$$\begin{pmatrix} 22 & 12 & 12 & 22 \\ 21 & 11+x & 11 & 21 \\ 1 & 2 & 11 & 11 \\ 2 & 2+x & 11 & 12 \end{pmatrix} \qquad \operatorname{rank} A = ?$$

B1

$$\sim \begin{pmatrix} 22 & 12 & 12 & 22 \\ 21 & 11+x & 11 & 21 \\ 1 & 2 & 11 & 11 \\ 2 & 2+x & 11 & 12 \end{pmatrix} \begin{matrix} -\mathrm{II}_n-\mathrm{III}_n \\ \\ \\ -2\cdot\mathrm{III}_n \end{matrix} \sim \begin{pmatrix} 0 & -1-x & -10 & -10 \\ 21 & 11+x & 11 & 21 \\ 1 & 2 & 11 & 11 \\ 0 & -2+x & -11 & -10 \end{pmatrix} \begin{matrix} \\ +\mathrm{I}. \\ +\mathrm{IV}. \\ \\ \end{matrix}$$

$$\sim \begin{pmatrix} 0 & -1-x & -10 & -10 \\ 21 & \cancel{11}0 & 1 & 11 \\ 1 & x & 0 & 1 \\ 0 & -2+x & -11 & -10 \end{pmatrix} \begin{matrix} \\ -21\,\mathrm{III}. \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 0 & -1-x & -10 & -10 \\ 0 & 10-21x & 1 & -10 \\ 1 & x & 0 & 1 \\ 0 & -2+x & -11 & -10 \end{pmatrix} \begin{matrix} -\mathrm{IV} \\ -\mathrm{IV} \\ \\ \cancel{+10\cdot\mathrm{III}} \end{matrix}$$

$$\sim \begin{pmatrix} 0 & 1-2x & 1 & 0 \\ 0 & 12-22x & 12 & 0 \\ 1 & x & 0 & 1 \\ 0 & -2+1\cancel{x} & -11 & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & x \\ 0 & 0 & 1 & 1-2x \\ 0 & 0 & \cancel{12-22x} & 12 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 0 & x \\ 0 & -10 & -11 & -2+x \\ 0 & 0 & 1 & 1-2x \\ 0 & 0 & 12 & 12-22x \end{pmatrix} \begin{matrix} \\ \\ \\ -12\cdot\mathrm{III} \end{matrix} \sim \begin{pmatrix} 1 & 1 & 0 & x \\ 0 & -10 & -11 & -2+x \\ 0 & 0 & 1 & 1-2x \\ 0 & 0 & 0 & 2x \end{pmatrix}$$

+ 11

$x \neq 0 \Rightarrow \mathrm{RANK}(A) = 4$

$$x = 0 \Rightarrow \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -10 & -11 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \mathrm{RANK}(A) = 3$$

+ 4

$$\int \frac{x^3+x^2-9x-9}{(x+2)(x^2+4x+5)}\,dx$$

B2

$$(x+2)(x^2+4x+5)=x^3+6x^2+13x+10$$

$$x^3+x^2-9x-9 : x^3+6x^2+13x+10 = 1$$
$$\underline{-x^3-6x^2-13x-10}$$
$$-5x^2-22x-19$$

$$= \int 1 - \frac{5x^2+22x+19}{(x+2)(x^2+4x+5)}\,dx \qquad +3$$

PARTIAL FRACTIONS : $\dfrac{Ax+B}{x^2+4x+5} + \dfrac{C}{x+2}$

$$(Ax+B)(x+2) + C(x^2+4x+5) = 5x^2+22x+19$$

x^2 : $A + C = 5$ $\qquad$ $x=-2$: $C\cdot 1 = 20-44+19$

$\boxed{A=10}$ $\qquad$ $\boxed{C=-5}$

1 : $2B+5C=19$ $\qquad$ $\boxed{B=22}$ $\qquad$ $+4$

$$= \int 1 + \frac{10x+22}{x^2+4x+5} + \frac{5}{x+2}\,dx = x + 5\ln|x+2| + \qquad +2$$

$$+ \int \frac{5(2x+4)}{x^2+4x+5} + \frac{2}{(x+2)^2+1}\,dx$$

$$\overset{c}{=} x + 5\ln|x+2| + 5\ln(x^2+4x+5) + 2\,\mathrm{arctg}(x+2) \qquad +4$$

$x \in (-\infty,-2)$ or $x \in (-2,+\infty)$ $\qquad +2$

$f(x,y,z) = x^2 + y \quad M = \{ \dots, x^2+y^2+2z^2 = 4, x>0 \}$ B3

M is not compact +2

$\bar{M} = \{ \quad , x^2+y^2+2z^2=4, x \geq 0 \}$ is compact

f continuous ... MAX & MIN attained on $\bar{M}$ +2

$M_1 : \{ x^2+y^2+2z^2 = 4, x > 0 \}$

$g(x,y,z) = x^2+y^2+2z^2-4$

(I) $\nabla g = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} = 0 \rightsquigarrow [0,0,0] \notin M_1 \quad$ x +1

(II) $\nabla f + \lambda \cdot \nabla g = 0$

$2x + 2\lambda x = 0$

$1 + 2\lambda y = 0$

$0 + 4\lambda z = 0$ $\quad \lambda = 0$ ↯ $1 + 2\lambda y = 0$; $\quad z = 0$

$2x(1+\lambda) = 0$: $x=0$ ↯ $x>0$; $\lambda = -1 \Downarrow y = \frac{1}{2}$ +4

$x^2+y^2+2z^2 = 4 \Rightarrow x = \pm \frac{\sqrt{15}}{2}$ $\quad \left[\frac{\sqrt{15}}{2}, \frac{1}{2}, 0 \right]$

$f = \frac{17}{4}$

$M2 : \{ x^2+y^2+2z^2 = 4, x = 0 \}$

(I) $\begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ LD $\Rightarrow y = z = 0$ $\quad x=0 \notin M_2$; $x \neq 0 \notin M_2$ $\quad$ x +2

(II) $2x(1+\lambda) + \mu = 0$

$\cancel{y}\ 1 + 2\lambda y = 0$

$4\lambda z = 0$ $\quad \lambda = 0$ ↯; $\quad z = 0$ & $x = 0 \Rightarrow y = \pm 2$

$x = 0$

$x^2+y^2+2z^2 = 4$

$[0, \pm 2, 0]$ +5

$f(0,2,0) = 2$ $\qquad f\left(\frac{\sqrt{15}}{2}, \frac{1}{2}, 0\right) = \frac{17}{4}$ MAX

$f(0,-2,0) = -2$ .. INF $= -2$, MIN DOES NOT EXIST +4