HOMEWORK 2

due date: October 18, 2017

a) Find supremum, infimum, minimum, and maximum (if they exist) of the set

$$M = \left\{ \frac{m+1}{n-2} \ : \ m, n \in \mathrm{N}, \ m \leq 5, \ n \geq 3 \right\}.$$

b) Prove that the infimum you have found is really an infimum, i.e. satisfies the properties in the definition of infimum.

Solution. a) The biggest number in M is 6, we take m as big as possible and n as small as possible. So, $\max M = \sup M = 6$. If $m = 1$ and n large, we get almost zero, so $\inf M = 0$ but minimum does not exist.

b) We verify the two properties from the definition of infimum.

   (i) If m, $n \in \mathrm{N}$, $m \leq 5$, $n \geq 3$, then $\frac{m+1}{n-2} > 0$, so 0 is a lower bound.

  (ii) Take $g' > 0$, we want to find $x \in M$ such that $x < g'$. Take $m = 1$ and n large. In particular, we need $\frac{2}{n-2} < g'$, so let us take $n > \frac{2}{g'} + 2$.