D1

$$2u + 3v + x + 5z = 0$$
$$2u + 6v + 2x + y + 2z = 0$$
$$2v + 3x - y - z = 0$$
$$2u - v + 2x - 3y + 10z = 0$$

$$\begin{pmatrix} 2 & 3 & 1 & 0 & 5 \\ 2 & 6 & 2 & 1 & 2 \\ 0 & 2 & 3 & -1 & -1 \\ 2 & -1 & 2 & -3 & 10 \end{pmatrix} \begin{matrix} \\ -\mathrm{I} \\ \\ -\mathrm{I} \end{matrix} \sim \begin{pmatrix} 2 & 3 & 1 & 0 & 5 \\ 0 & 3 & 1 & 1 & -3 \\ 0 & 2 & 3 & -1 & -1 \\ 0 & -4 & 1 & -3 & 5 \end{pmatrix} \begin{matrix} \\ \\ -3\,\mathrm{II} \\ -\mathrm{II} \end{matrix}$$

$$\sim \begin{array}{c} \begin{matrix} u & v & x & y & z \end{matrix} \\ \begin{pmatrix} 2 & 3 & 1 & 0 & 5 \\ 0 & 3 & 1 & 1 & -3 \\ 0 & -7 & 0 & -4 & 8 \\ \cancel{0} & \cancel{-7} & \cancel{0} & \cancel{-4} & \cancel{8} \end{pmatrix} \end{array} \quad +6$$

$$v = \frac{-4y + 8z}{7} \qquad y, z \text{ arbitrary} \quad +4$$

$$3v + x + y - 3z = 0$$

$$x = 3z - 3v - y = 3z - y + \frac{12}{7}y - \frac{24}{7}z$$
$$= -\frac{3}{7}z + \frac{5}{7}y$$

$$2u = -3v - x - 5z = \frac{12}{7}y - \frac{24}{7}z + \frac{3}{7}z - \frac{5}{7}y - 5z$$
$$= y - 8z$$
$$u = \frac{1}{2}y - 4z \qquad +5$$

$$\left[\frac{1}{2}y - 4z,\ -\frac{4}{7}y + \frac{8}{7}z,\ \frac{5}{7}y - \frac{3}{7}z,\ y,\ z\right],\ y, z \in \mathbb{R}$$

$$\int_1^5 \frac{x-7}{\sqrt{3x+1}+5}\,dx$$ (D2)

subst. $y = \sqrt{3x+1}$ $x = \frac{y^2-1}{3}$ $x=1 \Rightarrow y=2$

$dy = \frac{1}{2\sqrt{3x+1}} \cdot 3\,dx$ $dx = \frac{2}{3} y\,dy$ $x=5 \Rightarrow y=4$

$$= \int_2^4 \frac{\frac{y^2-1}{3} - 7}{y+5} \cdot \frac{2}{3} y\,dy = \frac{2}{9} \int_2^4 \frac{(y^2-1-21)y}{y+5}\,dy$$

+7

$$(y^3-22y):(y+5) = y^2-5y+3$$

$$\begin{array}{l} -(y^3+5y^2) \\ \hline -5y^2-22y \\ -(-5y^2-25y) \\ \hline 3y \\ -(3y+15) \\ \hline -15 \end{array}$$

$$= \frac{2}{9} \int_2^4 y^2 - 5y + 3 - 15\frac{1}{y+5}\,dy$$

+4

$$= \frac{2}{9} \left[\frac{1}{3}y^3 - \frac{5}{2}y^2 + 3y - 15\ln|y+5| \right]_2^4$$

+2

$$= \frac{2}{9}\left(\frac{64}{3} - 40 + 12 - 15\ln 9 - \frac{8}{3} + 10 - 6 + 15\ln 7 \right)$$

$$= \frac{2}{9} \cdot \left(\frac{56}{3} - 24 - 15\ln\frac{9}{7} \right) = \underline{\underline{-\frac{32}{27} - \frac{10}{3}\ln\frac{9}{7}}}$$

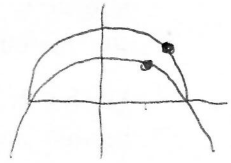
+2

$f(x,y) = x + y - 3 \qquad M = \{[x,y] \in \mathbb{R}^2 : x^2+y^2 \leq 1 \ \& \ y \geq 1-x^2\}$

D3

M is bounded
closed

+2 $\Rightarrow$ compact , f continuous $\Rightarrow$ MAX & MIN ATTAINED +3



$M1: \quad x^2+y^2 < 1 \ \& \ y > 1-x^2$

$\nabla f = \begin{pmatrix}1\\1\end{pmatrix} \neq 0$ NO SUSP POINTS +1

$M2: \quad x^2+y^2 = 1 \qquad g(x,y) = x^2+y^2-1$

(I) $\nabla g = \begin{pmatrix}2x\\2y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow x=y=0 \notin M2$

(II) $\nabla f + \lambda \nabla g = \begin{pmatrix}1\\1\end{pmatrix} + \lambda \cdot \begin{pmatrix}2x\\2y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ $\quad 2\lambda x = -1$, $2\lambda y = -1$, $x^2+y^2=1$

$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right], \cancel{\left[-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]}$ $\quad y < 1-x^2$ $\qquad x = y \Rightarrow x = y = \pm\frac{\sqrt{2}}{2}$ +5

$M3: \quad x^2+y^2 < 1 \qquad y = 1-x^2 \qquad g(x,y) = x^2+y-1$

(I) $\nabla g \neq 0$ $\quad \times$

(II) $\nabla f + \lambda \nabla g = \begin{pmatrix}1\\1\end{pmatrix} + \lambda \cdot \begin{pmatrix}2x\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \lambda = -1$, $1-2x=0$, $x = \frac{1}{2}$, $y = 1-\frac{1}{4} = \frac{3}{4}$ +5

$\left[\frac{1}{2}, \frac{3}{4}\right]$

$M4: \quad \cancel{g_1 = x^2+y^2-1 \qquad g_2 = x^2+y-1}$

$\cancel{\text{(I) } \nabla g_1, \nabla g_2 \text{ LD} \ldots \begin{pmatrix}2x\\2y\end{pmatrix} \begin{pmatrix}2x\\1\end{pmatrix} \Rightarrow y = \frac{1}{2} \Rightarrow x = \pm\frac{\sqrt{2}}{2}} \notin M_4$

(II) TWO POINTS: $[-1,0], [1,0]$ +2

$\boxed{f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \sqrt{2} - 3 \ \text{MAX}}$ $\qquad \boxed{f(-1,0) = -1-3 = -4 \quad \text{MIN}}$

$f\left(\frac{1}{2}, \frac{3}{4}\right) = \frac{5}{4} - 3 \qquad f(1,0) = 1-3$

+2