

NMAG 405 Universal Algebra 1, fall semester 2017–2018

## Homework 2

Deadline 16 Nov 2017, 10:40

**2.1.** (10 points) Let  $a$  and  $b$  be compact elements of an algebraic lattice. Prove that  $a \vee b$  is compact. Must  $a \wedge b$  be compact?

**2.2.** (10 points) Let  $\mathbf{L}$  be a complete lattice, and  $f : \mathbf{L} \rightarrow \mathbf{L}$  an order-preserving mapping (not necessarily a lattice homomorphism). Prove that there is some  $a \in L$  such that  $f(a) = a$ .

**2.3.** (10 points) Let  $C$  be a closure operator on a set  $A$ . Show that there is a Galois connection between  $A$  and some set  $B$  such that  $C$  is equal to the closure operator on  $A$  induced by that connection.