

## CSP lecture 16/17 winter semester – Problem Set 1

Consider the following relations on  $\{0, 1\}$ :

- $C_i := \{i\}$ , pro  $i \in \{0, 1\}$
- $R := \{(0, 0), (1, 1)\}$
- $N := \{(0, 1), (1, 0)\}$
- $S_{ij} := \{0, 1\}^2 \setminus \{(i, j)\}$ , pro  $i, j \in \{0, 1\}$
- $H := \{0, 1\}^3 \setminus \{(1, 1, 0)\}$
- $G_1 := \{(0, 0, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ ,  $G_2 := \{(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)\}$

**Problem 0.** Prove that the two definitions of  $\text{CSP}(\mathbb{A})$  (satisfiability of a list of constraints, homomorphism problem) are equivalent.

**Problem 1.** Find a polynomial-time algorithm for  $\text{CSP}(\mathbb{A})$ , where

1.  $\mathbb{A} = (\{0, 1\}; R)$
2.  $\mathbb{A} = (\{0, 1\}; R, C_0, C_1)$
3.  $\mathbb{A} = (\{0, 1\}; S_{10})$
4.  $\mathbb{A} = (\{0, 1\}; S_{10}, C_0, C_1)$
5.  $\mathbb{A} = (\{0, 1\}; S_{01}, S_{10}, C_0, C_1)$
6.  $\mathbb{A} = (\{0, 1\}; N)$
7.  $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1)$
8.  $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1, S_{00}, S_{01}, S_{10}, S_{11})$
9.  $\mathbb{A} = (\{0, 1\}; \text{všechny unární a binární relace})$

**Problem 2.** Find a polynomial-time algorithm for  $\text{CSP}(\{0, 1\}; H, C_0, C_1)$ .

**Problem 3.** Find a polynomial-time algorithm for  $\text{CSP}(\{0, 1\}; C_0, C_1, G_1, G_2)$ .

**Problem 4.** Find a polynomial-time algorithm for  $\text{CSP}(\mathbb{Q}; <)$ .