A1

$$\lim_{x\to 1}\left(\frac{1+x^2}{4-2x}\right)^{\frac{x}{\sin(\pi x)}} = \lim_{x\to 1} e^{\frac{x}{\sin \pi x}\cdot \log\left(\frac{1+x^2}{4-2x}\right)}$$

$$\lim_{x\to 1}\frac{x}{\sin \pi x}\log\left(\frac{1+x^2}{4-2x}\right) = \underbrace{\lim_{x\to 1} x}_{=1}\cdot \underbrace{\lim_{x\to 1}\frac{\log\left(\frac{1+x^2}{4-2x}\right)}{\frac{1+x^2}{4-2x}-1}}_{=1 \text{ CFT}}$$

$$\cdot \lim_{x\to 1}\frac{\frac{1+x^2}{4-2x}-1}{\sin \pi x} = 1\cdot 1\cdot \underbrace{\lim_{x\to 1}\frac{1}{4-2x}}_{=\frac{1}{2}}\cdot \lim_{x\to 1}\frac{1+x^2-(4-2x)}{\sin \pi x}$$

$$= \frac{1}{2}\cdot \lim_{x\to 1}\frac{x^2+2x-3}{\sin \pi x} \overset{L'H}{\underset{"\frac{0}{0}"}{=}} \frac{1}{2}\lim_{x\to 1}\frac{2x+2}{\pi\cos \pi x} = \frac{1}{2}\cdot\frac{4}{-\pi}$$

$$= -\frac{2}{\pi}$$

$$\lim e^{\frac{x}{\sin \pi x}\cdot \log\left(\frac{1+x^2}{4-2x}\right)} = e^{-\frac{2}{\pi}}$$

rewriting	1pt
log	3pts
x	1pt
sin	4pts
rational	2pts
result	1pt
CFT	3pts

$f(x) = x^2 + \log(2+x)$ (A2)

$D_f = (-2, +\infty)$ +1 $\quad \lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to -2^+} f(x) = -\infty$ +1

$f'(x) = 2x + \frac{1}{2+x}$ +2 $= \frac{2x^2+4x+1}{2+x}$

$f''(x) = 2 + \frac{0-1}{(2+x)^2}$ +2 $= \frac{2x^2+8x+7}{(2+x)^2}$

~~increasing~~ $f'>0$ $\iff 2x^2+4x+1>0$ $\quad x_{1,2} = \frac{-4 \pm \sqrt{16-8}}{4}$

$= -1 \pm \frac{\sqrt{2}}{2}$

increasing $(-2, -1-\frac{\sqrt{2}}{2}), (-1+\frac{\sqrt{2}}{2}, +\infty)$ LOCAL MAX $-1-\frac{\sqrt{2}}{2}$

decreasing $(-1-\frac{\sqrt{2}}{2}, -1+\frac{\sqrt{2}}{2})$ +4 LOCAL MIN $-1+\frac{\sqrt{2}}{2}$

convexity: $f''>0 \iff 2x^2+8x+7\geq 0$ $\quad x_{1,2} = \frac{-8 \pm \sqrt{64-56}}{4}$

$= -2 \pm \frac{\sqrt{2}}{2}$

convex on $(-2+\frac{\sqrt{2}}{2}, +\infty)$

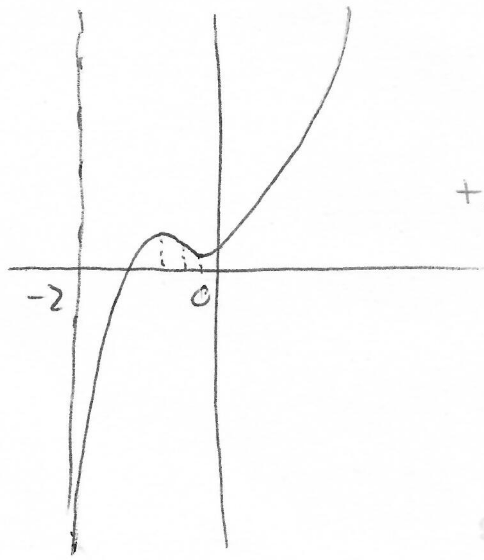
concave on $(-2, -2+\frac{\sqrt{2}}{2})$ INFLECTION $-2+\frac{\sqrt{2}}{2}$ +4

asymptotes:

in $-\infty$ $\nexists$

in $+\infty$ $\nexists$ +1

$\lim \frac{f(x)}{x} = +\infty$



+5

$g(x) = 2x^2 \operatorname{tg} x \cdot \operatorname{sgn}\left(\sin x - \frac{\sqrt{2}}{2}\right) \qquad x \sim \frac{\pi}{4}$ A3

$$g(x) = \begin{cases} 2x^2 \operatorname{tg} x & x > \frac{\pi}{4} \\ 0 & x = \frac{\pi}{4} \\ -2x^2 \operatorname{tg} x & x < \frac{\pi}{4} \end{cases}$$

+2

$g\left(\frac{\pi}{4}\right) = 0 \qquad \lim_{x \to \frac{\pi}{4}-} g = -\frac{\pi^2}{8} \sim -1{,}1$ NOT CONT.

+2

$\lim_{x \to \frac{\pi}{4}+} g = \frac{\pi^2}{8} \sim 1{,}1$

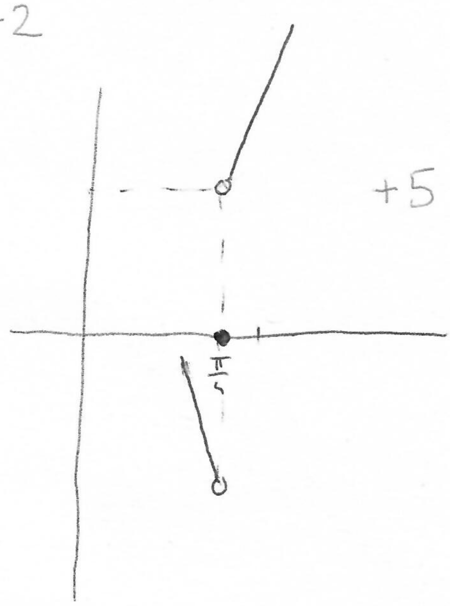
$$g'(x) = \begin{cases} 4x \operatorname{tg} x + 2x^2 \frac{1}{\cos^2 x} & x > \frac{\pi}{4} \\ -4x \operatorname{tg} x - 2x^2 \frac{1}{\cos^2 x} & x < \frac{\pi}{4} \end{cases}$$

+2

$\lim_{x \to \frac{\pi}{4}-} g' = -4 \cdot \frac{\pi}{4} \cdot 1 - 2\frac{\pi^2}{16} \cdot 2 = -\pi - \frac{\pi^2}{4} \sim -5$

+2

$\lim_{x \to \frac{\pi}{4}+} g' = \pi + \frac{\pi^2}{4} \sim +5$



+5

$g'_-\left(\frac{\pi}{4}\right) = +\infty$

$g'_+\left(\frac{\pi}{4}\right) = +\infty$ +2

$g'\left(\frac{\pi}{4}\right) = +\infty$