$A=\begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & -1 \\ -2 & -3 & x \end{pmatrix}$ $\qquad A^{-1} = ?$ $\qquad$ A1

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ -2 & -3 & x & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ -2\cdot\mathrm{I} \\ +\mathrm{II}. \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & -1 & -11 & -2 & 1 & 0 \\ 0 & 0 & x-1 & 0 & 1 & 1 \end{array}\right)$$

We can see that A is invertible

$\iff x \neq 1.$ $\qquad$ +4

If $x \neq 1$:

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & 11 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{x-1} & \frac{1}{x-1} \end{array}\right) \begin{array}{l} -2\cdot\mathrm{II} \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -17 & -3 & 2 & 0 \\ 0 & 1 & 11 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{x-1} & \frac{1}{x-1} \end{array}\right) \begin{array}{l} +17\cdot\mathrm{III} \\ +11\cdot\mathrm{III}. \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2+\frac{17}{x-1} & \frac{17}{x-1} \\ 0 & 1 & 0 & 2 & -1-\frac{11}{x-1} & -\frac{11}{x-1} \\ 0 & 0 & 1 & 0 & \frac{1}{x-1} & \frac{1}{x-1} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -3 & 2+\frac{17}{x-1} & \frac{17}{x-1} \\ 2 & -1-\frac{11}{x-1} & -\frac{11}{x-1} \\ 0 & \frac{1}{x-1} & \frac{1}{x-1} \end{pmatrix} \quad \text{for } x \neq 1$$

+10

A2

$$\int \frac{\sin x \cos^2 x}{3+\sin^2 x + 3\cos x}\,dx$$

$y = \cos x$

$dy = -\sin x$

+6

$$= -\int \frac{y^2}{3+(1-y^2)+3y}\,dy = +\int \frac{y^2}{y^2-3y-4}\,dy$$

$$= \int \frac{y^2-3y-4+3y+4}{y^2-3y-4}\,dy = \int 1 + \frac{3y+4}{(y-4)(y+1)}\,dy$$

+3

PARTIAL FRACTIONS:

$$\frac{3y+4}{(y-4)(y+1)} = \frac{A}{y-4} + \frac{B}{y+1} = \frac{A(y+1)+B(y-4)}{(y-4)(y+1)}$$

$A(y+1) + B(y-4) = 3y+4$

$y=-1 \;:\; -5B = 1 \quad B = -\frac{1}{5}$

$y = 4 \;:\; 5A = 16 \quad A = \frac{16}{5}$ +4

$$= \int 1 + \frac{16}{5}\frac{1}{y-4} - \frac{1}{5}\frac{1}{y+1}\,dy \stackrel{c}{=} y + \frac{16}{5}\ln|y-4| - \frac{1}{5}\ln|y+1|$$

$y \in (-\infty,-1)$ or $(-1,4)$ or $(4,+\infty)$

$$= \cos x + \frac{16}{5}\ln(4-\cos x) - \frac{1}{5}\ln(1+\cos x)$$ +2

$\cancel{x \in (\frac{\pi}{2}+2k\pi, \pi+2k\pi)}$ $\cancel{k\in\mathbb{Z}}$

$f(x,y,z) = 2x - y - 2z \qquad M = \{ \ldots, z^2 = x^2 + 2y^2, 0 \le z < 18 \}$ (A3)

not closed, but $\overline{M} = \{ \ldots \quad 0 \le z \le 18 \}$ +2

is compact, f continuous $\Rightarrow$ MAX & MIN attained on $\overline{M}$ +2

$M1$: $z^2 = x^2 + 2y^2, \ 0 < z < 18$

$g = x^2 + 2y^2 - z^2$

(I) $Dg = \begin{pmatrix} 2x \\ 4y \\ -2z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad [0,0,0] \notin M_1 \qquad \times \quad +1$

(II) $Df + \lambda Dg$

$2 + 2\lambda x = 0$
$-1 + 4\lambda y = 0$
$-2 - 2\lambda z = 0$
$x^2 + 2y^2 - z^2 = 0$

$(1) + (3) \Rightarrow 2\lambda(x - z) = 0$

$\lambda = 0$ ↯ (2) $\qquad x = z \Rightarrow y = 0$ ↯

$\times \quad +3$

$M2$: $z = 0 \Rightarrow x = 0 = y \quad [0,0,0] \quad \boxed{f = 0}$ +2

$M3$: $z = 18$ & $z^2 = x^2 + 2y^2$

$g_2(z) = z - 18 \quad g_1(z) = x^2 + 2y^2 - z^2$

(I) $Dg_1 = \begin{pmatrix} 2x \\ 4y \\ -2z \end{pmatrix} \ Dg_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ LD .. $x = 0, y = 0 \Rightarrow z = 0$ & $z = 18$ ↯ +2

(II) $Df + \lambda Dg_1 + \mu Dg_2 = 0$

$2 + 2\lambda x = 0$
$-1 + 4\lambda y = 0$
$-2 - 2\lambda z + \mu = 0$
$z^2 = x^2 + 2y^2$
$z = 18$

$(1) + 2(2) : 2\lambda(x + 4y) = 0$

$\lambda = 0$ ↯ $\qquad x = -4y$

$18^2 = 16y^2 + 2y^2$

$y = \pm\sqrt{18}$

$x = \mp 4\sqrt{18}$ +4

$[\pm 4\sqrt{18}, \mp\sqrt{18}, 18] \quad f(4\sqrt{18}, -\sqrt{18}, 18) = 9\sqrt{18} - 36 = 27(\sqrt{2} - \frac{4}{3}) > 0$ MAX ON $\overline{M}$

$f(-4\sqrt{18}, \sqrt{18}, 18) = -9\sqrt{18} - 36$ MIN ON $\overline{M}$

$\boxed{\text{SUP} = 27(\sqrt{2} - \tfrac{4}{3}) \qquad \text{INF} = -9\sqrt{18} - 36 \text{ NOT ATTAINED}}$ +4