification will permit us go into the geometric properties of these spaces.

This is a joint research with A.M. Naveria.

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[K] KOWALSKI, O. Spaces with volume-preserving symmetries and related classes of Riemannian manifolds. Rend. Sem. Mat. Univ. Politec. Torino 1983, Special Issue, 131-158(1984).

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Arslan, Kadri: On biharmonic curves and surfaces

Abstract:

In the present study we consider the curves and surfaces whose their mean curvature vectors are harmonic 1-type. We also consider weak biharmonic submanifolds in E^m.

Arvanitoyeorgos, Andreas: Flag manifold with homogenous geodesics

Abstract:

A geodesic in a Riemannian homogeneous manifold ($M = G/K, g$) is called a homogeneous geodesic if it is an orbit of an one-parameter subgroup of the Lie group G. We investigate G-invariant metrics with homogeneous geodesics (i.e. such that all geodesics are homogeneous) when $M = G/K$ is a flag manifold, i.e. an adjoint orbit of a compact semisimple Lie group G. We use an important invariant of a flag manifold, its T-root system, to give a simple necessary condition that M admits a non-standard G-invariant metric with homogeneous geodesics. For flag manifolds of classical Lie groups G we prove that the flag manifold $\mathrm{Com}(\mathbb{R}^{2n}) = SO(2n-1)/U(n-1)$ of all complex structures in $\mathbb{R}^{2n}$ is the only flag manifold which admits a non-naturally reductive $SO(2n-1)$-invariant metric with homogeneous geodesics. This manifold is a weakly symmetric space. It admits a 2-parametric family of invariant metrics. All of them have homogeneous geodesics, and only one of them (up to a homothety) is naturally reductive. It is the standard metric of the symmetric space $\mathrm{Com}(\mathbb{R}^{2n}) = \mathrm{SO}(2n)/U(n)$. In all other cases, the only G-invariant metric with homogeneous geodesics is the metric which is homothetic to the standard metric (i.e. the metric associated with the negative of the Killing form of the Lie algebra $\mathfrak{g}$ of G. We also describe the set of homogeneous geodesics and non-homogeneous geodesics for the flag manifold $\mathrm{SO}(2n-1)/U(p) \cdot \mathrm{SO}(2q-1)$ $(p + q = n)$ equipped with an arbitrary invariant metric. For the flag manifolds of exceptional Lie groups G we enumerate all spaces M that *may* admit a G-invariant metric with homogeneous geodesics, and which is not homothetic to the standard metric. There are only 9 such manifolds among 96 flag manifolds of exceptional Lie groups.

A common feature of these spaces is that their isotropy representation has two irreducible components.

Asada, Akira: Regularized volume form of the sphere of a Hilbert space with the determinant bundle

Abstract:

We have constructed regularized volume form of a flat infinite dimensional space (cf. Differential Geometry and Its Applications 2001, 165, Opava). Applying this method to the sphere of a Hilbert space H equipped a positive Schatten class operator whose zeta function is holomorphic at the origin, we compute its regularized volume form explicitely, by using polar coordinate. Polar coordinate of H has only latitude and does not have longitude. To get regualrized volume form of the sphere, we need to add longitude, which is interpreted to come from the determiannt bundle of H. So strictly speaking, our regularized volume form is defined on the sphere of H with the determinant bundle.

Balashchenko, Vitaly: Canonical structures on generalized symmetric spaces and generalized Hermitian geometry

Abstract:

The important role of invariant structures on homogeneous manifolds in differential geometry is well known. In this sense, *regular* Φ*-spaces (generalized symmetric spaces)* intensively studied by N.A.Stepanov, J.A.Wolf, A.Gray, O.Kowalski, A.S.Fedenko and many others have a distinguishing feature. More exactly, each such space $(G/H, \Phi)$ has the commutative algebra $\mathcal{A}(\theta)$ of *canonical structures*, which additionally compatible with the "symmetries" of G/H. The main goal of this talk is to show a fundamental role of canonical structures in the theory of homogeneous regular Φ-spaces as well as its applications.

The algebra $\mathcal{A}(\theta)$ contains a remarkable collection of classical structures, such as almost complex structures J, almost product structures P, f-structures of K.Yano ($f^3 + f = 0$), h-structures ($h^3 - h = 0$) (V.V.Balashchenko, N.A.Stepanov). Recently, an algebraic structure of the algebra $\mathcal{A}(\theta)$ has been explicitly described. This result partitions all regular Φ-spaces $(G/H, \Phi)$ into a countable set of disjoint classes with respect to their algebra $\mathcal{A}(\theta)$.

Further, the canonical structures P and h-structures on *homogeneous* k*-symmetric spaces* ($\Phi^k = id$) are compatible with the natural pseudo-Riemannian metrics in the "Riemannian" sense. As to the canonical structures J and f-structures, they are compatible in the sense of Hermitian and the *generalized Hermitian geometry* respectively. The concept of this geometry was created by V.F.Kirichenko in the 1980s. The objects of this geometry are *generalized almost Hermitian* (briefly, GAH*-structures*) of *rank* r. The case $r = 1$ includes classical AH-structures, metric almost contact structures, metric f-structures and h-structures. We dwell on the remarkable classes of metric f-structures: *Kähler* (Kf), *Hermitian* (Hf), *Killing* $(Killf)$, *nearly Kähler* (NKf), G_1f*-structures* (G_1f) (V.F.Kirichenko, A.S.Gritsans, V.V.Balashchenko). They contain the corresponding Gray-Hervella classes K, H, NK, G_1. The main result is that the canonical f-structures on homogeneous k-symmetric spaces provide a wealth of invariant examples for all the classes mentioned. In this respect the role of the canonical f-structures on 4-, 5-, and 6-symmetric spaces is very important (V.V.Balashchenko, Yu.D.Churbanov, O.V.Dashevich). The case of Killing f-structures is more difficult. In particular, the corresponding Riemannian metrics in the examples of invariant Killing f-structures are neither naturally reductive nor Einstein.

Moreover, the collections of canonical f-structures on homogeneous k-symmetric spaces provide a vast class of invariant examples for GAH-structures of arbitrary rank r (V.V.Balashchenko, D.V.Vylegzhanin).

Barros, Manuel: New Solitons in the O(3) nonlinear sigma model

Abstract:

We will give an algorithm to construct the moduli space of soliton solutions, that preserve a certain symmetry in the boundary, of the O(3) nonlinear sigma model. These solitons carry topological charges that can be holographically determined from the boundary conditions.

Benito, Roberto: Hidden simplecticity in Hamilton's principle algorithms

Abstract:

In a paper published in 1996 by H. Ralph Lewis and Peter J. Kostelec, Hamilton's principle is used to derive numerical algorithms. The basic idea is to consider a restricted class of admissible curves, usually polynomials, and then apply Hamilton's principle directly to the restricted function space (the space of approximating functions). The resulting algorithms fail to be symplectic in the usual way. We treat the vakonomic dynamics with higher-order constraints within a geometric framework which can be useful in the study of geometrical properties of variational integrators in the sense of Lewis-Kostelec. For instance, we show that Hamilton's principle algorithms are symplectic when considered within this higher-order geometric framework.

Benn, Ian: Symmetry Operators for the Dirac and Hodge-de-Rham Equation

Abstract:

All first order linear symmetry operators for the Dirac equation are obtained from conformal Killing-Yano tensors. This will be contrasted with the analogous case of the Hodge-de-Rham equation. Links between the algebra of these symmetry operators, and an algebra of the tensors that give rise to them, will be explored.

Birman, Graciela Silvia: Existence of developable hypersurfaces in semi-riemannian manifolds

Abstract:

The conditions for existence of a developable hypersurface in an euclidean 4-dimensional space are known. The main purpose of this article is to generalize that problem to semi-riemannian manifolds of dimension n equal or bigger than 4 and to find the system of differential equations for the existence of developable hypersurfaces containing a given 2-codimensional as well as a 3-codimensional semi-riemannian manifolds.

Bishop, Richard: See *Stephanie, Alexander*

Blair, David: Hyperbolic Twistor Spaces and Isotropic Kähler Manifolds

Abstract:

For indefinite almost Hermitian structures the vanishing of the square norm $||\nabla J||^2$ does not always imply the Kähler condition, $\nabla J = 0$. Thus an indefinite almost Hermitian structure is said to be *isotropic Kähler* if $||\nabla J||^2 = 0$ and *indefinite Kähler* if $\nabla J = 0$. The first examples of non-Kähler isotropic Kähler structures were recently constructed by Garcia-Rio and Matsushita. These examples are 4-dimensional and are related to certain Engel structures on $\mathbb{R}^4$ and their compact quotients.

Here we give a class of 6-dimensional examples. These arise as certain indefinite Hermitian structures on

hyperbolic twistor spaces. We first review the theory of these twistor spaces over 4-dimensional manifolds with neutral metrics and their two natural indefinite almost Hermitian structures. Only one of these indefinite almost Hermitian structures can be isotropic Kähler and we give precise geometric conditions on the base manifold ensuring this property. Finally we construct some 2-parameter families of neutral left-invariant metrics on some 4-dimensional Lie groups whose hyperbolic twistor spaces are isotropic Kähler but not indefinite Kähler. As time permits we will discuss these twistor spaces in higher dimensions.

This is joint work with J. Davidov and O. Muskarov.

Blazic, Novica: Conformally Osserman curvature

Abstract:

In this communication we study curvature characterizations of conformal class of complex and quaternionic projective spaces. The main tool is a conformal Jacoby operator J_W defined by $J_W(x)y := 3DW(y,x)x$, where W is the Weyl conformal curvature tensor. Manifold (M,g) is *conformally Osserman* if the eigenvalues of $J_W(x)$ are independent on the unit direction x , as suggested by Gilkey. For example, the following theorem was proved:

Theorem. *Let $(M,[g])$ be a conformally Osserman manifold = of dimension $m = 3D4n + 2 \geq 10$. If M is not conformally flat ($W \neq 0$ on M), then it is = conformally equivalent to a complex projective space or its = non-compact dual.*

As a byproduct, the classification of the conformally complex space = forms, if the dimension is at least 8, is also obtained.

These are joint results with Peter Gilkey.

Bless, Michael: Conformal Extremal Metrics for Surfaces of Genus 2

Abstract:

Sharp conformal isosystolic inequalities for the compact surface of genus two will be shown using Bavard's extremality criterium on the one hand, and on the other Strebel's theory of quadratic differentials in order to obtain a suitable representation of the conformal classes. Strebel introduced his theory of quadratic differentials on Riemann surfaces defining everything in local coordinates. In the first part we develop a formulation of this theory in terms of holomorphic differential forms of second order. The main tool will be a holomorphic 1-form, defined on a two-sheeted covering of the surface, which can be interpreted as the "'root"' of the quadratic differential. It's constructed in an analogous way like the analytic continuation of a germ of the square-root of a holomorphic function. We also construct a complete analytic continuation of this 1-form, which leads to a branched covering of the surface. Furthermore, we describe the Riemannian metric induced by the quadratic etric induced by the quadratic differential in terms of our formulation. Finally, for the case of a quadratic differential with closed trajectories, the surface will be mapped isometrically onto a surface, which consists of cylinders, which are identified isometrically along pieces of their boundaries. Thus, each conformal class of Riemannian metrics on a compact orientable surface contains a singular metric of cylindrical type. Using Strebel's existence theorems for quadratic differentials with closed trajectories, we will develop a representation of the conformal classes of Riemannian metrics on the compact surface of genus two by singular cylindrical metrics. Then, we will concretisize the criterium of Bavard for a Riemannian metric on a compact manifold to be extremal in the case of a surface of cylindrical type. Some of the latter, as will be shown, are already conformal extremal. Finally, based on this representation, conformal isosystolic inequalities will be shown.

Boeckx, Eric: Characteristic reflections in contact metric geometry

Abstract:

The notion of local symmetry is too strong a condition for K-contact or Sasakian spaces. Indeed, such manifolds have constant curvature 1. For this reason, Takahashi introduced the notion of local φ- symmetry for Sasakian spaces: these are Sasakian spaces for which the characteristic reflections (i.e., reflections with respect to the integral curves of the characteristic vector field of the Sasakian structure) preserve the Sasakian structure completely. Two generalizations of the notion of local φ-symmetry to the larger class of contact metric spaces have appeared. We report on the present state of affairs in this field and present our latest results concerning a possible classification of locally φ-symmetric contact metric spaces.

Bolsinov, Alexey: Integrability of geodesic flows on Riemannian manifolds

Abstract:

We construct new examples of integrable geodesic flows on homogeneous spaces and bi-quotients of compact Lie groups by using non-commutative integration concept suggested by A.Fomenko and A.Mishchenko. We also discuss topological obstructions to integrability of geodesic flows on smooth compact manifolds and some open problems.

Borbely, Albert: On the total absolute curvature of sphere eversions

Abstract:

It is a well known theorem of Smale that all immersions of the two-sphere into the three dimensional Euclidean space ($S^2 \to \mathbb{R}^3$) are regularly homotopic. In particular, there exists sphere eversions, that is, regular homotopies turning the sphere inside out (regular homotopies connecting two differently oriented two-spehers). In a recent paper by Tobias Ekholm (arXiv:math.GT/0310266v1) it was shown that it is possible to construct a spehere eversion such that for any $\epsilon > 0$ the total absolute curvature (the integral of the absolute value of the Gauss curvature) of the spheres remains below $8\pi + \epsilon$ during the eversion. In this talk it will be shown that 8π is the best possible value. In other words, we prove that for any sphere eversion there is an instant when the total absolute curvature of the immersed sphere is greater than 8π.

Brajercik, Jan: Variational principles on the frame bundles

Abstract:

We discuss the variational principles on the r-jet prolongations of the frame bundles, invariant with respect to the underlying structure group. We establish basic properties of the invariant Euler-Lagrange form and the Poincare-Cartan form of the invariant lagrangians. The corresponding Noether theorem is also discussed.

Bureš, Jarolím & Vanžura, Jiří: Orbits of multisymplectic 3-forms in dimension six.

Abstract:

On a 6-dimensional real vector space V there exist three types of multisymplectic 3-forms. Under the action of the general linear group $GL(V)$ they constitute three orbits. With each such multisymplectic 3-form (according to its type) we can associate a product, complex or tangent structure on V. Using them we obtain similar structures on the relevant orbits.

Bushueva, Galina: The higher order geometry of manifolds depending on parameters

Abstract:

We study trivial fiber bundles $E = M \times \mathbf{R}^m$ over $\mathbf{R}^m$. The generalized Weil bundle $T^A(E)$ of E is defined to be the set of A-velocities of sections of E. The structure group of the generalized Weil bundle is an extended higher order differential group. We define an infinite series of higher order frame bundles associated with $T^A(E)$ and study invariants of connections in these bundles. In particular, we find conditions under which a connection of such type is locally equivalent to the standard flat connection.

Calderbank, David: Conformal submanifold geometry

Abstract:

A manifestly conformally invariant theory of submanifolds of the conformal n-sphere is developed and used to obtain new insights and results, particularly for surfaces.

Calderbank, David: Weyl structures and Ricci corrected differentiation in parabolic geometries

Abstract:

Using a new approach to Weyl structures in parabolic geometries, the notion of Ricci corrected differentiation is developed and applied to the construction of invariant operators.

Calvaruso, Giovanni: Simmetry conditions on conformally flat Riemannian manifolds

Abstract:

Generalizing the classical result of P. Ryan concerning the classification of conformally flat locally symmetric spaces, we provide the complete classification of conformally flat semi-symmetric spaces and conformally flat pseudo-symmetric spaces of constant type.

Canqete, Antonio: Least-perimeter partitions of the disk

Abstract:

In the Calculus of Variations, problems related to isoperimetric partitions have multiple applications in physical sciences.

They can properly model multitude of natural phenomena, such as the shape of a cellular tissue or the interfaces separating several fluids.

In this talk we will consider the isoperimetric problem of partitioning a planar disk into n regions of given areas with the least possible perimeter; we will discuss specially the case of three regions, whose unique solution is the standard configuration, consisting of three circular arcs or segments meeting orthogonally the boundary of the disk, and meeting in threes at 120 degrees in an interior vertex. This is part of a joint work with Manuel Ritore.

Cap, Andreas: Infinitesimal automorphisms and deformations of parabolic geometries

Abstract:

Parabolic geometries are a particularly nice subclass of Cartan geometries. Assuming the conditions of regularity and normality, such geometries are equivalent to certain underlying structures, including conformal, almost quaternionic and certain almost CR structures.

The Cartan point of view easily leads to an interpretation of infinitesimal automorphisms and infinitesimal deformations of these structures in terms of a twisted de Rham sequence. The BGG machinery relates this sequence to a sequence of higher order operators, which can be nicely interpreted in terms of the underlying structure.

In the subclass of locally flat structures, this leads to a deformation complex which is well known in some special cases. For some of the structures, there are interesting integrability conditions related to torsion freeness and/or semi–flatness. One then obtains a subcomplex in the BGG sequence (to be explained in V. Soucek's lecture), which again can be interpreted as a deformation complex. In particular, we obtain an elliptic deformation complex for quaternionic structures, which generalizes the deformation complex for (anti–) self–dual conformal four manifolds.

Cavicchioli, Alberto: Volumes of some hyporbolic 3–MANIFOLDS

Abstract:

Let $\mathbb{H}^3$ be the three–dimensional hyperbolic space, and $\Gamma < \mathrm{Isom}(\mathbb{H}^3)$ a discrete torsion–free group of isometries acting without fixed points. A closed connected 3–manifold M^3 is said to be *hyperbolic* if it can be obtained as a quotient space $\mathbb{H}^3/\Gamma$. The Mostow Rigidity Theorem says that finite volume complete hyperbolic 3–manifolds are completely classified, up to isometry, by isomorphisms of their fundamental groups. Hence the finite *hyperbolic volume* is a good topological invariant for hyperbolic 3–manifolds. The structure of the set of finite volumes of hyperbolic 3–manifolds is described by the Thurston–Jorgensen Theorem. There are at most finitely many distinct hyperbolic 3–manifolds of a given finite volume. The finite volumes of hyperbolic 3–manifolds form a closed non–discrete well–ordered subset of the real line $\mathbb{R}$ with ordinal type ω^ω. The aim of the talk is to present some recent results on the topology and geometry of certain classes of closed 3–manifolds constructed by combinatorial methods as for example surgery on links and pairwise identifications of faces on the boundary of polyhedral 3–cells. We determine geometric structures, homeomorphism type, isometry groups, formulae for the volume, homology, and some covering properties. Here we list only essential references concerning with the arguments of the talk.

Chiossi, Simon: G_2 structures with torsion from nilmanifolds

Abstract:

In a joint work with Andrew Swann, we study the equations for a G_2 structure with torsion on a Riemannian product $M = N \times S^1$ in relation to the induced $SU(3)$ structure on N. All solutions are found in the case when the Lee form of the G_2 structure is non-zero and N is a nilmanifold with a half-integrable $SU(3)$ structure. Special properties of the torsion of these solutions are then discussed.

Chiricalov, Vladimir: Invariant toroidal manifolds and its application to the theory of multifrequency oscillations.

Abstract:

The report summarizes some results concerning mathematical theory of multifrequency oscillations. The central object of this theory is an invariant toroidal manifold of a dynamical system.In this report the invariant torus of a bilinear matrix differential system of equations on direct product of m-dimensional torus

and space of matrices will be considered. The existence of such a manifold is sufficient for multifrequency oscillations of the system to exist. We consider necessary and sufficient conditions for existence of invariant tori, the properties of Green's operator-function, which defines the main properties of invariant torus of a dynamical system, the method of the iterative construction of this torus.

Cho, JongTaek: η-parallel contact metric spaces

Abstract:

We prove that a contact metric manifold with η-parallel tensor h is K-contact or a (k, μ)-space. In the latter case, its associated CR-structure is in particular integrable.

Churbanau, Yury: Classical affinor structures and affine connections on homogeneous k-symmetric spaces

Abstract:

Let G/H be a homogeneous Φ-space of finite order (a homogeneous k-symmetric space [1]). Then it possesses the algebra of the canonical affinor structures [2]. This algebra contains classical affinor structures such as almost complex structures, almost product structures and f-structures.

The relationship between canonical structures acting on the tangent space $T_o(G/H)$ at the point $o = H$ and the Lie bracket on it was obtained. It gives the opportunity to indicate the algebraic criteria for integrability of these structures in some important cases.

The subspace $\mathfrak{m}^\varphi$ was first introduced in [3]. This subspace is important for the geometry of regular Φ-spaces. Denote by $\mathfrak{h}$ the Lie algebra of the subgroup H. The criterion for the inclusion $\mathfrak{m}^\varphi \subset \mathfrak{h}$ was proved. The example of the homogeneous 6-symmetric space, for which $\mathfrak{m}^\varphi \subset \mathfrak{h}$ is constructed. This space admits the invariant Killing f-structure.

Let G/H be a pseudo-Riemannian homogeneous Φ-space of finite order with a metric g and the Levi-Civita connection ∇. Under the assumption $\mathfrak{m}^\varphi \subset \mathfrak{h}$ the compatibility of the canonical classical affinor structures and the connection ∇ was investigated.

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Cicortas, Gratiela: See *Andrica, Dorin*

Comic, Irena & Grujic, Gabriela & Stojanov, Jelena: The spray theory in the subspaces of Miron's OsckM

Abstract:

The spray theory in OsckM was introduced in two papers written by R. Miron, Gh. Atanasiu and published in Rev. Roum. Math. Pures et Appl., 1996. Lately R. Miron gave the comprehend theory of higher order

geometry and the spray theory in two books published by Kluwer Acad. Publ, 1997. and 2001. The first author of this paper gave the relation between J structure, Liouville vector fields and the S-vector field in more general adapted basis then it was used by R. Miron and with slightly different variables , published in Studia Scientiarum Mathematicarum, 2003. Here the adapted basis is changed in such a way that the former mentioned relations have new, simpler and more elegant form. The combinatorial aspect was also used. Using the special adapted basis the spray theory in subspaces of OsckM is established. Mathematics Subject Classification (2000) : 53B40, 53C60, 53C15, 53C55, 58A20.

Costa, Joanna Nunes da: On Jacobi Manifolds and Dirac Structures

Abstract:

We will talk about some relations between Jacobi manifolds and Dirac structures:

Jacobi structures can be characterized by Dirac structures of Courant-Jacobi algebroids ([3] [5] [7]).

Reduction procedure on Jacobi manifolds can be established using Dirac structures ([6])

Jacobi and Dirac structures can be deformed by Nijenhuis operators to produce Jacobi-Nijenhuis and Dirac-Nijenhuis, respectively ([1] [2] [4]).

[1] J. F. Cariñena, J. Grabowski and G. Marmo, *Courant algebroids and Lie bialgebroids contractions*, J. Phys A: Math. Gen 37 (2004) 5189-5202

[2] J. Clemente-Gallardo and J. M. Nunes da Costa, *Dirac-Nijenhuis structures*, to appear in J. Phys A: Math. Gen (2004)

[3] J. Grabowski and G. Marmo, *The graded Jacobi algebras and (co)homology*, J. Phys. A : Math. Gen. 36 (2003) 161-181

[4] J. M. Nunes da Costa, *A characterization of strict Jacobi-Nijenhuis manifolds through the theory of Lie algebroids*, Rep. Math. Phys. 50(3) (2002) 339-347

[5] J.M. Nunes da Costa and J. Clemente-Gallardo, *Dirac structures for generalized Lie bialgebroids*, J. Phys. A : Math. Gen. 37 (2004) 2671-2692

[6] F. Petalidou and J. M. Nunes da Costa, *Reduction of Jacobi manifolds via Dirac structures theory*, Technical Report

[7] A. Wade, *Conformal Dirac structures*, Lett. Math. Phys. 53 (2000) 331-348

Crampin, Michael: On separation of variables in the Hamilton-Jacobi equation for geodesics

Abstract:

I shall review some recent work on separation of variables, concerned especially with Benenti tensors (also known as special conformal Killing tensors) and their generalizations.

Csikos, Balazs: A Schläfli-type formula in pseudo-Riemannian Einstein manifolds and its application to the Kneser-Poulsen conjecture

Abstract:

We present a formula for the variation of the volume of a polyhedron with curved faces lying in a pseudo-Riemannian Einstein manifold, which generalizes the classical Schlfli formula and the formulae of I. Rivin, J.-M. Schlenker and R. Souam. The formula is applied to attack the Kneser-Poulsen conjecture according to which the volume of the union of some balls (in the Euclidean, hyperbolic or spherical space) does not increase when the balls are rearranged in such a way that the center-center distances decrease.

Czarnecki, Maciej: Some properties of Hadamard foliations

Abstract:

An Hadamard foliation is a foliation of Hadamard manifold which all leaves are Hadamard.

If the curvature of manifold carrrying the foliation is $\leq -a^2$ and the norm of the 2nd fundamental form of foliation is $\leq a$ then the foliation is Hadamard. We introduce as in [1] a canonical embedding of the union of leaf boundaries into the boundary of hyperbolic space $\mathbb{H}^n$ carrying the foliation. Some properties of such embedding shall be considered: injectivity, continuity and dependence of the leaf boundary relation of the curvature of normal field in codimension 1.

[1] M. Czarnecki, Hadamard foliations of H^n, Diff. Geom. & Appl. 20(2004), 357–365

Czudkova, Lenka & Janova, Jitka: Non-holonomic physical systems and variational principle

Abstract:

In physics we often meet systems subjected to the various types of constraints. Naturally, it is advisable to formulate corresponding equations of motion and then to deal with a question of their variationality. While both equations of motion and variational theory for holonomic mechanical systems are well-known, non-holonomic situations are still studied by many authors. Besides other approaches, the geometrical theory of mechanical systems with non-holonomic constraints on jet manifolds was recently introduced by Krupková and a new concept of "constraint variationality" for so-called reduced equations of motion was proposed by Krupková and Musilová.

Using the results of these theories we discuss variational aspects of non-holonomic mechanical systems. On the illustrative examples we present some concrete physical problems (formulation of constraint variational principle, equivalence properties of constraint Lagrangians) and compare them with unconstrained cases.

Dehkordy, AzamEtemad: Some Properties of Minimal submanifolds in Hyperbolic Spaces

Abstract:

In this paper I study compact minimal submanifolds in hyperbolic space with an special estimate for scalar curvature. Furthermore we consider the closed minimal hypersurfaces in hyperbolic space when the scalar curvature of them belong to special closed interval and prove that their scalar curvature must be constant.

Djoric, Mirjana: CR submanifolds of maximal CR dimension of Kaehler manifolds and their (almost) contact structure

Abstract:

We treat n-dimensional real submanifold M of a Kaehler manifold $\tilde{M}$ in the case when the maximal holomorphic tangent subspace is $(n-1)$-dimensional. Under this hypothesis the submanifold M is necessarily

odd-dimensional and it admits a naturally induced almost contact metric structure (F, u, U, g). Consequently, there are two geometric structures: an almost contact structure F induced from the complex structure J of the ambient space $\tilde{M}$, and a submanifold structure represented by the second fundamental tensor h of M in $\tilde{M}$. We study certain conditions on the almost contact structure F and on the second fundamental tensor h of these manifolds when the ambient space is a Kaehler manifold. Moreover, we characterize several important classes of submanifolds in complex space forms and we give the condition when the holomorphic sectional curvature of the integral submanifold $M_{\mathcal{D}}$ of the distribution $\mathcal{D}$, which is spanned by all vectors orthogonal to U, is non–positive.

This talk is based on joint research with M. Okumura.

Doubrov, Boris: On local classification of three-dimensional homogeneous spaces

Abstract:

The local classification of n-dimensional homegeneous spaces is equivalent to the classification of transitive finite-dimensional Lie algebras of vector fileds on $\mathbb{R}^n$ or $\mathbb{C}^n$, or to the classification of effective subalgebras of codimension n in finite-dimensional Lie lagebras.

For $n \leq 2$ this problem was completely solved by Sophus Lie at the end of 19th century. He also presented a few ideas on how to classify Lie algebras of vector fileds in higher dimensions.

Based on his ideas, we describe the local structure of one-dimensional invariant foliations on homogeneous spaces of arbitrary dimension. Using this, we provide the complete local classification of all three-dimensional homogeneous spaces with a non-solvable transformation group. We aslo discuss the solvable case and, in particular, show that these homogeneous spaces can not be explicitely parametrized.

Doubrovin, Boris: Flat Hessian structures, Shephard groups, and Frobenius manifolds

Abstract:

Flat Hessian structure on an affine space is a zero curvature metric represented by the Hessian of smooth function. We will present a general construction of flat Hessian structures in terms of Frobenius manifolds. The general method will be illustrated for the examples coming from the theory of Shephard groups.

Doupovec, Miroslav & Mikulski, Wlodzimierz: Prolongation of connections to vertical bundles

Abstract:

Let G be a bundle functor defined on the category $\mathcal{FM}_{\Updownarrow,\backslash}$ of fibered manifolds with m-dimensional bases and n-dimensional fibres and locally invertible fiber respecting mappings. We study the prolongation of connections on $Y \to M$ into connections on $GY \to M$ from a general point of view. A special attention is devoted to the prolongation of connections and pairs of connections to vertical bundles. Some existence problems are solved and some classification results are obtained.

Dušek, Zdeněk, Kowalski O., Nikčević, S.: New examples of g.o. manifolds in dimension 7

Abstract:

A Riemannian g.o. manifold is a homogeneous Riemannian manifold (M, g) on which every geodesic is an orbit of a one-parameter group of isometries. It is known, that every simply connected Riemannian g.o.

manifold of dimension less or equal to 5 is naturally reductive. In dimension 6 there are simply connected g.o. manifolds which are not naturally reductive, and their full classification is known. In dimension 7, just one example was known up to recently (namely, a Riemannian nilmanifold constructed by C. Gordon). We classify g.o. manifolds using the notion of geodesic vectors and geodesic graphs and we introduce the notion of the degree of a g.o. manifolds. The degree is zero if and only if the manifold is naturally reductive. In all the examples of g.o. manifolds which are not naturally reductive and which were investigated up to recently the degree is equal to two.

In the present lecture I will describe compact irreducible 7-dimensional g.o. manifolds (together with their "noncompact duals") which are not naturally reductive. We conjecture that these examples together with the example constructed by C. Gordon are the only 7-dimensional g.o. manifolds which are not naturally reductive. The degree of these manifolds is also equal to two. Then I will present the example of a 13-dimensional H-type group which is a g.o. manifold of degree three.

Eastwood, Michael: Conjugate functions and semi-conformal mappings.

Abstract:

Suppose f is a smooth function of two variables. Is there a smooth function g such that $|\text{grad } f| = |\text{grad } g|$ and $\langle \text{grad } f, \text{grad } g \rangle = 0$? The answer is yes if and only if f is harmonic. What about the same question for a function of three or more variables? Joint work with Paul Baird derives a differential inequality that must be satisfied by f and a differential equation in the case of a function of three variables. When f admits a conjugate, the pair (f, g) provides a semiconformal mapping into $\mathbb{R}^2$. In particular, harmonic morphisms provide examples of conjugate pairs but there are more besides. The problem of finding a conjugate is conformally invariant so it not surprising that our constraints are also conformally invariant.

Eichhorn, Juergen: Index theory for generalized Dirac operators on open manidolds

Abstract:

For elliptic operators over closed Riemannian manifolds, there is a well defined index theory. On open manifolds, everything breaks down: operators are not Fredholm, the analytical index is not defined, the formula for the topological index does not make sense (since the integrals diverge). We present some special classes of geometric-analytic problems where classical index theory makes sense and thereafter a very general relative index theory for open manifolds of bounded geometry.

Erdem, Sadettin: Constancy of some maps into metric (para) f-manifolds and harmonicity via Lee forms and quadratic differentials

Abstract:

Constancy of maps into certain manifolds is discussed. Also some results provided in [2] and [15] concerning the relations between harmonicity of a map and minimality of its fibres, are extended to higher dimensions as well as to the "para" cases via Lee forms and quadratic differentials.

Etemad, Dehkordy: Some Properties of Minimal submanifolds in Hyperbolic Spaces

Abstract:

In this paper I study compact minimal submanifolds in hyperbolic space with an special estimate for scalar curvature. Furthermore we consider the closed minimal hypersurfaces in hyperbolic space when the scalar curvature of them belong to special closed interval and prove that their scalar curvature must be constant.

Ferapontov, Eugene: Geometric aspects of the integrability of multidimensional quasilinear systems

Abstract:

A multidimensional quasilinear system is said to be integrable if it can be decoupled in infinitely many ways into a family of commuting 1-dimensional systems in Riemann invariants. Classification results and geometric implications of this definition will be discussed.

Fernandez Delgado, Isabel: Maximal surfaces with conelike type singularities in complete flat three dimmensional space-times.

Abstract:

In this talk we study deal with the geometry of complete embedded maximal surfaces with isolated singularities in complete flat Lorentzian 3-manifolds. We also prove that a complete flat three dimensional space-time containing a surface of this kind must be the quotient of the Lorentz-Minkowski space under a translational group of rank less than or equal to two. At this point it has been crucial a result by Mess about the non existence a of certain type of Margulis space-times.

Fernandez, Luis Manuel: New formulas for the shape operator of slant immersions in S-space-forms(tentative).

Abstract:

Some inequalities between the shape operator and the sectional curvature for slant submanifolds in an S-space-form are obtained. In particular, they are applied for invariant and anti-invariant submanifolds.

Fernández, Marisa: Harmonic cohomology of Donaldson submanifolds

Abstract:

One of the main results of Hodge theory states that for any compact oriented Riemannian manifold, any de Rham cohomology class has an unique harmonic representative. Mathieu and, independently, Yan have proved that there is a similar property for symplectic manifolds satisfying the hard Lefschetz property, which asserts that any de Rham cohomology class has a (not unique, in general) *symplectically harmonic* representative.

We consider (M, ω) a symplectic manifold, that is, M is a differentiable manifold of dimension $2n$ with a closed non-degenerate 2–form ω, the *symplectic form*. As an obvious fact, whenever (M, ω) is not hard Lefschetz, there is some $r \geq 0$ such that (M, ω) satisfies the Lefschetz property up to the level r. For such a manifold we prove that every de Rham cohomology class in $H^k(M)$ contains at least one symplectically harmonic form for any $k \leq (r+2)$ and $k \geq (2n-r)$. Moreover, we show the relation between the harmonic cohomology of a Donaldson symplectic submanifold and that of its ambient space, and some consequences are discussed.

Ferreiro, Perez: On the Equivariant Variational Bicomplex

Abstract:

Let M be a compact n-manifold and let $E \to M$ be a bundle over M. We define an operator assigning to any (n+k)-differential form on the jet bundle J(E) a k-form on the manifold of sections of E and we analyze the properties of this map. This operator is closely related to the variational bicomplex theory, and provides a geometrical interpretation of the functional forms. We show how an extension of this operator to equivariant cohomology provides us with a method to obtain cohomololy classes on the space of sections modulo the action of a group. Finally, we analyze the results obtained applying this constructions to the particular cases of connections on principal bundles and Riemannian metrics.

Friedrich, Thomas: Special Geometries, Torsion and Holonomy

Abstract:

Fix a subgroup $\mathrm{G} \subset \mathrm{SO}(n)$ of the special orthogonal group and decompose the Lie algebra $\mathfrak{so}(n) = \mathfrak{g} \oplus \mathfrak{m}$ into the Lie algebra $\mathfrak{g}$ of G and its orthogonal complement $\mathfrak{m}$. The different geometric types of G-structures on a Riemannian manifold correspond to the irreducible G-components of the representation $\mathbb{R}^n \otimes \mathfrak{m}$. Indeed, consider a Riemannian manifold (M^n, g) and denote its Riemannian frame bundle by $\mathcal{F}(M^n)$. It is a principal $\mathrm{SO}(n)$-bundle over M^n. A G-structure is a reduction $\mathcal{R} \subset \mathcal{F}(M^n)$ of the frame bundle to the subgroup G. The Levi-Civita connection is a 1-form Z on $\mathcal{F}(M^n)$ with values in the Lie algebra $\mathfrak{so}(n)$. We restrict the Levi-Civita connection to $\mathcal{R}$ and decompose it with respect to the decomposition of the Lie algebra $\mathfrak{so}(n)$,

$$Z|_{T(\mathcal{R})} \ := \ Z^* \oplus \Gamma \, .$$

Then, Z^* is a connection in the principal G-bundle $\mathcal{R}$ and Γ is a 1-form on M^n with values in the associated bundle $\mathcal{R} \times_{\mathrm{G}} \mathfrak{m}$. If $\Gamma = 0$, then the Levi-Civita connection preserves the Γ-strucuture (integrable geometries). Some authors call Γ the *intrinsic torsion* of the G-structure. There is a second notion, namely the *characteristic connection* and the *characteristic torsion* of a G-structure. It is a G-connection ∇^{c} with totally skew symmetric torsion tensor. Not any type of geometric G-structures admits a characteristic connection. In order to formulate the condition, we embed the space of all 3-forms into $\mathbb{R}^n \otimes \mathfrak{m}$ using the morphism

$$\Theta \, : \, \Lambda^3(\mathbb{R}^n) \longrightarrow \mathbb{R}^n \otimes \mathfrak{m} \, , \quad \Theta(\mathrm{T}) \ := \ \sum_{i=1}^{n} e_i \otimes \mathrm{pr}_{\mathfrak{m}}(e_i \lrcorner \mathrm{T}) \, .$$

A G-structure admits a characteristic connection ∇^{c} if and only if the intrinsic torsion Γ belongs to the image of the Θ. In this case, the intrinsic torsion is given by the equation $2\,\Gamma = -\,\Theta(\mathrm{T}^{\mathrm{c}})$. For interesting geometric structures Θ is injective, i.e. the condition that the torsion is totally skew symmetric singles out a unique characteristic connection substituting the Levi-Civita connection. This characteristic torsion form has been computed explicitely in terms of the underlying geometric data. Formulas of that type are known for almost hermitian structures, almost metric contact structures, G_2-structures in dimension 7 and Spin(7)-structures in dimension 8. If $M^n = \mathrm{G}_1/\mathrm{G}$ is naturally reductive, the characteristic connection coincides with the *canonical* connection of the reductive space. In this sense, we can understand the characteristic connection of a Riemannian G-structure as a generalization of the canonical connection of a Riemannian naturally reductive space. The canonical connection of a naturally reductive space has parallel torsion form and parallel curvature tensor. For arbitrary G-structures and their characteristic connections, these properties do not hold anymore. Corresponding examples will be discussed. Non-integrable geometric structures and their characteristic connections are important in type II string theory. Indeed, their torsion forms serve as candidates for a NS-3-form involved in the so called Strominger model. This is a 6-tuple $(M^n, g, \nabla, \mathrm{T}, \Phi, \Psi)$ consisting of a Riemannian manifold (M^n, g), a metric connection ∇ with totally skew symmetric torsion form T, a dilation function Φ and a spinor field Ψ. The string equations can be written in the following way:

$$\mathrm{Ric}^{\nabla} + \frac{1}{2}\,\delta(\mathrm{T}) + 2\,\nabla^g(d\Phi) = 0\,, \quad \delta(\mathrm{T}) = \ 2\,(\mathrm{grad}(\Phi) \lrcorner \mathrm{T})$$
$$\nabla\Psi = 0\,, \quad (2 \cdot d\Phi - \mathrm{T}) \cdot \Psi \ = \ 0.$$

The first fermionic equation $\nabla\Psi = 0$ means that the spin holonomy of the connection preserves a spinor. We study the holonomy group of metric connections with totally skew symmetric torsion. For examples, in case of the flat euclidian space this group is always semisimple and does not preserve any non-degenerate 2-form or any spinor. On compact Riemannian manifolds we prove similar results using suitable integral formulas. Generalizations involving the torsion form T of the Schrödinger-Lichnerowicz formula and the Parthasarathy formula for the square of the Dirac operator will be discussed. In particular, these formulas yield an operator Ω acting on spinor fields and defined for any triple (M^n, g, ∇) with totally skew symmetric torsion T,

$$\begin{aligned}\Omega := (D^{1/3})^2 + \frac{1}{8}(d\mathrm{T} - 2\,\sigma_{\mathrm{T}}) + \frac{1}{4}\,\delta(\mathrm{T}) - \frac{1}{8}\,\mathrm{Scal}^g - \frac{1}{16}\,\|\mathrm{T}\|^2 \\ = \Delta_{\mathrm{T}} + \frac{1}{8}(3\,d\mathrm{T} - 2\,\sigma_{\mathrm{T}} + 2\,\delta(\mathrm{T}) + \mathrm{Scal})\end{aligned}$$

e call Ω the *Casimir operator* of the triple (M^n, g, ∇), since in case of a symmetric space it coincides with the group theoretical Casimir operator. Ω has some remarkable properties. For example, its kernel contains all ∇-parallel spinors, the 3-form T acts in its kernel etc. We investigate the integrability condition for parallel spinors as well as the Casimir operator for all the characteristic connections T^{c} of non integrable structures in dimension $n = 5, 6, 7$. Moreover, in these dimensions we will construct explicit solutions of the spinor Killing equation on naturally reductive spaces, for examples on Aloff-Wallach spaces. On the other side, any 7-dimensional 3-Sasakian manifold admits a two-parameter family of metric connections with totally skew symmetric torsion and parallel spinors. The parallelism $\nabla^{\mathrm{c}}\mathrm{T}^{\mathrm{c}} = 0$ of the torsion form of a characteristic connection is an important property. The first reason is that $\nabla^{\mathrm{c}}\mathrm{T}^{\mathrm{c}} = 0$ implies the conservation law $\delta(\mathrm{T}^{\mathrm{c}}) = 0$. Moreover, if the torsion is parallel, several formulas for differential operators acting on spinors simplify and it is possible to investigate the space of parallel or harmonic spinors in more detail. Sasakian structures or nearly Kähler structures (a Theorem of Kirichenko) have a parallel characteristic torsion form, even if they are not reductive. This motivates the investigation of Riemannian G-structures with a parallel characteristic torsion form in general. In dimension $n = 6$ we study the non integrable geometries with this property generalizing, in this sense, Kirichenko's result. Any almost hermitian manifold of type G_1 admits a unique characteristic connection. The U(3)-orbit type of the characteristic torsion is constant. It turns out that there exist only two orbits with a non abelian isotropy (holonomy) group in dimension six. The manifolds under consideration are torus fibrations over some special 4-manifold, twistor spaces or a non Kählerian hermitian structure on the Lie group $\mathrm{SL}(2, \mathbb{C})$. Finally we classify all naturally reductive hermitian $\mathcal{W}_3$-manifolds with small (abelian) isotropy group of the characteristic torsion.

In string theory, one likes to include into the model fluxes, i.e. an additional 4-form F. Then the spinor Killing equation reads as

$$\nabla^g_X\Psi + \frac{1}{4}\cdot(X\rfloor\mathrm{T})\cdot\Psi + \frac{1}{144}\cdot(X\rfloor\mathrm{F} - 8\cdot X\wedge\mathrm{F})\cdot\Psi = 0 .$$

Generalizing our previous results to this case we are able to construct solutions of this Killing equation depending on special non integrable geometries, for example depending on a nearly parallel Γ_2-structure, on a 3-Sasakian structure or on naturally reductive spaces.

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Th.Friedrich and S.Ivanov, *Parallel spinors and connections with skew-symmetric torsion in string theory*, Asian Journ. Math. 6 (2002), 303-336.

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B.Alexandrov, Th.Friedrich and N.Schoemann, *Almost hermitian 6-manifolds revisited*, to appear in Journ. Geom. Phys.

Fiedler, Bernd: Generators of algebraic curvature tensors based on a (2,1)-symmetry

Abstract:

We show that the spaces of algebraic curvature tensors $\mathfrak{R}$ and algebraic covariant derivative curvature tensors $\mathfrak{R}'$ are generated by Young symmetrized product tensors

$$\mathfrak{R}: \ y_t^*(U \otimes w)\ ,\ y_t^*(w \otimes U)\ ,\ \mathfrak{R}': \ y_{t'}^*(U \otimes W)\ ,\ y_{t'}^*(W \otimes U)$$

where w, W, U are covariant tensors of orders 1, 2, 3. Further W is a symmetric or alternating tensor, whereas U belongs to an irreducible symmetry class characterized by the partition $(2,1)$. y_t and $y_{t'}$ denote the Young symmetrizers of the Young tableaux

$$t = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \quad t' = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & \end{pmatrix}$$

If σ, τ are symmetric/alternating tensor fields of order 2 and ∇ is a torsion free covariant derivative, then

$$\nabla\sigma - \mathrm{sym}(\nabla\sigma) \quad \nabla\tau - \mathrm{d}\tau$$

are examples of tensors U.

The set of all symmetry classes from which U can be taken forms an infinite 1-parameter family $\mathfrak{S}_\alpha$, $\alpha \in (-\infty, \infty]$. For both $\mathfrak{R}$ and $\mathfrak{R}'$, there exists exactly one value α_0 such that the expressions (1) vanish for all $U \in \mathfrak{S}_{\alpha_0}$. For all other $\alpha \neq \alpha_0$ the tensors $U \in \mathfrak{S}_\alpha$ lead to generators (1) of algebraic tensors $\mathfrak{R}$ and $\mathfrak{R}'$.

The number of summands in the coordinate representations of (1) depend on the value of α. For $\mathfrak{R}$, $\mathfrak{R}'$ we determine finite sets $\mathcal{A}, \mathcal{A}' \subseteq (-\infty, \infty]$ such that the length of an expression (1) can be minimized iff $\alpha \in \mathcal{A}$ or $\alpha \in \mathcal{A}'$. Furthermore, if the length of an expression (1) is minimal then U admits an index commutation symmetry.

Foundation of our investigations is a theorem of S. A. Fulling, R. C. King, B. G. Wybourne and C. J. Cummins which says that the above Young symmetrizers y_t, $y_{t'}$ generate the symmetry classes of algebraic

tensors $\mathfrak{R}$, $\mathfrak{R}'$. Furthermore we apply ideals and idempotents in group rings $\mathbb{C}[\mathcal{S}_r]$, the Littlewood-Richardson rule and discrete Fourier transforms for symmetric groups $\mathcal{S}_r$. For certain symbolic calculations we used the Mathematica packages *Ricci* and *PERMS*.

Frigioiu, Camelia: The Lagrangian Geometrization in Mechanics

Abstract:

M. C. Munoz-Lecenda and F. J. Yaniz-Fernandez formulated the problem of geometrization of mechanical systems using the geometry of tangent bundle, [3]. They introduced the dissipative mechanical systems and using geometric methods, they applied the theory to dynamical systems and control problems. In this paper, starting with the notion of mechanical system, introduced in their paper, we give a method to study such of problems, using Lagrange Geometry. We shall constructe the Lagrange space for a mechanical system, the semispray and the non-linear connection who derived from the Lagrangian of this space.

Fukaya, Kenji: Homology of Loop space and Floer homology of Lagrangian submanifold

Abstract:

Chen's iterated integral provide a homomorphism from homology of Bar complex of a de Rham complex manifold to a cohomology of its Loop space. We can show that this respects Lie algebra structure defined on them. The Maurer Cartan equation of the Lie algebra structure of Bar complex gives a deformation of cohomology of the manifold. The theorey of Floer homology of Lagrangian submanifold (developped jointly with Oh Ohta Ono) proved such a deformation. We can lift this solution of Mauer Cartan equation to loop space homology. It gives some application to symplectic topology.

Futaki, Akito: Asymptotic Chow semistability and integral invariants

Abstract:

I will discuss the relationship between the exsitence of canonical Kaehler metrics and asymptotic Chow semistability. The role of integral invariants related to holomorphic vector fields is emphasized.

Garcia, Pedro & Lopez, Marco & Rodrigo, Cesar: Euler-Poincar Reduction in Principal Fibre Bundles and Lagrange Multipliers

Abstract:

One of the main reasons of that renewed interest in the calculus of variations with constraints of the last years is, without doubt, the problem of Lagrangian reduction, according to which a certain kind of variational problems, called "reducible", can be "reduced" to lower order constrained variational problems. Unlike the classical treatment of constrained problems, where the "infinitesimal admissible variations" are those induced by deformations satisfying the constraint, for this new class of problems, infinitesimal variations can be more general than the classical ones (vakonomic and non-holonomic mechanics, graviting relativistic fluids, H-minimal Lagrangian submanifolds, etc.). In this situation, it seems natural to revise the traditional Lagrange multiplier method with the object to explore its possible validity within this new context. The subject is still more relevant, taking into account the lack at the present time of a reasonable definition of Cartan form for constrained problems, a concept that, as is known, has been central in the traditional calculus of variations.

In this talk we shall deal with this subject in the case of Euler-Poincar reduction for principal fibre bundles,

which in fact constitutes the first generalization to field theory of this kind of classical reduction from analytical mechanics.

We begin with the statement of the problem, we shall see next how this kind of reduction can be formulated as a constrained variational problem, and compare the results so obtained with those arising from an application of the Lagrange multipliers method which, as we will see, has a nice geometrical interpretation for this case. We give a Cartan formalism for this class of problems, obtaining a corresponding regularity condition for the Lagrangian density, and studying its multisymplectic formalism.

Garcia-Rio, Eduardo: New examples of complete locally conformally flat manifolds of negative Ricci curvatures

Abstract:

Complete locally conformally flat Riemannian manifolds of nonnegative Ricci curvature are known to be in the conformal class of $\mathbb{R}^n$, $\mathbb{R} \times S^{n-1}$ or S^n. Our purpose is to construct new examples of complete locally conformally flat manifolds with nonpositive Ricci curvature, or even negative sectional curvature. Those examples are described as certain warped and multiply warped product spaces.

Garrido Bullon, Angel: Classification Theorems in Lorentzian Manifolds

Abstract:

We make one more complete classification, by the introduction of many causality conditions, such that the Horismoidally Ciclyc or Simple, the Acausality or Achronallity and so, with their recyprocal implications, if there exists.

Gilkey, Peter: Complete k curvature pseudo-Riemannian manifolds which are not locally homogeneous.

Abstract:

We discuss a family of complete simply connected pseudo-Riemannian manifolds of neutral signature $p+3, p+3$ which are $p+2$-curvature homogeneous but which are not $p+3$ affine curvature homogeneous and hence not locally homogeneous. All the local Weyl invariants of these manifolds vanish. This is joint work with Stana Nikcevic.

Gil-Medrano, Olga: A critical radius for unit Hopf vector fields on spheres

Abstract:

The volume of a unit vector field V of the sphere S^n (n odd) is the volume of its image $V(S^n)$ in the unit tangent bundle. Unit Hopf vector fields, that is, unit vector fields that are tangent to the fibre of a Hopf fibration $S^n \to \mathbb{C}\,\mathbb{P}^{\frac{n-1}{2}}$, are well known to be critical for the volume functional. Moreover, Gluck and Ziller proved that these fields achieve the minimum of the volume if $n = 3$ and they opened the question of whether this result would be true for all odd dimensional spheres. It was shown to be inaccurate on spheres of radius one. Indeed, Pedersen exhibited smooth vector fields on the *unit* sphere with less volume than Hopf vector fields for a dimension greater than five. In this article, we consider the situation for any odd dimensional spheres, but not necessarily of radius one. We show that the stability of the Hopf field *actually depends on radius,* instability occurs precisely if and only if $r > \frac{1}{\sqrt{n-4}}$. In particular, the Hopf field cannot be minimum in this range. On the contrary, for r small, a computation shows that the volume of vector

fields built by Pedersen is greater than the volume of the Hopf one thus, in this case, the Hopf vector field remains a candidate to be a minimizer. We then study the asymptotic behaviour of the volume; for small r it is ruled by the first term of the Taylor expansion of the volume. We call this term the *twisting* of the vector field. The lower this term is, the lower the volume of the vector field is for small r. It turns out that unit Hopf vector fields are absolute minima of the twisting. This fact, together with the stability result, gives two positive arguments in favour of the Gluck and Ziller conjecture *for small* r. This is a joint work with V. Borrelli.

Goldberg, Vladislav: See *Akivis, Maks*

Goto, Midori: Global stractures of compact conformally flat semi-symmetric spaces in dimension 3

Abstract:

Let M be a conformally flat manifold of dimension n. The universal covering of M admits a conformal immersion to S^n, called a developing map.

The developing map and the homomorphism of $\pi_1(M)$ into the Moebious group of S^n, whose image is called a holonomy group, form the most important invariants for the study of conformally flat manifolds. When a developing map is a covering map, we can relate the developing image with the limit set of the holonomy group.

We study global structures of compact conformally flat semi-symmetric spaces of dimension 3 whose developing maps are covering maps.

Gover, Rod: Conformally Einstein manifolds and the Fefferman-Graham tensor

Abstract:

The existence of a conformally Einstein structure is equivalent to the existence of a parallel section of a certain vector bundle – the so called standard conformal tractor bundle. From this one can show that the Fefferman-Graham tensor (which is an obstruction to the Fefferman-Graham ambient metric) is an obstruction to a Riemannian (or pseudo-Riemannian) manifold being conformal to Einstein. Other obstrustions to Einstein will also be derived as well as implications of the conformally Einstein condition for conformally invariant differential operartors.

Grebenyuk, Marina: Equipment of the Charakteristic Distributions for the H(M(V)) Distribution in Affine Space.

Abstract:

A triple of distributions of affine space An+1, consisting of the basic distribution of the first kind r-dimensional linear elements r = V (V-distribution), equipping distribution of the first kind m-dimensional linear elements dm = M (M-distribution) and equipping distribution of the first kind of hyperplane elements dn = H (H-distribution) with the relation of the incidence of their corresponding elements in the common center A are called H(M(V))-distribution [1]. Three-component distributions allowing the generalization of the theory of regular and vanishing hyperzone, zone, hyperzone distribution, surfaces of full and non-full range, tangential-equipped surfaces in multidimensional spaces have been studied. H(M(V))-distribution is defined in the frame where we place the individual vectors in the plane Xn-r [2] - charakteristic of the hyperplane dn, when center A moves along the curves, which belong to the basic distribution, and the determined

vectors - in the plane ćn-m [2] - charakteristic of the hyperplane n, when center A moves along the curves, which belong to the equipping M-distribution The obtained geometrical objects determine the inner invariant normals of the first kind for the charakteristic distributions Xn-r and n-m respectively in the differential neighborhood of the second order of the three-component distribution forming element. The normals of the second kind for the charakteristic distributions Xn-r É ć n-m - the planes Nn-r-1(á) and Nn-m-1(á) - are defined by the subsidiary vectors. The tenzors of the second order, which determine the fields of the normals of the second kind for the characteristic distributions, are constructed by an inner invariant way. The results of the research can be applied to a general theory of the distributions in multi-dimensional spaces. The research method is based on the Differential-Geometrical Method developed by Prof. G.Laptev [3].

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Grujic, Gabriela: See *Comic, Irena*

Hansoul, Sarah: On the existence of Natural and Projectively Equivariant Quantization

Abstract:

We show the existence of a natural map depending on the projective class of a linear torsion-free connection between the space of symbols of the differential operators mapping p-forms on a manifold into smooth functions on this manifold, and the space of these differentials operators. We discuss possible generalisations of the construction of this map to other spaces of differential operators.

Harris, Adam: Asymptotic approximation of contact-holomorphic curves

Abstract:

The name "Contact-holomorphic curve" refers to the natural analogue in contact geometry of pseudoholomorphic curves in symplectic geometry. We will discuss aspects of the approximation of these contact curves by holomorphic curves, which leads to a topological description of their asymptotic behaviour near periodic orbits of the Reeb vector field.

Heintze, Ernst: Involutions of affine Kac-Moody algebras and infinite dimensional symmetric spaces

Abstract:

In finite dimensions, compact Lie groups with a biinvariant metric are important examples of Riemannian manifolds. They are in turn special examples of the so called symmetric spaces G/K where K is the fixed point set of an involution on G.

The closest analogue of a compact Lie group in infinite dimensions is an affine Kac-Moody group and thus of a symmetric space, the quotient of an affine Kac-Moody group by the fixed point set of an involution.

The purpose of this talks is to outline a new classification of these infinite dimensional symmetric spaces or equivalently of the involutions of affine Kac-Moody algebras. We show in particular that it can be reduced to well known problems in finite dimensions.

Hineva, Stefka: Ricci Curvature and the Invariants of the Second Fundamental Form

Abstract:

We obtain estimates for the Ricci curvature at a point in the direction of the unit tangent vector to a non-totally geodesic submanifold in an Riemannian manifold by the dimension of the submanifold, the square of the length of the second fundamental form and the square of the mean curvature and show when these estmates are achieved. In the case when the submanifold is a hypersurface in a space form we prove that the equality in the lower bound of the Ricci curvature is held only when the hypersurface is a rotation one.

Hrimiuc, Dragos: Sub-Finslerian Geometry.

Abstract:

The concept of sub-Riemannian geometry is extended to Finsler metrics. We will focus on giving geometric descriptions for normal and abnormal minimizers in terms of some special connections.

Hurtado, Ana: Volume, energy and generalized energy of unit vector fields on Berger's spheres. Stability of Hopf vector fields

Abstract:

This is a joint work with Olga Gil-Medrano. We study to what extent the known results concerning the behaviour of Hopf vector fields, with respect to volume, energy and generalized energy functionals, on the round sphere are still valid for the metrics obtained by performing the canonical variation of the Hopf fibration.

Igonin, Sergei: Coverings and the fundamental group for PDEs

Abstract:

Following I. S. Krasilshchik and A. M. Vinogradov, we regard systems of PDEs as manifolds with involutive distributions and consider their special morphisms called differential coverings, which include constructions like Lax pairs and Backlund transformations in soliton theory. We show that, similarly to usual coverings in topology, at least for some PDEs differential coverings are determined by actions of a sort of fundamental group. This is not a discrete group, but a certain system of Lie groups. From this we deduce an algebraic necessary condition for two PDEs to be connected by a Backlund transformation. We compute the fundamental group for several well-known PDEs and prove that certain PDEs are not connected by any Backlund transformation.

Iordanescu: Radu-Sorin

Abstract:

Jordan structures in differential geometry This talk is based on the second chapter of my recent book "Jordan Structures in Differential Geometry and Physsics" (Editura Academiei Romane,201pp.,2003).I take this opportunity to emphasize :

1)the most important applications of the Jordan structures to differential geometry;

2)how the important contributions of the outstanding Romanian mathematicians Dan Barbilian,Gheorghe Tzitzeica,and Gheorghe Vranceanu are related to Jordan structures;

3)the importance of the results obtained in Romania in this topic,as well as the fact that this topic is at present an important field of research ith a lot of elements to be further developed;

4)some open problems selected from the long list existent in the field.

Ishikawa, Goo: Singularities of improper affine spheres and their duals

Abstract:

We study the equation "Hessian = const" for improper affine spheres from the view point of contact geometry and provide the generic classification of singularities appearing in geometric solutions to the equation as well as their duals. We also mention similar results for developable surfaces and surfaces of constant Gaussian curvature. (A joint work with Y. Machida).

Ivanov, Stefan: Einstein G_2-manifolds with closed fundamental 3-form

Abstract:

We give an answer to a question posed recently by R.Bryant, namely we show that a compact 7-dimensional manifold equipped with a G_2-structure with closed fundamental form is Einstein if and only if the Riemannian holonomy of the induced metric is contained in G_2. This could be considered to be a G_2 analogue of the Goldberg conjecture in almost Kähler geometry. The result was generalized by R.L.Bryant to closed G_2-structures with too tightly pinched Ricci tensor. We extend it in another direction proving that a compact G_2-manifold with closed fundamental form and divergence-free Weyl tensor is a G_2-manifold with parallel fundamental form. We introduce a second symmetric Ricci-type tensor and show that Einstein conditions applied to the two Ricci tensors on a closed G_2-structure again imply that the induced metric has holonomy group contained in G_2.

Itoh, Jin-ichi: On the cut loci of generalized quadratic surfaces

Abstract:

Recently we proved that the cut locus of any point on any ellipsoid is an arc on the curvature line through the antipodal point. Also, we prove that the conjugate locus has exactly four cusps, which is known as the last geometric statement of Jacobi ([1]).

In this talk we study the cut loci of some types of compact Liouville surfaces and determine them on the other quadratic surfaces (two sheeted hyperboloid and elliptic paraboloid), i.e. the cut locus of any point is an arc or two arcs on a curvature line.

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to appear *manuscripta math.*

Janova, Jitka: See *Czudkova, Lenka*

Janyska, Josef: Higher order reduction theorems and their applications

Abstract:

It is well known that natural operators of classical (linear and symmetric) connections on manifolds and of tensor fields with values in natural bundles of order one can be factorized through the curvature tensors, the tensor fields and their covariant differentials. These theorems are known as the first (operators on classical connections only) and the second reduction theorems.

Recently, the reduction theorems were generalized for general linear connections on vector bundles. In this gauge-natural situation we need auxiliary classical connections on the base manifolds. It was proved that natural operators with values in gauge-natural bundles of order (1,0) defined on the space of general linear connections on a vector bundle, on the space of classical connections on the base manifold and on certain tensor bundles can be factorized through the curvature tensors of linear and classical connections, the tensor fields and their covariant differentials with respect to both connections.

In the lecture we shall present another generalization of the reduction theorems (both classical and gauge cases). Namely, we show that the reduction theorems can be proved for operators with values in higher order natural or gauge-natural bundles.

As an application of higher order classical reduction theorems we classify all natural $(0,2)$-tensor fields on the cotangent bundle of a manifold with a linear (non-symmetric) connection. As applications of higher order gauge version of reduction theorems we classify all classical connections on the total space E of a vector bundle $p : E \to M$ and all connections on $J1E$ induced naturally by a given general linear connection on E and a classical connection on M.

Jensen, Gary: Isoparametric Hypersurfaces in Spheres with Four Principal Curvatures

Abstract:

Let M be an isoparametric hypersurface in the sphere S^n with four distinct principal curvatures. Münzner showed that the four principal curvatures can have at most two distinct multiplicities m_1, m_2, and Stolz showed that the pair (m_1, m_2) must either be $(2,2)$, $(4,5)$, or be equal to the multiplicities of an isoparametric hypersurface of FKM-type, constructed by Ferus, Karcher and Münzner from orthogonal representations of Clifford algebras. In this talk I will present joint work with Thomas Cecil and Quo-Shin Chi in which we prove that if the multiplicities satisfy $m_2 \geq 3m_1 - 1$, then the isoparametric hypersurface M must be of FKM-type. Together with known results of Takagi for the case $m_1 = 1$, and Ozeki and Takeuchi for $m_1 = 2$, this handles all possible pairs of multiplicities except for 10 cases, for which the classification problem remains open.

Jimenez, Sonia: Higher dimensional symmetries of PDE systems

Abstract:

We will introduce the notion of higher–dimensional symmetry of a system of partial differential equations in analogy with that of higher–order symmetry. For a wide class of PDE systems we prove that every internal (infinitesimal) symmetry comes from a higher–dimensional external symmetry.

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P.J. Olver, Applications of Lie Groups to Differential Equations, Springer-Verlag, New York, 1986.

Juhl, Andreas: Polynomial families of conformally invariant differential operators

Abstract:

We construct polynomial families of conformally invariant local (differential) operators from functions on a sphere to functions on a subsphere using suitable polynomials in the GJMS-operators of both spheres. The relevant polynomials arise from families of homomorphism of generalized Verma modules and are related to Gegenbauer polynomials. We present some conjectures which extend that picture to the curved case. Among other things it provides a new perspective on the Q-curvature.

Jung, Seoung Dal: Transversal Killing and twistor spinor on the Riemannian foliation.

Abstract:

In this talk, we give the definition of the transversal twistor spinor on the Riemannian foliation. And we study the lower bound for the basic Dirac operator on the Riemannian foliation by using the transversal twistor operator.

Junkl, Marek: See *Lakoma, Lenka*

Kapustina, Tatjana: Infinitesimal holomorphically-projective transformations in tangent bundles of higher order (of Riemannian spaces)

Abstract:

In my work I introduce "sinectic" metric in tangent bundles of higher order as sum of vertical lifting of base metric and intermediate liftings of k (2,0)-type symmetric tensor fields. In this paper I consider infinitesimal holomorphically-projective transformations of this metric.

Kashiwada, Toyoko: Study on a class of almost contact structures in connection with l.c.a. K-structures

Abstract:

An l.c.(a.) K.-manifold is a (an almost) Hermitian manifold $M^{2n}(J,g)$ whose metric is locally conformal to a (an almost) Kähler metric. It is characterized by existence of a closed 1-form α such that

$$d\Omega = 2\alpha \wedge \Omega$$

where Ω is the fundamental form.

L.c.K.-manifolds have been mainly investigated under the condition α to be parallel, as the Hopf manifold is a typical example of such a class. Pertinent example of an l.c.a.K.-manifold with parallel α is a product manifold of a contact manifold N^{2n-1} and the real line. If in particular N^{2n-1} is a Sasakian manifold, the product manifold is an l.c.K.-manifold.

Another notable 1-form on an l.c.a.K.-manifold is $\beta := \alpha \circ J$. In an l.c.K.-manifold with parallel β, we can show that an integral manifold of the distribution $B^{\perp}$ $(B := \beta^{\sharp})$ admits Kenmotsu structure, that is, an almost contact metric structure (φ, η, ξ, g) such that

(i) normal, (ii) $d\Phi = 2\eta \wedge \Phi$, (iii) $d\eta = 0$.

Since a product manifold of a Kenmotsu manifold and the real line is an l.c.K.-manifold with the parallel β, so, we can study a Kenmotsu manifold in connection with the geometry of an l.c.K.-manifold. This angle will give us a direction on the study of almost contact manifolds.

In this talk, we shall further go ahead with this discussion:

We consider l.c.a.K.-manifold satisfying

$(\sharp) \nabla\beta = 0, \mathcal{L}_A J = 0. (A := \alpha^{\sharp})$

Then an integral manifold of $B^{\perp}$ admits an almost contact structure satisfying i

$$(\flat)\Phi = 2\eta \wedge \Phi, \mathcal{L}_{\xi}\varphi = 0$$

where $\Phi(X,Y) = g(X, \varphi Y)$.

The class of almost contact metric manifolds satisfying $(\flat)$ seems then to be meaningful.

From properties of l.c.a.K.-manifolds of $(\sharp)$, we can deduce

Theorem. *Let $N^{2n-1}(\varphi, \eta, g)$ be an almost contact metric manifold satisfying $(\flat)$. If it is conformally flat and the scalar curvature is constant, then N^{2n-1} is Kenmotsu manifold, and hence, of constant sectional curvature* $(= -1)$.

Example: Let $L(J, g_L)$ be an almost Kähler manifold and put $N = \mathbf{R}^1 \times L$. Then, $(\varphi, \xi, \eta, g_0)$ on N defined by

$$\varphi := \begin{pmatrix} 0 & 0 \\ 0 J_L & \end{pmatrix}, \xi := \frac{d}{dt}, \eta := dt, := dt \otimes dt + e^{2t} g_L$$

is an almost contact metric structure satisfying $(\flat)$. In case when $L(J, g_L)$ is a Kähler manifold, it is a Kenmotsu structure. (Cf. Kenmotsu [Tohoku Math.J., 24(1972), 93-103])

Keller, Julien: Asymptotic of Generalized Bergman kernel on a compact Khler manifold

Abstract:

Let E be a holomorphic vector bundle and L an ample line bundle on a smooth Khler manifold M. We give an asymptotic expansion in k of the orthonormal projection of the space of sections $(M, E * L^k)$ towards the space of holomorphic sections $H^0(M, E * L^k)$. The idea is to use the d-bar resolution introduced by Hormander and Demailly. The result is linked to the holomorphic Morse inequalities.

Kim, Eui-Chul: Some extensions of the Einstein-Dirac equation via the variational method

Abstract:

Via the principle of least action, we derive several new types of gene ralizations of the Einstein-Dirac equation. To derive the new Einstein-Dirac sys tems, we consider to apply the variational principle in a non-standard way.

Klokova, Lubov: Quadric hyperstrip and connections of a submanifold in the conformal space

Abstract:

It is known, that Darbouz correspondence maps the confornal space , into a hyperquadric in the projective space. If a submanifold is given, then by means of its image a hypersrtrip is determined which consists of the tangent hyperplanes to in the points of this image and is called the quadric hypership QHm of the submanifold. It is shown that the invariant rigging of a regular hyperstrip, constructed in [2], gives the invariant rigging of, erected by M.I.Akivis [1], by applying it to this QHm.

Kowalski, Oldrich: On 3-dimensional Riemannian manifolds with prescribed Ricci eigenvalues.

Abstract:

In this lecture we deal with 3-dimensional Riemannian manifolds where some conditions are put on their principal Ricci curvatures. First we classify locally all Riemannian 3-manifolds with prescribed distinct Ricci eigenvalues, which can be given as arbitrary real analytic functions. This is a generalization of the result published in [4]. Further we recall, for the CONSTANT distinct Ricci eigenvalues,an explicit solution of the problem, but in a more compact form than it was presented in [2]. Finally, we give a survey of related results where the precribed Ricci eigenvalues are not all distinct. These results were published earlier in various journals. (See, for instance [1], [3]). Last but not least, we compare various PDE methods used for solving problems of this kind.

Keywords: Riemannian manifold, principal Ricci curvatures, PDE methods, curvature homogeneous spaces.

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[2] O.Kowalski, F.Prfer: On Riemannian 3-manifolds with distinct constant Ricci eigevalues.

Kozlowski, Wojciech: Flat functions on manifold.

Abstract:

A goal of my lecture would be a presentation of some results about flat functions on manifold equipped with affine connection, especially on Riemannian manifold. Rougly speaking, a smooth function on manifold

is called flat if its kth covariant derivative is equal to zero for some k. I would show that the set of all flat functions on manifold is a ring that has interesting properties similar to holomorphic or polynomial functions i.e. Identity Principle, Louville type theorem, on compact Riemannian manifold each flat function is constant, on connected manifold the ring of flat functions is an integral domain.

Kozma, Laszlo: Sub-Finslerian geometry

Abstract:

In the talk first the notion and basic questions of sub–Finslerian geometry will be explained. A sub-Finslerian manifold is, roughly speaking, is manifold M which is equipped with a cone (or distribution) $D \subset TM$ and sub-Finslerian function $F : D \to R^+$ with some regularity conditions. This notion, initiated by C. López and E. Martínez in 2000, generalizes the notion of sub–Riemannian geometry, called also Carnot–Carathéodory theory. We discuss how it is possible to introduce the parallelism, covariant derivations and geodesics in this circumstance. Regular and abnormal geodesics will be explained in details. Then we show the relationships and the applicability of this new notion to control theory.

Krbek, Michael: Some representations of the differential group

Abstract:

We shall study representations of the differential group L^r_n of invertible r-jets of maps of $\mathbb{R}^n \to \mathbb{R}^n$ mapping the point 0 to 0. We shall focus our attention to representations realized as spaces of differential forms on the $(r-1)$ prolongation of a given fibered manifold $\pi\colon Y \to X$, $\dim X = n$.

Kristaly, Alexandru: Multiplicity results for elliptic eigenvalue problems on Riemann manifolds

Abstract:

In this talk we present some multiplicity results for elliptic eigenvalue problems which are formulated on Riemann manifolds, involving concave and convex nonlinearities.

Krupkova, Olga: Lepagean forms in the calculus of variations, mechanics and field theory

Abstract:

A survey of the theory of Lepagean forms will be presented, with a particular stress on Lepagean equivalents of Lagrangians and dynamical forms. Various applications of Lepagean forms in the calculus of variations, as well as in higher-order mechanics and field theory will be discussed.

Kurek, Jan: Canonical affinors on the tangent bundle of symplectic manifolds

Abstract:

We describe all canonical tensor fields of type (1,1) (affinors) on the tangent bundle of a symplectic manifolds.

Lakoma, Lenka & Mikes, Josef & Junkl, Marek: The e-decomposition of tensor spaces of the type (2,2)

Abstract:

Decomposition of tensor spaces with almost complex structuresis a standard task in representation theory and in differential geometry. We deduced explicit formulas for tensors of the type (2,2) which have some special algebraic properties.

Langerock, Bavo: Holonomy for generalised connections

Abstract:

The concept of holonomy for generalised connections over a bundle map on a principal fibre bundle is introduced. We show some basic properties of these holonomy groups and briefly discuss possible fields of applications.

Lauret, Jorge: Geometric structures on nilmanifolds: a distinguished compatible metric.

Abstract:

Let (N, S) be a nilpotent Lie group endowed with an invariant geometric structure S (cf. symplectic, complex, hypercomplex). We define a left invariant Riemannian metric on N compatible with S to be minimal, if it minimizes the norm of the invariant part of the Ricci tensor along all compatible metrics with the same scalar curvature. We prove that minimal metrics (if any) are unique up to isometry and scaling, develop soliton solutions for the invariant Ricci flow and are characterized as the critical points of a natural curvature functional. The uniqueness allows us to distinguish two geometric structures with Riemannian data, giving rise to a great deal of invariants.

Our approach proposes to vary Lie brackets rather than inner products; our tool is the moment map for the action of GL(n) on the algebraic variety of all nilpotent Lie algebras of dimension n, which is proved to coincide in this setting with the Ricci operator. This gives us the possibility to use strong results from geometric invariant theory. We describe the moduli space of all isomorphism classes of geometric structures on nilpotent Lie groups of a given class and dimension admitting a minimal compatible metric, as the disjoint union of semi-algebraic varieties which are homeomorphic to categorical quotients of suitable linear actions of reductive Lie groups. Such special geometric structures can therefore be distinguished by using invariant polynomials.

Leandre, Remi: White noise analysis, filtering equation and the ¿Index theorem for families

Abstract:

Bismut pioneered the relation between the Index theorem of families and the filtering equation of stochastic analysis. We define the Bismut-Witten current over the loop space as an Hida distribution with values in the space of L^2-forms on the free loop space of the based manifold. Connection is done with Bismut representation of the Chern character of a family of Dirac operators by using superconnections.

Loubeau, Eric: The stability of biharmonic maps

Abstract:

Biharmonic maps are the critical points of the bienergy functional and generalise harmonic maps. We investigate the index of a class of biharmonic maps, derived from minimal Riemannian immersions into spheres. We also consider the Hopf map and modify it into a nonharmonic biharmonic map which we show to be unstable and estimate its biharmonic index and nullity.

Macias-Virgos, Enrique: Diffeological methods in Foliations theory

Abstract:

J.M. Souriau's diffeological spaces are a useful tool for the study of geometrical properties of the space of leaves of a foliated manifold.

Manno, Gianni: On symmetries and covering of the equation of minimal surfaces.

Abstract:

The geometry of minimal submanifolds is studied in the framework of jets of submanifolds. Symmetries and covering are discussed in the case of minimal surfaces and geodesics.

Matveev, Vladimir: Lichnerowicz-Obata conjecture

Abstract:

Let a connected Lie group act on a complete Riemannian manifold by diffeomorphisms that take (unparameterized) geodesics to geodesics. Then, the group acts by affine transformations, or the metric has constant positive sectional curvature.

Matsumoto, Koji: Twisted warped product CR-submanifolds in a Kaehler manifolds

Abstract:

Let R^4 be a 4-dimensional real number space with a coordinate system (x^1, x^2, x^3, x^4). We define a Riemannian metric $g = (g_{ji})$ on R^4 as

$$(g_{ji}) = \begin{pmatrix} 1 & 0 & -kx^1 & 0 \\ 0 & A_1 & kx^2A_1 & kx^1 \\ -kx^1 & kx^2A_1 & A_1A_2 & k^2x^1x^2 \\ 0 & kx^1 & k^2x^1x^2 & 1 \end{pmatrix}$$

for certain constant k, where we put

$$A_1 = 1 + k^2(x^1)^2, \quad A_2 = 1 + k^2(x^2)^2.$$

We write this Riemannian manifold as $\tilde{R}^4(k)$. Then, by the straightfoward calculation, the manifold $\tilde{R}^4(k)$ is of the negative constant scalar curvature r,

$$r = -\frac{5}{2}k^2$$

for non-zero constant k.

Remark. Of course, if $k = 0$, then our manifold is Euclidean.

Next, if we define a (1,1) tensor $F = (F_j{}^i)$ as

$$(F_j{}^i) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & -kx^2 & 1 & 0 \\ 0 & -A_2 & kx^2 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

then we can show that (F, g) is a Hermitian structure on $\tilde{R}^4(k)$, that is,

$$F^2(X) = -X, \quad g(FX, FY) = g(X, Y), \quad N_F(X, Y) = 0$$

for any $X, Y \in \Gamma T\tilde{R}^4(k)$, where N_F denotes the Nijenhuis tensor with respect to F.

We define a new Riemannian metric g^* as

$$g^* = Ce^{-\frac{1}{2}kx^3}g$$

for any positive constant C. Then we can show that (F, g^*) is a Kaehlerian structure on $\tilde{R}^4(k)$.

Matsyuk, Roman:

Abstract:

As an example of higher-order constrained differential systems we show that the motion of the quasi-classical relativistic spinning particle may be described by a fourth-order constrained differential equation generated by a Lagrangian function on a submanifold with physical constraints. This dynamical system is rigorously obtained from the so-called Dixon equations in flat space-time.

Mestdag, Tom: A Lie algebroid approach to Lagrangian systems with symmetry.

Abstract:

The Euler-Lagrangian equations of a Lagrangian which is invariant under a group action can be reduced to the so-called Lagrange-Poincare equations. If, in addition, we assume the presence of a connection, these equations can be seen to fall apart into a 'horizontal' and a 'vertical' equation. In this talk, we will obtain an intrinsic characterization of both equations, based on a decomposition of the Lie algebroid structure of the carrying space. The method can be extended to nonholonomic systems with symmetry.

Michor, Peter: Riemannian metrics on spaces of plane curves and of immersions.

Abstract:

We study some weak Riemannian metrics on spaces of immersions from a compact manifold M into a higher dimensional Riemannian manifold, modulo the group of diffeomorphisms of M. Of particular interest is the special case of spaces of regular smooth curves in the plane, viewed as orbit space under the group of diffeomorphisms of the circle acting as reparameterizations. There the most interesting metric is (for a constant $A > 0$):

$$G_c^A(h, k) := \int_{S^1} (1 + A\kappa_c(\theta)^2)\langle h(\theta), k(\theta)\rangle |c'(\theta)|\, d\theta$$

where κ_c is the curvature of the curve c.

Big surprise: For $A = 0$ this induces 0 as geodesic distance function on the orbit space. But for $A > 0$ is induces a point separating metric. We give some estimates for the distance function, derive the geodesic equation and the sectional curvature, solve the geodesic equation with simple endpoints numerically, and pose some open questions. Most of the results for plane curves hold also in the general situation.

Mihai, Adela-Gabriela: Geometric Inequalities for Submanifolds in Generalized Complex Space Forms

Abstract:

We consider several classes of submanifolds (invariant, totally real, slant, CR-submanifolds) in generalized complex space forms. Geometric estimates of intrinsic invariants (e.g., Ricci curvature, scalar curvature, Chen invariants) in terms of the main extrinsic invariant, namely the squared mean curvature, are obtianed for such submanifolds. Equality cases are investigated.

Mikes, Josef: F-planar mappings onto special Riemannian spaces (with O. Pokorna, V.Kiosak, V. Berezovski)

Abstract:

F-planar mappings and transformations onto special Riemannian spaces are studied, for instance symmetric, semisymmetric, Kählerian and Hermitian spaces. Relations between F-planar mappings and almost geodesic mappings are founded.

Minguzzi, Ettore: Maximizing curves for the charged particle action in globally hyperbolic spacetimes

Abstract:

In a globally hyperbolic spacetime given two events, the second in the chronological future of the first, there is always a connecting geodesic. We present two theorems which prove that, more generally, under weak assumptions, given a charge-to-mass ratio there is always connecting solution of the Lorentz force equation having that ratio.

Mira, Pablo: The Cauchy problem for Bryant surfaces

Abstract:

Given a regular analytic curve in the hyperbolic 3-space H^3 together with an analytic distribution of tangent planes along it, we construct the only surface in H^3 with mean curvature one (i.e. the only Bryant surface) that spans this configuration. As applications, we treat symmetries, period problems and planar geodesics of Bryant surfaces. This is a joint work with J.A. Gálvez.

Modugno, Marco & Janyska, Josef: Special Lie algebras and Hermitian vector fields.

Abstract:

In the framework of covariant quantum mechanics, special quadratic functions play a fundamental role. They constitute a Lie, which generates classical and quantum symmetries. In particular, this Lie algebra is naturally isomorphic to the Lie algebra of Hermitian vector field of the quantum bundle.

In this talk we analyse the basic properties of this Lie algebra and compare the two different cases aring in the Galilei and Einstein frameworks.

Molnar, Emil: Projective models of homogeneous 3-orbifolds and -manifolds

Abstract:

Recently, I have given the projective interpretation of the eight maximal homogeneous 3-geometries: E3,

S3, H3, S2 X R, H2 X R, SL2R, Nil, Sol (Beitrage Alg. Geom 38/2 (1997), 261-268). Thus we have the possibility to search for discretely (or freely) acting groups G with finite covolume, and to investigate some properties of the factor space H3 /G where H3 denotes any of the above Thurston geometries. Some interesting phenomena occur: as parallel realizations with different metrics, but also the opposite: finite volume orbifold with cusp of H2 X R metric, and not of SL2R metric. This last phenomenon contradicts to a Thurston "theorem (?)" (Th. 4.7.10 in Thurston W.P., Three-dimensional geometry and topology I., Ed. by Silvio Levy, Princeton Univ. Press, Princeton, New Jersey, (1997)).

Munteanu, Marian-Ioan: CR-submanifolds of CR-codimension 2 on hypersurfaces of Sasakian manifolds

Abstract:

Let us consider a hypersurface in a Sasakian manifold. It is possible, in a natural way, to define a CR-structure of CR-codimention 2 on the hypersurface. We defined a torsion free connection associated in a certain way to the f-structure subordinated to the CR structure. We analysed the properties of this connection as a generalization of the adapted connection in dimension 1.

Murathan, Cengizhan: Ricci generalized Pseudo-parallel Hypersurfaces

Abstract:

In the present paper we investigate Ricci generalized pseudo-parallel hypersurfaces M in a space form $N^{n+1}(c)$. Using the obtained results we show aditional curvature properties of investigated hypersurfaces. We prove that Ricci generalized pseudo-paralel hypersurface M in a space form $N^{n+1}(c)$ is either a hypersurface of E^{n+1}, or pseudo-paralel, or totally-umbilical, or has at most two distinct principal curvatures.

Nagy, Paul-Andi: Algebraic reduction and local structure of certain almost Kaehler manifolds

Abstract:

We study almost Kähler manifolds whose curvature tensor satisfies the third curvature condition of Gray. We show that the study of manifolds within this class reduces to the study of a subclass having the property that the torsion of the first canonical Hermitian connection has the simplest possible algebraic form. This allows to understand the structure of the Kähler nullity of an almost Kähler manifold with parallel torsion. We show that the Einstein coupled to the third Gray condition implies the vanishing of the Ricci tensor; therefore the Golberg conjecture holds true inside this special class of manifolds. Moreover, under some other extra assumptions, mainly of algebraic nature, we provide a local description of all possible almost Kaehler manifolds suject to the third Gray condition.

Nakanishi, Nobutada: Nambu-Poisson Cohomology

Abstract:

We compute Nambu-Poisson cohomology for Nambu-Poisson tensors of order 3 defined on $\mathbb{R}^4$. In particular, we show that Nambu-Poisson ohomology group of exact Nambu-Poisson tensors is isomorphic to so-called "relative cohomology".

Nikcevic, Stana: Complete curvature homogeneous pseudo-Riemannian manifolds

Abstract:

We exhibit 3 families of complete curvature homogeneous pseudo-Riemannian manifolds which are modeled on irreducible symmetric spaces and which are not locally homogeneous. Some of the manifolds in the family are Jordan Osserman and Jordan Ivanov-Petrova. This is joint work with Peter B. Gilkey.

Nikolayevsky, Yuri: On Osserman Conjecture

Abstract:

Let M^n be a Riemannian manifold, with the curvature tensor R. For a point $p \in M^n$ and a tangent vector $X \in T_pM^n$, the Jacobi operator $R_X : T_pM^n \to T_pM^n$ is defined by $R_XY = R(X,Y)X$.

The manifold M^n is called *pointwise Osserman* if, for every $p \in M^n$, the eigenvalues of the Jacobi operator R_X do not depend of a unit vector $X \in T_pM^n$, and is called *globally Osserman* if they do not depend of the point p either. R. Osserman conjectured that globally Osserman manifolds are flat or locally rank-one symmetric. This Conjecture is proved for all n, except for $n = 16$:

Theorem. In each of the following cases a Riemannian manifold M^n is flat or locally rank-one symmetric:

1) M^n is globally Osserman and $n \neq 16$.

2) M^n is pointwise Osserman and $n \neq 2, 4, 16$.

3) $n = 16$, the manifold M^{16} is (pointwise or globally) Osserman, and its Jacobi operator has no eigenvalues of multiplicity $m \in \{7, 8, 9\}$.

The proof follows the two-step approach suggested in [P.Gilkey, A.Swann, L.Vanhecke, Quart. J. Math. Oxford (2), 46(1995), 299 – 320]:

(i) find all Osserman algebraic curvature tensors;

(ii) classify Riemannian manifolds having curvature tensor as in (i).

The key idea is the proof of the fact that in all the cases covered by the Theorem, an Osserman algebraic curvature tensor has a *Clifford structure* introduced in [P.Gilkey, J. Geom. Anal., 4(1994), 155 – 158] and in the paper cited above.

Okayasu, Takashi: On hypersurfaces with constant scalar curvature

Abstract:

We show that in Euclidean space there is no closed hypersurface with constant scalar curvature which is of cohomogeneity 2 other that the spheres. In noncompact case, we show there are many complete hypersurfaces with constant scalar curvature. We also study hypersurfaces in the hyperbolic spaces.

Olszak, Karina: Curvature properties of Kahler manifolds with norden metrics

Abstract:

Using the one-to-one correspondence between Kahler-Norden (precisely, Kahler structures with Norden metrics), it is possible to describe many curvature properties of Kahler-Norden metrics. Especially, it is interesting to know relationships between holomorphic Weyl conformal curvature and Bochner curvature.

Olszak, Zbigniew: On almost cosymplectic manifolds

Abstract:

Almost cosymplectic manifolds (in the sense of S.I.Goldberg and K.Yano) are those almost contact metric manifolds for which both structure forms are closed. A special class of such manifolds (the so-called almost cosymplectic (κ, μ, ν)-spaces) is investigated. The leaves of their canonical foliation are always Kahlerian manifolds. It is possible to describe the local structure of such manifolds. For, models of such spaces are constructed. In the case when the function μ is constant, the models are left invariant with respect to special Lie groups actions.

Ortacgil, Ercument: Characteristic algebras of second order almost structures

Abstract:

We define second order almost structures (SOAS's) on a differentiable manifold M and bundle maps between them. Such structures exist in abundance and are particular examples of diffieties. Some SOAS defines a general principle bundle. We raise the question whether two SOAS's which are isomorphic as general principle bundles are also isomorphic in our restricted sense. Some SOAS x and its projection px determine algebras A(1), pA(2) which are invariant Lie algebra cohomologies of the automorphism bundles of x and px. A(1) and pA(2) fit into a short exact sequence. pA(2) coincides with the Pontryagin algebra of the general principle bundle defined by x via Chern- Weil construction. We believe that A(2) is a diffeomorphism invariant.

Ozdemir, Fatma: On L-Structures and Nearly Kahlerian Structures on Weyl Spaces

Abstract:

In this work , we give the definitions of the L-Structures and nearly Kahlerian structures on Weyl spaces and prove some theorems concerning these structures.

Palese, Marcella: Covariant gauge-natural conservation laws and equivalence classes in finite order variational sequences

Abstract:

When a gauge-natural invariant variational principle is assigned, to get *canonical* covariant conservation laws, the vertical part of gauge-natural lifts of infinitesimal principal automorphisms – defining infinitesimal variations of sections of gauge-natural bundles – must satisfy generalized Jacobi equations for the gauge-natural invariant Lagrangian. *Vice versa* all vertical parts of gauge-natural lifts of infinitesimal principal automorphisms which are in the kernel of generalized Jacobi morphisms are generators of canonical covariant currents and superpotentials. The role of the geometric structure of variational calculus on fibered manifolds - and specifically on gauge-natural bundles - as worked out within the Krupka's *finite order* variational sequences framework is here crucial and our results provide a structural example of the power of such a geometric framework for understanding of Physics. In particular, only a few gauge-natural lifts can be considered as canonical generators of covariant gauge-natural physical charges. (Joint with E. Winterroth)

Pavlov, Maxim: Solution of classical problem in Differential Geometry. Description of surfaces determined by affine Blaschke metric

Abstract:

The components of radius-vector in affine diffenential geometry in particular, but important case (affine Blaschke metric) satisfy for linear system in partial derivatives with variable coefficient. We prove that this system can be interpreted as integrable system with special constraint. This system was solved exactly. Three-parametric family of solutions was found. Variable coefficient and radius-vector are described by two elliptic functions. Corresponding surfaces are constructed explicitely.

Peralta-Salas, Daniel: On the geometry and topology of the equilibrium shapes on manifolds

Abstract:

Let $(P1)$ be certain elliptic free-boundary problem on a Riemannian manifold (M,g). In this work [1] we study the restrictions on the topology and geometry of the fibres $f^{-1}(t)$, $t \in f(M)$, of the solutions f to $(P1)$. We study these restrictions with geometrical rather than with analytical techniques. The form of $(P1)$ is a generalization of the equations modeling static self-gravitating fluids. Firstly we prove a remarkable property of the fibres: they can be represented by analytic functions across the free-boundary. This property is a consequence of the ellipticity of the problem and the regularity of the solutions at the free-boundary. Then we prove [1] that the partition of (M,g) induced by these solutions is an equilibrium partition.

Definition. An analytic function I is an equilibrium function if I, $||\nabla I||^2$ and ΔI agree fibrewise. The partition induced by an equilibrium function is called an equilibrium partition.

For general Riemannian manifolds we prove [1] the following theorem

Theorem. Any equilibrium partition of a Riemannian manifold possesses a fibre bundle local structure, each one of its fibres has constant mean curvature and locally the neighbour fibres are geodesically parallel.

We also give [1] further properties of the equilibrium partitions in more specific spaces, i. e. locally symmetric, conformally flat and constant curvature Riemannian manifolds. We apply [1] these results to the classical problem in physics of classifying the equilibrium shapes of both Newtonian and relativistic self-gravitating fluids. Our techniques are completely different to the previous ones appeared in the literature of this topic. We recover and generalize the classical theorems of Lichtenstein, Lindblom, ... without any physical assumption and lay down the foundations of a general theory of equilibrium shapes in general spaces [1].

Reference

[1] Pelayo, A. and Peralta-Salas, D.: Topological and geometrical restrictions, free-boundary problems and self-gravitating fluids. Preprint available at math-ph/0305038.

Popescu, Liviu Octavian: New aspects in sub-Finslerian geometry

Abstract:

I intend to extend the sub-riemannian geometry to the more general case of sub-finslerian geometry with applications in control theory, nonholonomic mechanics. The associated connections, geodesics and some examples will be studied.

Prince, Geoff: EDS and the inverse problem in the calculus of variations

Abstract:

The inverse problem in the calculus of variations for a given set of second order ordinary differential equations consists of deciding whether their solutions are those of Euler–Lagrange equations and exhibiting the

nonuniqueness of the resulting Lagrangians when they occur.

This paper discusses recent advances in the use of exterior differential systems theory in this problem to produce a classification scheme for second order ordinary differential equations and to solve some cases.

Pugliese, Fabrizio: Symmetries of an equation arising in General Relativity

Abstract:

Local symmetries of an elliptic equation of Euler-Daboux type describing some Ricci-flat metrics with non-abelian bidimensional Killing algebra are computed. The structure of the symmetry algebra is also described.

Purcaru, Monica: On N-linear Connections Compatible with Conformal Structures

Abstract:

The purpose of the present paper is to introduce the concept of conformal d-structure in a Lagrange geometry of order 2. One studies some properties of this notion. One finds the transformations for the coefficients of an N-linear connection in the Lagrange geometry of order 2, by a transformation of nonlinear connection. One solves the problem of determining the set of all conformal metrical and almost symplectic n-linear connections in the case when the nonlinear connection is arbitrary and treats also some important particular cases. Finally one finds the group of transformations of the conformal metrical and almost symplectic N-linear connections, which preserve the nonlinear connection N.

Ratiu, Tudor: Momentum maps and redcution

Abstract:

Several aspects of symplectic reduction theory will be addressed, such as the optimal momentum map, singular reduction, their relation to groupoid theory, and their link to Poisson reduction and dual pairs.

Rodionov, Eugene: On the geometry of homogeneous Riemannian manifolds

Abstract:

In these abstracts we discuss some questions from the theory of homogeneous Riemannian manifolds. This paper contains the following topics. 1. Geodesics lines of homogeneous Riemannian manifolds. Homogeneous Riemannian manifolds with closed geodesics. The clousers of geodesic lines of homogeneous Riemannian manifols. 2. Homogeneous Riemannian manifolds of positive curvature. The case of the sectional curvature. Homogeneous Riemannian manifolds with positive Ricci curvature. One-dimensional curvature of homogeneous Riemannian spaces. 3. Conformal deformations of the homogeneous Riemannian metrics. Locally conformally homogeneous Riemannian and pseudo-riemannian spaces. 4. Homogeneous Riemannian manifolds with Einstein metric. General results. Homogeneous Killing manifolds with Einstein metric. The case of low dimension (dim ¡ 8).

Rodriguez, Jesus: Differential correspondences and characteristics (joint work with S. Jimenez and R. J. Alonso)

Abstract:

In a long paper published in 1895 (see [4]), S. Lie attempted to reduce, as far as possible, the theory of

systems of partial differential equations to that of systems of first order with only one unknown function. Such a reduction is a consequence of certain correspondences between jet spaces, which allow to transform PDE systems of a certain type into PDE systems of another kind.

Following this ideas we started in [3] a theory of differential correspondences based in a theory of jets developed in [5], which is the natural continuation of the theory of Weil's near points [6]. The main idea is: a jet of a smooth manifold is an ideal of its ring of smooth functions, and the tangent spaces, prolongations and other processes are described in terms of the same ring. The natural relation of inclusion of ideals gives rise to canonical correspondences between jet spaces.

These correspondences allow to associate with each PDE system of a suitable class another PDE system which contains as solutions that of the original one as well as its intermediate integrals (see [3] for details). Similar ideas are used in [1] to give the first intrinsic formulation and proof of Drach's theorem.

In this talk we apply the theory of differential correspondences to the study of the characteristics of PDE systems. We compute the tangent space of the Lie correspondence at a point and describe the characteristics of PDE systems in terms of differential correspondences, clarifying the meaning of the characteristic covectors. As an example we describe the characteristic directions of a PDE system with two independent variables in terms of differential correspondences.

1. R. Alonso, *A transformation of PDE systems related to Drach theorem*, to appear in J. Math. Anal. Appl.

2. E. Goursat, *Leçons sur l'integration des équations aux dérivées partialles du second ordre à deux variables indépendantes,* Tom. II, Hermann, Paris, 1898.

3. S. Jiménez, J. Muñoz, and J. Rodríguez, *On the reduction of some systems of partial differential equations to first order systems with only one unknown function*, in: Proceedings of the VIII Conference on Diff. Geom. and its Appl. (Opava 2001), Silesian Univ., Opava, 2002, 187-196.

4. S. Lie, *Zur allgemeine Theorie der partiellen Differentialgleichungen beliebiger Ordnung*, Ges. Abh., Bd IV, 320-389.

5. J. Muñoz, F. J. Muriel, and J. Rodríguez, *Weil bundles and jet spaces*, Czech. Math. J. **50** (125) (2000), 721-748.

6. A. Weil, *Théorie des points proches sur les variétés différentiables*, Colloque de Géometrie Différentielle, C.N.R.S. (1953), 111–117.

Rosado, Eugenia: Covariant Hamiltonians for first-order variational problems

Abstract:

Let γ be a first-order Ehresmann connection on the linear frame bundle of a manifold N. The notion of torsion associated to γ is defined. Then, the covariant Hamiltonian Λ^{γ} associated with a first-order Lagrangian is constructed and Λ^{γ} is shown to be diffeomorphism-invariant for every diffeomorphism-invariant Lagrangian density Λ if and only if the torsion associated with γ is constant. The case of a linear connection is also studied.

Rosales, Cesar: Stable constant mean curvature hypersurfaces inside convex domains

Abstract:

Constant mean curvature (CMC) surfaces are geometric objects that model multitude of physical phenomena; they appear, for example, as the interfaces between two inmiscible fluids or between two gases at different pressures. In the context of the Calculus of Variations, CMC surfaces arise as critical points of the area for *volume preserving variations.*

In this talk we study *stable CMC hypersurfaces* inside a domain D: by definition, these are local minima of the area among the hypersurfaces in D separating a given amount of volume. By using variational and geometric arguments, we give a complete description of compact, stable CMC hypersurfaces inside certain convex domains C which are invariant under the action of a group of dilations (convex cones, for example). As a consequence, we also characterize the *isoperimetric hypersurfaces* in C -global minima of the area inside C for fixed volume.

Rybicki, Tomasz: On the perfectness of groups of homeomorphisms with no restriction on support

Abstract:

It is well known that the identity component of the group of all compactly supported homeomorphisms of a manifold is perfect and simple. The same is true for the identity component of the group of all compactly supported C^r-diffeomorphisms provided $1 \leq r\ leq\infty$, $r \neq n+1$, and n is the dimension of the manifold. Several generalizations for the automorphism groups of geometric structures are known. The problem of the perfectness of analogous groups with no restriction on support is studied. By making use of deformation principles of Siebenmann we investigate under what conditions homeomorphism and diffeomorphism groups are perfect provided so are their compactly supported subgroups.

(joint with Jacek Lech)

Samiou, Evangelia: Homogeneous spaces with sections (joint work with Andreas Kollross)

Abstract:

We study homogeneous Riemannian manifolds all of whose geodesics can be mapped by some isometry into a fixed homogeneous, connected, totally geodesic submanifold Σ; we call such a submanifold Σ a section. We show that these spaces are locally symmetric if the section is two-dimensional and give non-symmetric counterexamples with higher-dimensional sections. However, we can still show that the manifold is locally symmetric also for higher-dimensional sections if we require in addition that the orbits of the isotropy group at some point $p \in \Sigma$ all meet Σ orthogonally and that the isotropy representation has no trivial submodules.

Samokhin, Alexey: Symmetries of first order PDEs

Abstract:

The structure of the full symmetry algebra fo a general first order partial differential equation is described. Exaqmples are provided.

Sarlet, Willy: Adjoint symmetries in non-holonomic mechanics

Abstract:

The type of non-holonomic mechanical systems we have in mind is quite general: the system can be time-dependent and the non-holonomic constraints need not be linear or affine. The constraints are simply

modelled by a given submanifold C of the first-jet bundle $J^1\tau$ of the evolution space $\tau : E \to \mathbb{R}$, and the dynamical system is considered to be a second-order differential equation field Γ, living directly on C. We discuss how the fact that this Γ comes from non-holonomic mechanics in the sense of the d'Alembert-Chetaev principle, is essentially encoded in the availability of a projection operator P, which maps arbitrary vector fields along the projection $\pi_C : C \to E$ onto those having the property that their vertical lift is tangent to C. The geometrical benefit coming from P is that it gives rise to an inherited vertical endomorphism-type tensor field on the constraint submanifold C, which then in turn leads to a natural construction of a non-linear connection, associated to the dynamics Γ.

The main purpose of the talk is to show how in this fairly general set-up, the theory of adjoint symmetries can be developed, with the aid of the tools referred to above. We shall discuss the general mechanism by which all first integrals of the system can be obtained, in principle, through an algorithmic search for adjoint symmetries. The computational aspects of the method will be illustrated by some simple examples.

Savo, Alessandro: Some eigenvalue estimates for the Hodge Laplacian

Abstract:

We plan to discuss some new estimates for the first positive eigenvalue of the Hodge Laplacian acting on differential forms of a compact manifold. Special attention will be given to manifolds with boundary (in particular, convex Euclidean and spherical domains) and to isometric immersion in some Euclidean space.

Sekizava, Masami: On curvatures of linear frame bundles with naturally lifted Riemannian metrics (joint work with O. Kowalski)

Abstract:

Inspired by an earlier paper by L.Cordero and M. de Leon (J.math.pure et appl.), we study curvature properties of a linear frame bundle LM with special Riemannian metrics "naturally lifted" from a Riemannian metric of the base manifold M. We are mainly interested in the scalar curvature. Whereas the "canonical metric" studied by Cordero and de Leon has the "rigidity" property saying that the constant scalar curvature on LM always implies flatness of M, we give examples of natural metrics on LM with the "opposite" property: If M is a space of nonzero constant sectional curvature K, then the scalar curvature of LM is a nonzero constant multiple of K and hence a nonzero constant. A completely analogous situation occured before in the theory of natural metrics on tangent bundles. There is a result by E.Musso and F.Tricerri about the "rigidity" of Sasaki metric and the construction of a new natural metric on TM by V.Oproiu which has the analogous property as our metric on LM.

Shurygin,Vadim V.: Brylinski cohomology of poisson manifolds and quantum de Rham cohomology

Abstract:

In the present paper, we study the cohomology of the canonical double complexes of Poisson manifolds. This cohomology was introduced by J.-L. Brylinski in [1]. We prove that, for any Poisson manifold (M, w), this cohomology is isomorphic to the de Rham cohomology of M. In the case of a symplectic manifold, this was shown in [1]. As an application of the result obtained, we prove that the quantum de Rham cohomology of a Poisson manifold introduced by H.-D. Cao and J. Zhou in [2] is a deformation quantization of the ordinary de Rham cohomology.

[1] J.-L. Brylinski, *A differential complex for Poisson manifolds*, J. Diff. Geom. **28** (1988), 93–114.

[2] H.-D. Cao, J. Zhou, *On quantum de Rham cohomology*, preprint math.DG/9806157, 1998.

Simon, Udo: Affine hypersurface theories revisited: gauge invariant structures

Abstract:

Following Felix Klein's program, several differential geometries were developed (projective, equiaffine, centroaffine etc.) in the first half of the last century. In the 80's K. Nomizu initiated a more structural approach w.r.t. the unimodular group. Considering arbitrary normalizations of a non-degenerate hypersurface, in most cases the invariance group is unknown. This suggests the study of such invariants of hypersurfaces in affine space w. r. t. the general affine transformation group which are independent of a normalization. We presented first results of this type at the Brno conference in 1986, considering the conformal structure of relative metrics, the projectively flat structure of conormal connections, and further invariants, and studied their geometric properties. We are now able to give a complete description of the geometry of non-degenerate hypersurfaces in affine space in terms of such invariants; one essential tool is a Weyl structure together with its gauge transformations on a given affine hypersurface; the gauge transformations are equivalent to a change of the normalization. We give a geometric test of this approach, discussing our invariants in the context of well known problems in affine hypersurface theory, e.g. recent solutions of the affine Bernstein conjectures of Calabi and Chern.

Škodová Marie (with J. Mikeš and O. Pokorná): On holomorphically projective mappings from equiaffine symmetric and recurrent spaces onto Kählerian spaces

Abstract:

In this contribution holomorphically projective mappings from the semisymmetric, symmetric and recurrent equiaffine spaces A_n onto (pseudo-) Kählerian spaces $\bar{K}_n$ are studied. We proved that in this case space A_n is holomorphically projective flat and $\bar{K}_n$ is space with constant holomorphic curvature.

These results are the generalization of results by T. Sakaguchi, J. Mikeš, V.V. Domashev, which were done for holomorphically projective mappings of symmetric, recurrent and semisymmetric Kählerian spaces.

Smetanova, Dana: On Regularizable Lagrangians in fiel theory

Abstract:

Hamilton equations based not only upon the Poincar–Cartan equivalent of a first order Lagrangian, but rather upon its Lepagean equivalent are investigated. The case of an second order Lagrangian is discussed. Lagrangians which are singular within the Hamilton–De Donder theory, but regularizable in this generalized sence are studied.

Smirnov, Roman: A new invariant theory: Invariants, covariants and joint invariants of Killing tensors

Abstract:

Multiple factors have contributed to the recent resurgence of interest among mathematicians in the classical invariant theory (CIT) of homogeneous polynomials.

The factor that brought my collaborators and me to the area is that the underlying ideas of CIT can be naturally incorporated into the geometric study of (generalized) Killing tensors defined in pseudo-Riemannian

spaces of constant curvature. The resulting invariant theory of Killing tensors (ITKT) shares many of the same features with the original CIT.

I target to review some of the results obtained so far.

Stepanov, Sergey E.: The seven classes of equiaffine mappings of pseudo-Riemannian manifolds

Abstract:

Let M be a differentiable manifold M with an *equiaffine structure* that is a pair (η, ∇) where η is a volume element and ∇ is a linear connection with zero torsion such that $\nabla\eta = 0$ (see [1]). Then $divX = trace(\nabla X)$ for an arbitrary $X \in TM$.

Suppose that M and M' are two manifolds with the equiaffine structures (η, ∇) and (η', ∇') respectively. We say that a differential mapping $f : M \to M'$ is *equiaffine* if it satisfies the equation $divX = div'(f_*X)$ for an arbitrary $X \in TM$. If $dimM = dimM'$, then f is an equiaffine diffeomorphism if and only if $trace(\nabla' - \nabla) = 0$ where $\nabla' - \nabla$ is the deformation tensor $T \in (f^*TM') \otimes S^2M$.

For example, if we assume that $f : (M, g) \to (M', g')$ is an equivolume diffeomorphism (also known as volume-preserving diffeomorphism) of pseudo-Riemannian manifolds then $trace(\nabla' - \nabla) = 0$ for the Levi-Civita connections ∇' and ∇ respectively. Hence f is equiaffine.

THEOREM(see also [2]). *For $dimM \geq 3$, $E = T_xM$, $E^* = T_x^*M$, $q = g_x$ and a given orthonormal basis $\{e_1, \ldots, e_n\}$ in E the tensor space*

$$\mathcal{J}(E) = \{\tau \in E^* \otimes S^2E | \tau(a,b,c) = \tau(a,c,b); \sum\nolimits_{i=1,\ldots,dimM} \tau(e_i, e_i, c) = 0\}$$

is the orthogonal direct sum of the subspaces

$$\mathcal{J}_1(E) = \{\tau \in \mathcal{J}(E) | \tau(a,b,c) = \tau(b,a,c)\};$$

$$\mathcal{J}_2(E) = \{\tau \in \mathcal{J}(E) | \tau(a,b,c) + \tau(b,c,a) + \tau(c,a,b) = 0\};$$

$$\mathcal{J}_3(E) = \{\tau \in \mathcal{J}(E) | \tau(a,b,c) = (n^2 + n - 2)^{-1} \times$$
$$[(n+1)\tau_{23}(a)q(b,c) - \tau_{23}(b)q(a,c) - \tau_{23}(c)q(a,b)]\},$$

where $\tau_{23}(a) = \sum\limits_{i=1,\ldots,n} \tau(a, e_i, e_i) = 0$ *and a, b and c are arbitrary elements of E.*

Moreover these spaces are invariant and irreducible under the action of the orthogonal group $O(q)$.

This theorem implies that there are in general seven invariant subspaces of $\mathcal{J}(E)$ and so this leads to

DEFINITION. *Let $J(E)$ be an invariant subspaces of $\mathcal{J}(E)$ and $\tilde{T}(X,Y,Z) = g(X, T(Y,Z))$ for all $X, Y, Z \in TM$. We say that an equiaffine diffeomorphism $f : (M,g) \to (M', g')$ is of type J when $\tilde{T}_x \in J(T_xM)$ for all $x \in M$.*

Hence we may consider seven classes of equiaffine diffeomorphisms between pseudo-Riemannian manifolds. For example, if $f \in \mathcal{J}_3$ then f belongs to a special class of subgeodesic maps which was investigated in [3].

References:

[1] Nomizu K. On completeness in affine differential geometry. *Geometriae Dedicata*, **20**(1986), no. 1, 43-49.

[2] Stepanov S.E. On a group approach to studying the Einstein and Maxwell equations. *Theoretical and Mathematical Physics*, **11**(1997), no. 1, 419-427.

[3] Nicolescu Liviu. Les espaces de Riemann en representation subgeodesique // Tensor, N.S., **32**(1978), no. 2, 182-187.

Stojanov, Jelena: See *Comic, Irena*

Swaczyna, Martin: Constraint Lepage equivalents of Lagrangians of higher order

Abstract:

The geometric theory covering higher order mechanical systems subjected to general non-holomomic constraints of arbitrary order(i.e.depending on time, positions, velocities, accellerations, and higher derivatives) is presented. Higher order constraint structure on fibered manifold is defined to be a fibered submanifold endowed with the canonical distribution or higher order Chetaev bundle. The investigation of variational aspects of this theory leads to introducing of constraint Lepage equivalent of Lagrangian as certain equivalence class of differential 1-forms. Coordinate representations of constraint Lepage equivalents of higher order Lagrangians and for constraints of different order are presented.

Szenthe, Janos: On generalizations of Birkhoff's theorem

Abstract:

Birkhoff's theorem states that a spherically symmetric space-time with vanishing Ricci curvature is static [B]. Generalizations of this theorem to spherically symmetric Einstein space-times and to more general ones were given by M. Cahen, R. Debever, H. Goenner, A. Barnes and others [K-S-MacC]. The basic idea of these generalizations was to consider the Killing equation in order to obtain an additional Killing field, and to give sufficient conditions for the solvability of the equation in terms of the Einstein tensor. However, as Goenner has pointed out it is not likely that by the Einstein tensor alone a condition is obtaiable which is both necessary and sufficient [G]. Recently, a global approach to spherically symmetric space-times was presented [Sz] which naturally yields such a condition.

References

[B] Birkhoff, G. D., *Relativity and Modern Physics*, Harvard Univ. Press, 1923.

[G] Goenner, H., Einstein tensor and generalizations of Birkhoff's theorem, *Commun. Math. Phys.* **16**(1970), 34-47.

[K-S-MacC] Kramer, D., Stephani, H., MacCallum, M., *Exact Solutions of Einstein's Field Equations*, Cambridge Univ. Press, 2003.

[Sz] Szenthe, J., On the global geometry of spherically symmetric space-times, *Math. Proc. Camb. Phil. Soc.* To appear.

Tamaru, Hiroshi: A class of noncompact homogeneous Einstein manifolds

Abstract:

The abstract of the lecture: We consider certain one-dimensional solvable extensions of the nilradicals of parabolic subalgebras of semisimple Lie algebras, and study their structures, curvatures and Einstein condi-

tions. Our solvmanifold is Einstein if the nilradical is of two-step. New examples of Einstein solvmanifolds whose nilradicals are of three and four-step are also given.

Tanaka, Makiko: Geometry of finite symmetric spaces

Abstract:

A finite group is considered as a symmetric space with point-symmetries defined by $s_x(y) = xy^{-1}x$. We investigate monomorphisms (in the sense of the category of symmetric spaces) $f : G \to M$ from a finite group G into a compact symmetric space M and apply the gemetrical theory of polars and meridians to G.

Toda, Kouichi: Study of soliton equations on noncommutative spaces

Abstract:

We present a powerful method to generate various equations which possess the Lax representations on noncommutative $(1 + 1)$ and $(2 + 1)$-dimensional spaces. The generated equations contain noncommutative integrable equations obtained by using the bicomplex method and by reductions of the noncommutative (anti-)self-dual Yang-Mills equation. This suggests that the noncommutative Lax equations would be integrable and be derived from reductions of the noncommutative (anti-)self-dual Yang-Mills equations, which implies the noncommutative version of Ward's conjecture.

These results was reported in 1. Journal of High Energy Physics PRHEP-unesp2002/038 (2002)

2. Physics Letters A, vol 316, 77(2003)

3. Journal of Physics A, vol 36, 11981(2003)

4. hep-th/0309265.

Toman, Henrietta: Canonical coordinate-systems and exponential maps of differentiable n-ary loop

Abstract:

This lecture is devoted to the study of canonical coordinate systems and the corresponding exponential maps of n-ary differentiable loops. The notion of canonical coordinate-systems plays very important role in the theory of Lie-groups. It can be generalized in many ways for differentiable loops, which are non-associative generalizations of Lie-groups. The canonical coordinate systems of n-ary loops can be derived from the local normal forms of(n+1)-webs. We will discuss their differentiability properties. We give a characterization of n-ary loops having one-parameter subgroups in each direction.

Tran, Quoc Binh: Isometric or harmonic mappings of complete Riemannian manifolds.

Abstract:

It is well known that there is no compact minimal submanifold in the Euclidean space. Let M^n be a compact Riemannian manifold and $\varphi : M^n \longrightarrow E^{n+m}$ be an isometric immersion, then $\varphi(M^n)$ is compact, thus bounded and so contained in a ball $B(R)$ for sufficiently big radius R. In 1998 X. S. Yang (Publ. Math. Debrecen 52(1998), no. 1-2, 78-83; MR 99a:53074) proved that if the lenght of the mean curvature H of $\varphi(M^n)$ is less than $\frac{1}{\sqrt{mr}}$, then no ball in $E^(n+m)$ can contain $\varphi(M^n)$. In this talk we consider a complete

Riemannian manifold M^n whose Ricci curvature satisfies the condition $Ric(X)(x) > c(1+\rho^2 log^2(\rho+2))$, where $x \in M^n, X \in T_xM^n$ unit vector , c is a negative constant and ρ is the distant function on M^n from a fixed point x_0 to x. We show that under these weaker conditions if $|H| < \frac{1}{r}$, then $\varphi(M^n)$ can not be contained in a ball $B(r)$ of radius r. Finally we prove that if M^n is complete Riemannian manifold whose Ricci curvature satisfies the above inequality then for any $\varphi M^n \longrightarrow E^{n+m}$ harmonic map, whose energy density $e(\varphi)$ satisfies $inf_M e(\varphi) > 0$, then $\varphi(M^n)$ cannot be contained in any ball of Euclidean space.

Tripathi, Mukut Mani: Certain basic inequalities and its applications

Abstract:

Some basic inequalities between intrinsic invariants and the main extrinsic invariant (squared mean curvature) for submanifolds in a Riemannian manifold are established. The well known intrinsic invariants are scalar curvature, sectional curvatures and Ricci curvature and the first Chen invariant. Some obstructions for minimal immersion are established. We also introduce a new intrinsic invariant and find a new inequality. This inequality gives in particular cases, the inequality for first Chen invariant and the inequality for maximum Ricci curvature for submanifolds in a real space form. This inequality also has several applications for submanifolds in complex space forms and Sasakian space forms.

Uysal, Aynur: D-recurent spaces with Ricci quarter-symmetric metric connection

Abstract:

In 1980,Mishra and Pandey defined a Ricci quarter-symmetric metric connection in a Rieman manifold and studied its proporties.Recently Uysal and Dogan,in 2003,definedD-recurrent spaces with semi-symmetric metric connections and constructed an example of these spaces. In this paper,we define D-recurrent spaces with Ricci quarter-symmetric metric connection and obtain some proporties of curvature tensors of these connections.Further we establish an example of D-recurrent spaces which have Ricci quarter-symmetric metric connection.

Uysal, Mitat: Building 3D models in computer environment

Abstract:

In this paper,standart techniques from computer vision and computational geometry are reviewed and a new approach is proposed. A simple model aquisition system built from web cams and digital cameras is proposed and discussed.

Vavrikova, Hana: On almost geodesic mappings onto Riemannian spaces

Abstract:

We study almost geodesic mappings onto Riemannian spaces. We found more precise fundamental equations of these almost geodesic mappings of type $\pi_1(e)$ and $\pi_2(e)$.Among other things we proved that the set of all Riemannian spaces, which admits almost geodesic mappings of type $\pi_2(e)$, where $e = \pm 1$, depends on at most $\frac{1}{2}n^2(n+1)+2n+3$ real parameters.

Velimirovic, Ljubica: On infinitesimal bending of a subspace of a generalized riemannian space

Abstract:

Infinitesimal bending of a subspace of a generalized Riemannian space (with nonsymmetric basic tensor), as a particular case of an infinitesimal deformation, is defined. Necessary and sufficient conditions for infinitesimal deformation to be infinitesimal bending are obtained. The presentation of the bending field in tangent and normal components is studied. Some particular cases are discussed.

Vylegzanin, Denis: Generalized Hermitian geometry on manifolds with f-structures

Abstract:

In the early 1980s V.F.Kirichenko introduced the general notions of the generalized almost Hermitian structure and the generalized almost Hermitian manifold (or GAH-structure and GAH-manifold) of rank r. In spite of the fact that generalized almost Hermitian structures are defined for an arbitrary rank r, almost all papers on generalized Hermitian geometry deal with the study of GAH-structures of rank 1. In this work we consider GAH-structures of rank r, smooth manifolds with some pairwise commutative f-structures and homogeneous k-symmetric spaces. Homogeneous k-symmetric spaces and structures on them were studied by N.A.Stepanov, J.A.Wolf, A.Gray, O.Kowalski, A.S.Fedenko and many others. One of the results in this field is the description of all canonical structures of classical type on k-symmetric spaces, which was obtained by V.V.Balashchenko and N.A.Stepanov.

The main results are the following:

it is proved that the set of the pairwise commutative f-structures on a smooth manifold can be deduced to the set of the f-structures with the trivial pairwise product, which completely characterize the initial set of the f-structures;

the construction of the generalized almost Hermitian structure of rank greater than 1 on a manifold with an arbitrary number of the pairwise commutative f-structures have been obtained;

it is proved that the homogeneous k-symmetric spaces with canonical f-structures are the generalized almost Hermitian manifolds of arbitrary rank;

the generalized almost Hermitian structures have been constructed and their main properties have been investigated for several particular homogeneous k-symmetric spaces considered before by O.Kowalski, M.Bozhek, J.A.Jimenez.

Walczak, Szymon: Collapse of the foliated manifolds

Abstract:

Geometry of foliated Riemannian manifold with conformal deformation of the metric along the leaves of the foliation. Limits of such manifolds in Gromov-Hausdorff topology.

Winterroth, Ekkehart: Generalized Bianchi–Bergman identities in field theories and the curvature of variational principles

Abstract:

By resorting to Noether's Second Theorem, we relate the generalized Bianchi-Bergman identities for field theories on gauge-natural bundles with the kernel of the associated gauge-natural Jacobi morphism. A suitable definition of curvature of gauge-natural variational principles can be consequently formulated.

Wolak, Robert: Maps between manifolds foliated by Riemannian foliations (joint work with Jerzy Konderak, Bari, Italy)

Abstract:

Harmonic G-invariant maps between Riemannian G-manifolds have played an important role in the theory of harmonic maps. For many classes of Riemannian manifolds harmonic maps are rare. Therefore taking the advantage of the existence of a Riemannian foliation we have introduced the concept of a transversally harmonic map. We demonstrate basic properties of these maps and study the relations and differences between transversally harmonic and harmonic maps. By constructing suitable examples we show that these two classes are distinct. We investigate the conditions which ensure that transversally harmonic maps are harmonic and vice versa, and at the same time we discover some fundamenthal differences between these two classes of maps.

In the case of Riemannian foliations with compact leaves, both regular and singular transversally harmonic maps can be as harmonic maps between spaces of leaves. These spaces are singular stratified Riemannian spaces which include orbifolds. Therefore our results can be considered as the extension of the work of J. Eells, A. Verjovsky, A. El Kacimi, and E.G. Gómez.

[1] J. Eells, A. Verjovsky, *Harmonic and Riemannian foliations*, Bol. Soc. Mat. Mexicana (3) **4** (1998) 1 - 12

[2] A. El Kacimi, E.G. Gómez, *Applications harmoniques feuilletées*, Illinois J. Math., Vol. **40** (1) (1996), 115 - 122

[3] J. Konderak, R. Wolak, *Transversally harmonic maps between manifolds with Riemannian foliations* , Quart. J. Math. **54** (2003), 335-354

[4] J. Konderak, R. Wolak, *Some remarks on transversally harmonic*

Wood, John: Jacobi fields along harmonic maps

Abstract:

A *Jacobi field* is an infinitesimal deformation of a harmonic map. With L. Lemaire, the author described the space of harmonic 2-spheres in CP^2 as a smooth submanifold of the space of all C^k maps $(k > 1)$. There remained the question of whether all Jacobi fields along such harmonic maps are integrable, i.e. do they arise from variations through harmonic maps? The authors answered this affirmatively for harmonic 2-spheres in the complex projective plane. The affirmative answer implies that the Jacobi fields form the tangent bundle to each component of the manifold of harmonic maps from S^2 to CP^2, thus giving the nullity of any such harmonic map; it also has bearing on the behaviour of weakly harmonic E-minimizing maps from a 3-manifold to CP^2 near a singularity and the structure of the singular set of such maps from any manifold to CP^2. In contrast, we show that the answer is negative for harmonic 2-spheres in an n-sphere.

Xenos, Philippos: Some classes of 3- dimensional contact metric manifolds Karatsobanis J. and Xenos Ph. J.

Abstract:

D. E. Blair [B02] constructed non-compact examples of conformally flat contact metric 3-manifolds and proved that on these spaces the equation curl B=—B—B holds, where B is a smooth vector field. We first construct compact examples of conformally flat contact metric 3-manifolds and we prove that the equation $P(g) = r_g$ can be solved in these spaces, provided that r_g is negative at some point. Here P is the operator acting on the metric g and producing the scalar curvature r_g of g. Next we give an example of non-homogeneous, (k, μ)-space which satisfies some further properties. Finally we give an example of homogeneous semi-K contact 3-manifold.

[B02] Blair, D. E., Riemannian geometry of contact and symplectic manifolds, Progress in Mathematics, 203, Birkhauser Boston, Inc., Boston, MA, 2002.

Yoshinori, Machida: Legendre geodesics and their characterization

Abstract:

We define Legendre geodesics by normal Cartan connections of projective contact structures. We also characterize Legendre geodesics among all systems of second order ODEs on contact structures.

Yoshio, Matsuyama: On Einstein totally real submanifolds of a complex projective space

Abstract:

In the present paper we study Einstein totally real submanifolds. We show that if M is an n-dimensional complete Einstein totally real minimal submanifolds of a complex projective space CPn, then M is parallel and M is one of the following conditions holds: M is totally geodesic, M is a flat torus in CP2 with parallel second fundmental form and M is congruent to the standard embedding of the symmetric space of rank 2. We also study a totally real surface of a complex projective space.

Yumaguzhin, Valeriy: Classification of linear ordinary differential equations.

Abstract:

The talk is devoted to a local classification of linear ordinary differential equations of order $n \geq 3$ up to a contact transformation. We consider linear ODEs as geometric structures. We calculate differential invariants of the action of the pseudogroup of all contact transformation on the bundle of these structures. Obtained invariants give possible to get the local classification of linear ODEs.

Yumaguzhin, Valeriy: Differential invariants of 2-order ordinary differential equations

Abstract:

We suggest an approach to construct differential invariants for ordinary differential equations of the form

$$y'' = u^0(x,y) + u^1(x,y)y' + u^2(x,y)y'^2 + u^3(x,y)y'^3 \qquad (1)$$

with respect to point transformations. This approach is based on direct investigation of the action of pseudogroup of all point transformations on the bundle of equations (1).

An equation (1) is identified with the section

$$(x^1, x^2) \longrightarrow (u^0(x^1, x^2),\ u^1(x^1, x^2),\ u^2(x^1, x^2),\ u^3(x^1, x^2))$$

of the product bundle

$$\pi : \mathbb{R}^2 \times \mathbb{R}^4 \to \mathbb{R}^2 .$$

Thus the set of all equations (1) is identified with the set of all sections of π.

It is well known that an arbitrary point transformation f transforms equation (1) to the equation of the same form. This means that the coefficients of the transformed equation can be expressed in terms of the coefficients of the initial one and the derivatives of order ≤ 2 of f. In this way a lift of the point transformation to the diffeomorphism $f^{(0)}$ of the total space of π is defined. $f^{(0)}$ is lifted in the natural way to diffeomorphism $f^{(k)}$ of the bundle $J^k\pi$ of k-jets of sections of π, $k = 1, 2, \ldots$. Thus the pseudogroup of all point transformations of the base of π acts by its lifted transformations on every $J^k\pi$.

First nontrivial differential invariant of this action appears on $J^2\pi$. It is a horizontal 2-form with values in some algebra. This invariant is an obstruction to a linearizability of equation (1) by a point transformation.

We construct the next differential invariant in $J^3\pi$. It is a covariant tensor field of a type $(0, 3)$ with values in the same algebra as above.

Finally, in $J^4\pi$, we construct a complete collection of differential generators for the algebra of scalar differential invariants.

The invariants obtained here constitute a complete collection of differential invariants to solve the local equivalence problem for equations (1).

Zelenko, Igor: Differential geometry of curves in Lagrange Grassmannian with application to invariants of rank 2 vector distributions

Abstract:

It turns out that the problem of finding differential invariants of rather wide classes of geometric structure on manifolds can be reduced to the problem of finding differential invariants of curves in the Lagrange Grassmannian (w.r.t. the action of the linear Symplectic Group). These invariants are far going generalization of the curvature tensor on a Riemannian manifold. In our talk we will describe two principal invariants of the curves in the Lagrange Grassmannian: the generalized Ricci curvature, which is an invariant of the parametrized curve in the Lagrange Grassmannian providing the curve with a natural projective structure, and a fundamental form, which is a degree 4 differential on the curve. Among the realizations of the fundamental form are the constructed by E. Cartan invariant of rank 2 distributions in $\mathbb{R}^5$ (Cartan's tensor) and also Weyl conformal and projective tensors in Riemannian geometry. Applying the notion of the fundamental form to geometric structures given by rank 2 vector distributions in $\mathbb{R}^n$ (where $n > 4$), we obtain the generalization of Cartan's tensor to arbitrary dimension $n > 5$. Some results of the talk were obtained in collaboration with Prof. A. Agrachev.