Financial and Insurance Mathematics - Final Exam

Requirements for Master Degree in Financial and Insurance Mathematics Oral part of the Final Exam

1. Probability and statistics

Random variables, characteristics of their distributions

Distribution, probability density, distribution function, expected value, variance, quantile, median, skewness, kurtosis, characteristic function, probability generating function, moment generating function.

Random vectors, joint distribution, covariance, correlation, dependence modelling and measurement

Distribution, probability density, distribution function, relation between joint and marginal distributions, variance matrix, covariance, linear correlation. Dependence modelling via copulas, sklar theorem, fundamental copulas. Rank correlation, coefficients of tail dependence.

Conditional distribution

Conditional conditional density, conditional expected value, and conditional variance

Probability distributions of insurance mathematics

Distribution of the number of claims and claim amount distribution . Test of goodness of fit and Kolmogorov-Smirnov test as a diagnostic tool for the model choice. Collective risk models, compound distributions and their properties, numerical calculation and approximations of the distributions of aggregate loss. Extreme value distribution and its use in block maxima. Generalized Pareto distribution and its use for excess claims modelling.

Parameter estimators and their properties

Definition of the estimator, consistency, unbiased estimators, bias. Estimation of normal distribution. Parameter estimation methods.

Confidence interval

Exact and approximate confidence intervals, their construction and relationship to hypothesis testing. Delta-method.

Principles of hypothesis testing

Null and alternative hypothesis, statistical test, test statistic, critical region, critical values, level of significance, power of the test, p-value.

Maximum-likelihood principle and the method of moments

Likelihood function, scores, Fisher information. Regular families of distributions. Asymptotic properties of moment and maximum-likelihood estimators.

One-sample, paired and two-sample tests

t-tests, tests of Kolmogorov-Smirnoff type, Wilcoxon test. Assumptions, hypothesis, alternative, test statistics, critical values.

Analysis of variance (ANOVA)

One-way ANOVA: Assumptions, hypothesis, alternative, decomposition of the sum of squares, distribution of the sum of squares, F-test.

Model of linear regression

Assumptions, formulation of the model, meaning of the parameters, least squares method, properties of the estimators, correlation coefficient, coefficient of determination, hypothesis testing of the regression coefficients. Testing submodels. Diagnostics and model building.

Bayes principle

A priori and a posteriori distributions. Bayes theorem. Loss function and Bayes estimators. Bayes central limit theorem and credibility interval. Conjugate priors.

Law of large numbers and central limit theorem

Sequences of independent random variables. Chebychev inequality. Weak and strong law of large numbers. Central limit theorem for independent identically distributed random variables. Ljapunov, Feller-Lindeberg central limit theorem. Slutsky theorem.

Markov chains

Discrete time Markov chains. Transition matrices. Limiting distributions. Stationary distribution. Discrete processes in discrete state space. Transition intensities. Kolmogorov differential equations. Poisson process. Pólya process.

Stationary processes

Stationary processes, strict and weak stationarity, covariance function, spectral density, periodogram, their properties and relationships. AR, MA, ARMA processes, linear process and their analysis. Prediction of finite and infinite series. Ergodic theorem and its application.

Time Series

Decomposition: trend, seasonality, and periodicity. Tests of randomness. Box-Jenkins methodology: ARMA models, identification, estimation, verification, and prediction. ARIMA and sesonality models. Financial time series: models of volatility (GARCH), models nonlinear in the expected value. Multidimensional time series: vector autoregression, Kalman filter.

Credibility theory

Bühlmann model. Exact credibility. (Bühlmann model with conditionally independent and identically distributed claims amounts.)

Collective risk models

Description. Ruin probability, Lunberger inequality, Cramér relationship. Adjustment coefficient.

Essentials of stochastic analysis

Stochastic integral, Îto formula, elementary properties of martigales. Wiener process. Geometric Bownian motion process.

2. Life and general insurance

Demographic model of life insurance

Residual lifetime. Force of mortality. Application of life tables and commutation numbers. Lee-Carter model of mortality modelling and forecasting.

Capital and life policies

Death, survival, and mixed capital insurance with variable premium. Life insurance with constant and variable payments. Net and gross premiums for life benefits.

Reserves of life insurance

Net and gross reserves of the standard policies, zillmerizing. Loss decomposition.

Multiple decrement models

Demographic model of multiple decrement insurance. General multiple decrement capital insurance.

Multiple life insurance

Demographic model of multiple lifes. The joint-life status. The last survival status. Extended definition of commutation numbers.

Solvency and reinsurance

Solvency of an insurance company. Elementary forms of reinsurance. Quoting.

Technical reserves of general insurance

Legal environment. Claims reserves. Survey of reserves. Development triangles. Mack model. Chain ladder. Bornhuetter-Ferguson method. Separation methods. Application of the generalized linear model to reserve assessment.

Tariff rates

Bühlmann-Straub model. Bonus-malus systems and their modelling by Markov chains. Statistical methods for the selection of the tariff variables. Statistical methods for tariff classes construction. Premium calculation in the tariff structure. Application of the generalized linear model to premium calculation.

3. Finance and accounting

Finance essentials

Time value of money. Simple, compound, and continuous interest, valuation interest rate (cost of capital). Decomposition of the interest rate. Cash flows valuation: present value, future value, duration, convexity, internal rate of return, profitability index, payback period, internal value of a cash flow. Theory of immunization. Comparison of investment projects. Effect of inflation.

Securities and their pricing

Stocks. Coupon bonds, zero coupon bonds, callable bonds. Clean and dirty price of a bond, yield to maturity,current yield, accrued interest. Spot and forward yield curves and their estimation. Financial derivatives, forwards, futures, options, swaps, PUT-CALL parity. Self-financing and replicating portfolio. Risk neutral measure and risk netral environment, arbitrage. Black-Scholes model, implied volatility. Hedging. Stochastic models of interest rates (Ho-Lee and Vasicek).

Financial risk

Standard deviation. at risk (VaR), conditional value at risk (CVaR), spectral measures, expectiles. Credit risk. Rating.

Stock market analysis

Markowitz theory of portfolio. Return, expected return, and risk of the portfolio. Construction of a minimum risk portfolio with prescribed expected return, both for short sales allowed and not allowed, and under the existence or nonexistence of a riskless asset. Sharpe measure of a portfolio. Capital asset pricing model (CAPM). Security market line (SML). Capital market line (CML). Arbitrage pricing theory (APT).

Accounting

Accounting for insurance contracts, the concept of deferral and amortization, valuation of assets and liabilities. Basic valuation methods according to International Financial Reporting Standards. IFRS4 Insurance Contracts, liability adequacy test. Final accounts and the interpretation of financial reports. Czech accounting standards.