Preliminaries 1

Definition 1.

- Let A be a set and let n be a natural number, $n \geq 1$. By the *n-th Cartesian power of A*, denoted by A^n, we mean the set of all n-tuples of elements of A.
- By an *n-ary relation* on A (or, a relation of arity n on A) we mean a subset of A^n.
- A *relational structure* is a tuple $\mathbb{A} = (A; R_1, \ldots, R_k)$, where A is a set, called the *universe* of $\mathbb{A}$, and $R_1, \ldots, R_k$ are relations on A (arities may vary). A relational structure with a single binary relation is also called a *digraph*.
- A mapping from A^n to A is called an n-ary operation on A.
- Let f be an n-ary operation on A and let R be an m-ary relation on A. We say that f is a *polymorphism* of R, if, for every $a_{11}, a_{12}, \ldots, a_{1m}, a_{21}, \ldots, a_{2m}, \ldots, \ldots, a_{nm} \in A$ such that $(a_{i1}, a_{i2}, \ldots, a_{im}) \in R$ (for all $i = 1, 2, \ldots, n$),
  we have $(f(a_{11}, a_{21}, \ldots, a_{n1}), \ldots, f(a_{1m}, \ldots, a_{nm})) \in R$.
- We say that f is a polymorphism of a relational structure $\mathbb{A} = (A; R_1, \ldots, R_k)$, if f is a polymorphism of R_i for every $i = 1, 2, \ldots, k$.

Remark 2.

- The universe of a relational structure can also be called the *underlying set*, *domain*, *base set*, etc.
- Instead of "f is a polymorphism of R" we can also say that "f preserves R", or "R is preserved by f", or "f is compatible with R", or "R is compatible with f", or "R is invariant under f", etc.

Example 3.

- For $A = \{0, 1\}$ we have
  $$A^3 = \{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}.$$
- The set $R = \{(0,0,1), (1,1,1)\}$ is a ternary relation on A.
- An n-ary operation preserves a unary relation B on A, iff $f(a_1, \ldots, a_n) \in B$ whenever $a_1, \ldots, a_n \in B$.
- The binary operation max on the set $\{0, 1\}$ is a polymorphism of $(\{0,1\}; \{(1,0), (0,1), (1,1)\})$. The binary operation min is not a polymorphism of this structure. Also, the ternary addition modulo 2 is not a polymorphism of this structure.