(1)

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+\sin x} - \sqrt{x^2+\sqrt{x}}}{\sin\frac{1}{\sqrt{x}+1}} = \lim_{x\to+\infty} \frac{\frac{\sin x - \sqrt{x}}{\sqrt{x^2+\sin x} + \sqrt{x^2+\sqrt{x}}}}{\frac{\sin\frac{1}{1+\sqrt{x}}}{\frac{1}{1+\sqrt{x}}} \cdot \frac{1}{1+\sqrt{x}}} =$$

$$= \frac{1}{\underbrace{\lim_{x\to+\infty} \frac{\sin\frac{1}{1+\sqrt{x}}}{\frac{1}{1+\sqrt{x}}}}_{=1 \text{ CFT}}} \cdot \lim_{x\to+\infty} \frac{\sqrt{x}\left(\frac{\sin x}{\sqrt{x}} - 1\right)}{x\left(\sqrt{1+\frac{\sin x}{x^2}} + \sqrt{1+\frac{\sqrt{x}}{x^2}}\right)} \cdot \sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)$$

$$= \lim \frac{\frac{\sin x}{\sqrt{x}} - 1}{\sqrt{1+\frac{\sin x}{x^2}} + \sqrt{1+\frac{\sqrt{x}}{x^2}}}\left(1+\frac{1}{\sqrt{x}}\right) = \frac{0-1}{1+1}(1+0) = -\frac{1}{2}$$

sqrt 2pts

sin 3pts

~~take~~

biggest terms 6pts

result 1pt

CFT 3pts

$$f(x) = \sqrt{x + \frac{2}{x}}$$ C2

$D_f = (0, +\infty)$ +1 $\lim_{x \to +\infty} f(x) = +\infty$, $\lim_{x \to 0^+} f(x) = +\infty$ +1

$$f' = \frac{1}{2\sqrt{x+\frac{2}{x}}} \cdot \left(1 - \frac{2}{x^2}\right) \quad +2$$

$$f'' = \frac{\frac{4}{x^3} 2\sqrt{x+\frac{2}{x}} - \left(1-\frac{2}{x^2}\right)\frac{2}{2\sqrt{x+\frac{2}{x}}} \cdot \left(1-\frac{2}{x^2}\right)}{4\left(x+\frac{2}{x}\right)} \, {}^{+2} =$$

$$= \frac{8\frac{1}{x^3}\left(x+\frac{2}{x}\right) - \left(1-\frac{2}{x^2}\right)^2}{4\left(x+\frac{2}{x}\right)\sqrt{x+\frac{2}{x}}} = \frac{\frac{8}{x^2} + \frac{16}{x^4} - 1 + \frac{4}{x^2} - \frac{4}{x^4}}{4\left(x+\frac{2}{x}\right)\sqrt{x+\frac{2}{x}}}$$

$$= \frac{12 + 12x^2 - x^4}{4x^4\left(x+\frac{2}{x}\right)\sqrt{x+\frac{2}{x}}}$$

increasing on $(\sqrt{2}, +\infty)$, decreasing on $(0, \sqrt{2})$

LOCAL MIN AT $\sqrt{2}$ +4

convexity : $f'' > 0 \Rightarrow x^4 - 12x^2 - 12 < 0$ $\quad x^2 = \frac{12 \pm \sqrt{12^2 + 4 \cdot 12}}{2}$

$= 6 \pm 2\sqrt{12} = 6 \pm 4\sqrt{3}$

convex on $(0, 6+4\sqrt{3})$

concave on $(6+4\sqrt{3}, +\infty)$ +4

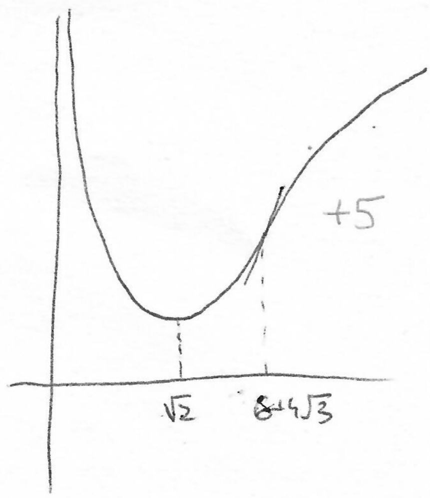
asymptotes

$-\infty$ $\nexists$

$+\infty$ $\nexists$ +1

$\lim \frac{f(x)}{x} = 0$

$\lim f(x) = +\infty$



$$g(x) = \begin{cases} \operatorname{arctg}\left(\frac{x^2+1}{x-3}\right) & x \neq 3 \\ \frac{\pi}{2} & x = 3 \end{cases}$$

C3

$x \sim 3$

$g(3) = \frac{\pi}{2}$ $\quad \lim_{x \to 3^-} g = -\frac{\pi}{2}$ $\quad \lim_{x \to 3^+} g = \frac{\pi}{2}$ $\quad +2$

$\Rightarrow$ NOT CONTINUOS

$$g'(x) = \frac{1}{1 + \left(\frac{x^2+1}{x-3}\right)^2} \cdot \frac{2x(x-3) - (x^2+1)}{(x-3)^2} = \frac{2x(x-3) - (x^2+1)}{(x-3)^2 + (x^2+1)^2}$$

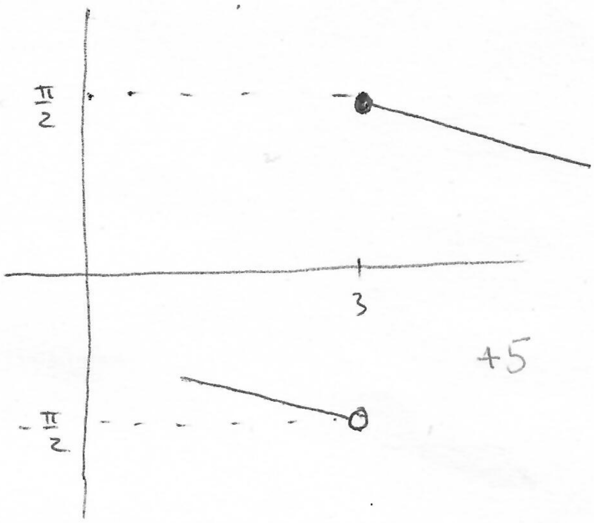
$+3$

$\lim_{x \to 3^-} g' = -\frac{1}{10}$ $\quad \lim_{x \to 3^+} g' = -\frac{1}{10}$ $\quad +2$

$g'_-(3) = +\infty$

$g'_+(3) = -\frac{1}{10}$ $\quad +3$

$g'(3) = \cancel{\exists}$



$+5$