$$A = \begin{pmatrix} 1 & 2 & 3 & 45 \\ 6 & 7 & 8 & 90 \\ 3 & 2 & 2 & 32 \\ 3 & 2 & 2 & 21 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{5} \\ 1 & 1 & 7 & 0 \\ 1 & 17 & 0 & 0 \\ 0 & 10 & 0 & 0 \end{pmatrix}$$

C1

$$\det(AB) = \det A \det B \qquad +2$$

$$\det B = -\frac{1}{5} \begin{vmatrix} 1 & 1 & 7 \\ 1 & 17 & 0 \\ 0 & 10 & 0 \end{vmatrix} = -\frac{1}{5} \cdot 7 \cdot \begin{vmatrix} 1 & 17 \\ 0 & 10 \end{vmatrix} = -\frac{70}{5} \qquad +3$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 & 45 \\ 6 & 7 & 8 & 90 \\ 3 & 2 & 2 & 32 \\ 3 & 2 & 2 & 21 \end{vmatrix} \begin{matrix} \\ -2\cdot\mathrm{III} \\ -\mathrm{IV} \\ -3\cdot\mathrm{I} \end{matrix} = \begin{vmatrix} 1 & 2 & 3 & 45 \\ 0 & 3 & 4 & 26 \\ 0 & 0 & 0 & 11 \\ 0 & -4 & -7 & \underset{=\,-114}{21-135} \end{vmatrix} \begin{matrix} \\ \\ \\ +4\cdot\mathrm{II} \end{matrix}$$

$$= -1 \cdot \begin{vmatrix} 3 & 4 & 26 \\ -4 & -7 & -114 \\ 0 & 0 & 11 \end{vmatrix} + \cancel{4\cdot\mathrm{I}} = -1 \cdot \cancel{\begin{vmatrix} 1 & 4 & 26 \\ 0 & 9 & -10 \\ 0 & 0 & 11 \end{vmatrix}}$$

$$= +11 \begin{vmatrix} 3 & 4 \\ +4 & +7 \end{vmatrix} = 11 \cdot (21 - 16) = 55 \qquad +8$$

$$\det(AB) = -\frac{70}{5} \cdot 55 = \underline{\underline{-770}} \qquad +2$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2\sin x} (\sin x + 1) \cos x \, dx$$

C2

$y = \sin x$

$dy = \cos x \, dx$ +6

$$= \int_{-1}^{1} e^{2y} (y+1) \, dy = \left[\frac{1}{2} e^{2y} (y+1)\right]_{-1}^{1} - \int_{-1}^{1} \frac{1}{2} e^{2y} \, dy$$

$f = y+1$ $\quad g' = e^{2y}$

$f' = 1$ $\quad g = e^{2y} \cdot \frac{1}{2}$ +6

$$= e^2 - 0 - \left[\frac{1}{4} e^{2y}\right]_{-1}^{1} = e^2 - \frac{1}{4} e^2 + \frac{1}{4} e^{-2}$$

$$= \frac{3}{4} e^2 + \frac{1}{4e^2}$$ +3

$f(x,y,z) = 3y - x + 5z \qquad M = \{ \qquad z = x^2 + (x-y)^2,\ z \le 13 \}$

M is closed and bounded $\Rightarrow$ compact

C3

f cont $\Rightarrow$ MAX & MIN ATTAINED +5

$M1: \quad z < 13,\ z = x^2 + (x-y)^2 = 2x^2 - 2xy + y^2 \qquad g(x,y,z) = 2x^2 - 2xy + y^2 - z$

(I) $Dg = \begin{pmatrix} 4x - 2y \\ -2x + 2y \\ -1 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \times$

(II) $Df + \lambda Dg:$

$-1 + \lambda \cdot (4x - 2y) = 0$

$3 + \lambda(-2x + 2y) = 0$

$5 + \lambda \cdot (-1) = 0$

$z = 2x^2 - 2xy + y^2$

$\lambda = +5$

$+20x - 10y = 1$

$-10x + 10y = -3$

$+10x = -2$

$x = -\frac{1}{5},\ y = -\frac{1}{2}$

$z = \frac{2}{25} - \frac{1}{5} + \frac{1}{4} = \frac{13}{100}$ +5 (1+4)

$\boxed{\left[-\frac{1}{5}, -\frac{1}{2}, \frac{13}{100}\right] \quad f = \frac{1}{5} - \frac{3}{2} + \frac{65}{100} = -\frac{65}{100} = \boxed{-\frac{13}{20}} \quad \text{MIN}}$

$M2: \quad g_2 = z - 13 \qquad g_1 = 2x^2 - 2xy + y^2 - z$

(I) $Dg_1 = \begin{pmatrix} 4x - 2y \\ -2x + 2y \\ -1 \end{pmatrix} \quad Dg_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{matrix} 4x - 2y = 0 \\ -2x + 2y = 0 \end{matrix} \Rightarrow \begin{matrix} x = y = 0 \\ z = 13 \ ↯ \ \times \end{matrix}$

(II)

$-1 + \lambda(4x - 2y) = 0 \qquad z = 13$

$3 + \lambda(-2x + 2y) = 0 \qquad 2x^2 - 2xy + y^2 = z$

$5 - \lambda + \mu = 0$

$2 + 2\lambda x = 0 \qquad x = -\frac{1}{\lambda} \qquad -1 - \frac{4}{\lambda} - 2y\lambda = 0$

$y = -\frac{5}{2\lambda} = \frac{5}{2}x$

$z = 13 = 2x^2 - 2x \cdot \frac{5}{2}x + \left(\frac{5}{2}x\right)^2 = 2x^2 - 5x^2 + \frac{25}{4}x^2 = \frac{13}{4}x^2$

$x^2 = 4 \quad x = \pm 2 \quad y = \pm 5 \quad [\pm 2, \pm 5, 13]$ +8 (2+6)

$\boxed{f(2,5,13) = -2 + 15 + 65 = 78 \quad \text{MAX}}$

$f(-2,-5,13) = 2 - 15 + 65 = 52$

ANSWER +2