HOMEWORK 3

due date: October 25, 2017

a) Compute the limit
$$\lim_{n\to\infty} \frac{5^n + n^7}{n^2 + n + 7^n}.$$

b) Compute the limit
$$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{n^5 + n^2} - \sqrt{n^5 - 4}\right).$$

Solution.
a)
$$\lim_{n\to\infty} \frac{5^n + n^7}{n^2 + n + 7^n} = \lim_{n\to\infty} \frac{5^n}{7^n} \cdot \lim_{n\to\infty} \frac{1 + \frac{n^7}{5^n}}{\frac{n^2}{7^n} + \frac{n}{7^n} + 1} = 0 \cdot \frac{1+0}{0+0+1} = 0,$$
since by comparing the orders of growth we have
$$\lim_{n\to\infty} \frac{5^n}{7^n} = 0, \quad \lim_{n\to\infty} \frac{n^7}{5^n} = 0, \quad \lim_{n\to\infty} \frac{n}{7^n} = 0, \quad \lim_{n\to\infty} \frac{n^2}{7^n} = 0$$

b)
$$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{n^5 + n^2} - \sqrt{n^5 - 4}\right) = \lim_{n\to\infty} \sqrt{n}\frac{n^5 + n^2 - (n^5 - 4)}{\sqrt{n^5 + n^2} + \sqrt{n^5 - 4}} = \lim_{n\to\infty} \sqrt{n}\frac{n^2\left(1 + \frac{4}{n^2}\right)}{n^{5/2}\left(\sqrt{1 + \frac{1}{n^3}} + \sqrt{1 - \frac{4}{n^5}}\right)}$$
$$= \lim_{n\to\infty} \frac{1 + \frac{4}{n^2}}{\sqrt{1 + \frac{1}{n^3}} + \sqrt{1 - \frac{4}{n^5}}} = \frac{1+0}{1+1} = \frac{1}{2}.$$