HOMEWORK 7

due date: November 22, 2017

a) Compute the limits

$$\lim_{x\to 0} \frac{\log(\cos x)}{\cos x - 1} \qquad \text{and} \qquad \lim_{x\to 3} \frac{\sin(x^2-8)}{x^2-8}.$$

b) Compute the limit

$$\lim_{x\to 2} \frac{e^{x^3-8}-1}{x^2-4}.$$

I both a) and b), if you use the compound function theorem (Theorem 25), write what is the inner function g, the outer function f and write the relevant limits from Theorem 25 (what is c, A, B). Voluntarily: Verify condition (I).

Solution.
a) In the first limit, we apply CFT (compound function theorem) with inner function $y = g(x) = \cos x \to 1$ as $x \to 0$ and outer function $f(y) = \frac{\log y}{y-1} \to 1$ as $y \to 1$, i.e. $c = 0$, $A = 1$, $B = 1$. The answer is 1. Condition (I) in CFT says $\cos x \neq 1$ on $P(0, \eta)$. It holds since $\cos x < 1$ on $(-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})$.
In the second limit, it is enough to insert $x = 3$ (the function is continuous in $x = 3$) and we obtain the result $\sin 1$.
b) Aritmetics of limits yields

$$\lim_{x\to 2} \frac{e^{x^3-8}-1}{x^2-4} = \lim_{x\to 2} \frac{e^{x^3-8}-1}{x^3-8} \cdot \lim_{x\to 2} \frac{x^3-8}{x^2-4}.$$

The first limit on the right-hand side equals 1 by CFT with inner function $y = g(x) = x^3 - 8 \to 0$ as $x \to 2$ and the outer function $f(y) = \frac{e^y-1}{y} \to 1$ as $y \to 0$, so $c = 2$, $A = 0$, $B = 1$. The second limit on the right-hand side equals

$$\lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{x^2+2x+4}{(x+2)} = 3.$$

So, the result is 3. Condition (I) in CFT says $x^3 - 8 \neq 0$ on $P(2, \eta)$. It holds since $x^3 = 8$ only for $x = 2$.