$$\boxed{\frac{1-q^2}{1-2q\cos x+q^2}=f(x) \qquad |q|\leq 1}$$

$$\cos x=\frac{e^{ix}+e^{-ix}}{2}$$

$$f(x)=\frac{1-q^2}{1-2q\left(\frac{e^{ix}+e^{-ix}}{2}\right)+q^2}=\frac{1-q^2}{1-qe^{ix}-qe^{-ix}+q^2}=\frac{1-q^2}{(1-qe^{ix})(1+e^{-ix}q)}$$

$$\frac{1}{1-qe^{ix}}=1+(qe^{ix})+(qe^{ix})^2+(qe^{ix})^3+\dots=\sum_{n=0}^{\infty}(qe^{ix})^n$$

geometrická řada

$$\operatorname{Re}\sum_{n=0}^{\infty}(qe^{ix})^n=\operatorname{Re}\frac{1}{1-qe^{ix}}$$

$$\|\qquad\qquad\qquad\|$$

$$\sum_{n=0}^{\infty}q^n\cos(nx) \qquad \operatorname{Re}\frac{1}{1-qe^{ix}}\,\frac{1-qe^{-ix}}{1-qe^{-ix}}=\frac{1-qe^{-ix}}{1-qe^{ix}-qe^{-ix}+q^2}$$

$$=\frac{1-q\cos x}{1-2q\cos x+q^2}$$

$$\Rightarrow \frac{1-q\cos x}{1-2q\cos x+q^2}=\sum_{n=0}^{\infty}q^n\cos nx$$

$$\frac{1-q^2}{1-2q\cos x+q^2}=a\cdot\frac{1-q\cos x}{1-2q\cos x+q^2}+b \qquad a,b=?$$

$$a \underline{-aq\cos x} + b \underline{-2bq\cos x} + bq^2 = 1-q^2$$

$$-a-2b=0$$

$$\underline{a+b=1}$$

$$-b=1$$

$$b=-1 \qquad a=2$$

$$\frac{1-q^2}{1-2q\cos x+q^2} = -1+2\cdot\frac{1-q\cos x}{1-2q\cos x+q^2} = -1+2\sum_{n=0}^{\infty} q^n\cos(nx)$$

$$\boxed{f(x) = -1+2\sum_{n=0}^{\infty} q^n\cos(nx)}$$