

Exam test C

for Mathematics 1, WS 2017/18

1. (15 points) Compute the limit

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + \sin x} - \sqrt{x^2 + \sqrt{x}}}{\sin \frac{1}{1+\sqrt{x}}}.$$

2. (20 points) Investigate the function

$$f(x) = \sqrt{x + \frac{2}{x}}$$

(find local extrema, intervals of monotonicity, convexity, inflections, limits in endpoints of D_f , asymptotes and draw graph of f).

3. (15 points) Investigate the function

$$g(x) = \begin{cases} \operatorname{arctg} \left(\frac{x^2+1}{x-3} \right) & x \neq 3 \\ \frac{\pi}{2} & x = 3 \end{cases}$$

in a neighborhood of 3 (compute limits of $g(x)$ and $g'(x)$ as $x \rightarrow 3+$, $x \rightarrow 3-$, decide, whether g is continuous at 3, and draw graph of g in a neighborhood of 3) and compute $g'_+(3)$, $g'_-(3)$, $g'(3)$.