

Parciální derivace

3. cvičení

Matematika 2, NMMA702, Ondřej Bouchala

Příklady:

1. Spočtete parciální derivace následujících funkcí všude, kde existují:

a) $x^m y^n$

b) e^{xy}

c) $xy + yz + zx$

d) $\sqrt{x^2 + y^2}$

e) $\sqrt[3]{x^3 + y^3}$

f) $|x| \cdot |y|$

g) $\sqrt[3]{xy}$

h) $|y - \sin x|$

i) $|\sin y - \sin x|$

j) $\sqrt[3]{x + y^2}$

k) $f(x, y) = \sqrt[3]{x^2 + y} \cdot \ln(x^2 + y^2),$
 $f(0, 0) = 0$

l) $f(x, y) = e^{\frac{-1}{x^2 + xy + y^2}}, f(0, 0) = 0$

m) $\left(\frac{x}{y}\right)^z$

n) $x^{\frac{y}{z}}$

o) x^{y^z}

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Výsledky:

1. a) $\frac{\partial f}{\partial x} = mx^{m-1}y^n$, $\frac{\partial f}{\partial y} = nx^m y^{n-1}$ pro $(x, y) \in \mathbb{R}^2$.
- b) $\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$ pro $(x, y) \in \mathbb{R}^2$.
- c) $\frac{\partial f}{\partial x} = y + z$, $\frac{\partial f}{\partial y} = x + y$, $\frac{\partial f}{\partial z} = x + y$ pro $(x, y, z) \in \mathbb{R}^3$.
- d) $\frac{\partial f}{\partial x}(x, y) = \frac{x}{\sqrt{x^2+y^2}}$, $\frac{\partial f}{\partial y}(x, y) = \frac{y}{\sqrt{x^2+y^2}}$, pokud $(x, y) \neq (0, 0)$. $\frac{\partial f}{\partial x}(0, 0)$ a $\frac{\partial f}{\partial y}(0, 0)$ neexistují.
- e) $\frac{\partial f}{\partial x}(x, y) = \frac{x^2}{\sqrt[3]{(x^3+y^3)^2}}$, $\frac{\partial f}{\partial y}(x, y) = \frac{y^2}{\sqrt[3]{(x^3+y^3)^2}}$, pokud $y \neq -x$. $\frac{\partial f}{\partial x}(0, 0) = \frac{\partial f}{\partial y}(0, 0) = 1$, $\frac{\partial f}{\partial x}(x, -x)$ a $\frac{\partial f}{\partial y}(x, -x)$ neexistují pro $x \neq 0$.
- f) $\frac{\partial f}{\partial x}(x, y) = |y| \operatorname{sign} x$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x, y) = |x| \operatorname{sign} y$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0, 0) = \frac{\partial f}{\partial y}(0, 0) = 0$. $\frac{\partial f}{\partial x}(0, y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x, 0)$ pro $x \neq 0$ neexistují.
- g) $\frac{\partial f}{\partial x}(x, y) = \frac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x, y) = \frac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0, 0) = \frac{\partial f}{\partial y}(0, 0) = 0$. $\frac{\partial f}{\partial x}(0, y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x, 0)$ pro $x \neq 0$ neexistují.
- h) $\frac{\partial f}{\partial x}(x, y) = -\operatorname{sign}(y - \sin x) \cdot \cos x$, $\frac{\partial f}{\partial y}(x, y) = \operatorname{sign}(y - \sin x)$, pokud $y \neq \sin x$. $\frac{\partial f}{\partial y}(x, \sin x)$ neexistuje pro $x \in \mathbb{R}$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, (-1)^k) = 0$ pro $k \in \mathbb{Z}$. $\frac{\partial f}{\partial x}(x, \sin x)$ neexistuje pro $x \neq \frac{\pi}{2} + k\pi$.
- i) $\frac{\partial f}{\partial x}(x, y) = \cos x \operatorname{sign}(\sin x - \sin y)$, $\frac{\partial f}{\partial y}(x, y) = -\cos y \operatorname{sign}(\sin x - \sin y)$, pokud $\sin x \neq \sin y$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = \frac{\partial f}{\partial y}(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = 0$. V ostatních bodech parciální derivace neexistují.
- j) $\frac{\partial f}{\partial x}(x, y) = \frac{1}{3\sqrt[3]{x+y^2}}$, $\frac{\partial f}{\partial y}(x, y) = \frac{2y}{3\sqrt[3]{x+y^2}}$, pokud $x \neq -y^2$, $\frac{\partial f}{\partial x}(-x^2, x)$ a $\frac{\partial f}{\partial y}(-x^2, x)$ neexistují pro $x \in \mathbb{R}$.
- k) V $(x, -x^2)$ neexistují parciální derivace pokud $x^2 + x^4 \neq 1$, pokud $x^2 + x^4 = 1$, jsou obě parciální derivace nulové.
- l) $\frac{\partial f}{\partial x} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{2x+y}{(x^2+xy+y^2)^2}$, $\frac{\partial f}{\partial y} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{x+2y}{(x^2+xy+y^2)^2}$ pro $(x, y) \neq (0, 0)$; v bodě $(0, 0)$ jsou obě parciální derivace nulové.
- m) Pokud $x, y > 0$ nebo $x, y < 0$, pak $\frac{\partial f}{\partial x} = \frac{z}{y} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial y} = -\frac{zx}{y^2} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^z \cdot \log \frac{x}{y}$.
- n) Pokud $x > 0$ a $y \neq 0$, pak $\frac{\partial f}{\partial x} = \frac{y}{z} \cdot x^{\frac{y}{z}-1}$; $\frac{\partial f}{\partial y} = x^{\frac{y}{z}} \cdot \log x \cdot \frac{1}{z}$; $\frac{\partial f}{\partial z} = -x^{\frac{y}{z}} \cdot \log x \cdot \frac{y}{z^2}$.
- o) Pokud $x, y > 0$, pak $\frac{\partial f}{\partial x} = y^z \cdot x^{y^z-1}$; $\frac{\partial f}{\partial y} = x^{y^z} \cdot \log x \cdot zy^{z-1}$; $\frac{\partial f}{\partial z} = x^{y^z} \cdot \log x \cdot y^z \cdot \log y$.