D1

$$\lim_{x\to 2} \frac{x^2+5}{\sqrt{x+6}-\sqrt{x^3}} (\log x^2 - \log 2x)$$

$$= \lim_{x\to 2} x^2+5 \cdot \underbrace{\lim \frac{\log \frac{x^2}{2x}}{\frac{x^2}{2x}-1}}_{=1 \text{ CFT}} \cdot \lim \frac{\frac{x^2}{2x}-1}{\sqrt{x+6}-\sqrt{x^3}}$$

$$= 9 \cdot \lim \frac{\frac{x-2}{2}}{\frac{x+6-x^3}{\sqrt{x+6}+\sqrt{x^3}}} = \frac{9}{2} \cdot \lim \left(\sqrt{x+6}+\sqrt{x^3}\right) \cdot$$

$$\cdot \lim \frac{x-2}{x+6-x^3} = \frac{9}{2} \cdot 2\sqrt{8} \cdot \lim_{x\to 2} \frac{1}{1-3x^2} =$$

l'H $\frac{0}{0}$

$$= 18\sqrt{2} \cdot \left(-\frac{1}{11}\right) = -\frac{18}{11}\sqrt{2}$$

CFT

inner $g(x)=\frac{x^2}{2x} \to 1$

outer $f(y)=\frac{\ln y}{y-1} \to 1$ as $y \to 1$

logarithms	6 pts
sqrt	2 pts
x^2+5	1 pt
rational function	2 pts
results	1 pt
CFT details	3 pts

$f(x) = (x+1)e^{1-x^2}$ (D2)

$D_f = \mathbb{R}$ $\lim_{x\to+\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$ +1

$f'(x) = e^{1-x^2} + (x+1)e^{1-x^2}\cdot(-2x) = e^{1-x^2}(1-2x-2x^2)$ +2

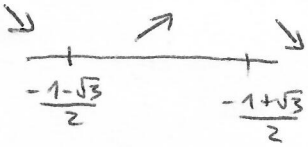
$f''(x) = e^{1-x^2}(-2x)(1-2x-2x^2) + e^{1-x^2}(-2-4x)$ +2

$= e^{1-x^2}(-2x+4x^2+4x^3-2-4x)$

$= e^{1-x^2}(4x^3+4x^2-6x-2)$

$= 2e^{1-x^2}(x-1)(2x^2+4x+1) = 2e^{1-x^2}$

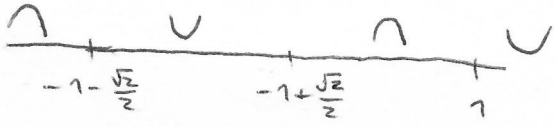
MONOTONICITY: $x_{1,2} = \frac{2\pm\sqrt{4+8}}{-4} = -\frac{1}{2}\pm\frac{1}{2}\sqrt{3}$



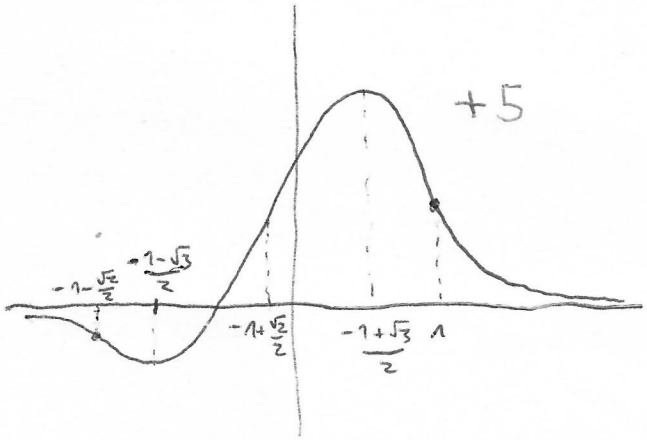
LOCAL MIN $-\frac{1-\sqrt{3}}{2}$

LOCAL MAX $-\frac{1+\sqrt{3}}{2}$ +4

CONVEXITY: $x_{1,2} = \frac{-4\pm\sqrt{16-8}}{4} = -1\pm\frac{\sqrt{2}}{2}$



INFLECTIONS: $-1\pm\frac{\sqrt{2}}{2}$, 1 +5



ASYMPTOTES:

$y = 0$ +1

($\lim f(x) = 0$)

$g(x) = \frac{1}{4}\left[x^2+1\right]\sin\left(\pi x^2 + \sin(\pi x)\right) \qquad x \sim 1$ (D3)

$$g(x) = \begin{cases} \frac{1}{2}\sin\left(\pi x^2 + \sin(\pi x)\right) & x \geq 1 \\ \frac{1}{4}\sin\left(\pi x^2 + \sin \pi x\right) & x < 1 \end{cases} \quad +2$$

$$g'(x) = \begin{cases} \frac{1}{2}\cos\left(\pi x^2 + \sin \pi x\right)\left(2\pi x + \pi\cos \pi x\right) & x > 1 \\ \frac{1}{4}\cos\left(\pi x^2 + \sin \pi x\right)\left(2\pi x + \pi\cos \pi x\right) & x < 1 \end{cases} \quad +2$$

$\lim_{x\to 1^+} g'(x) = \frac{1}{2}\cdot(-1)\left(2\pi - \pi\right) = -\frac{\pi}{2} \sim -1{,}57$ $\quad +2$

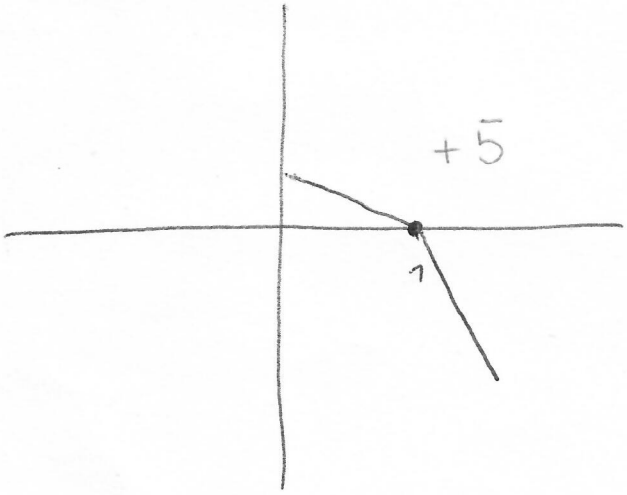
$\lim_{x\to 1^-} g'(x) = \frac{1}{4}\,(-1)\,(2\pi - \pi) = -\frac{\pi}{4} \sim -0{,}78$

$\lim_{x\to 1^-} g(x) = \frac{1}{4}\sin(\pi + \sin\pi) = 0$

$\lim_{x\to 1^+} g(x) = \frac{1}{2}\sin(\pi + \sin\pi) = 0$

$g(1) = 0$ $+2$

$\Rightarrow$ ~~continuous~~



$g'_+(1) = -\frac{\pi}{2}$

$g'_-(1) = -\frac{\pi}{4}$

$g'(1)$ ~~$\exists$~~ $+2$