Exam test B

for Mathematics 2, SS 2017/18

1. (15 points) Find the rank of the following matrix depending on the parameter $x \in \mathrm{R}$:

$$\begin{pmatrix} 22 & 12 & 12 & 22 \\ 21 & 11+x & 11 & 21 \\ 1 & 2 & 11 & 11 \\ 2 & 2+x & 11 & 12 \end{pmatrix}.$$

2. (15 points) Compute the antiderivative and find its domain

$$\int \frac{x^3+x^2-9x-9}{(x+2)(x^2+4x+5)}\,dx.$$

3. (20 points) Find supremum and infimum (and maximum and minimum if they exist) of the function f on the set M, where

$$f(x,y,z) = x^2+y, \qquad M = \{[x,y,z] \in \mathrm{R}^3 :\ x^2+y^2+2z^2 = 4,\ x > 0\}.$$