KONFIDENČNÉ OBLASTI PRE KOEFICIENTY KALIBRAČNEJ FUNKCIE

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Úvod

(i) vytvorenie a vyhodnotenie kalibračného modelu
(ii) meranie kalibrovaným meradlom

Majme m rôznych objektov $V_1, V_2, ..., V_m$, každý z nich meráme n-krát dvomi rôznymi meracími prístrojmi $\mathcal{X}$ a $\mathcal{Y}$, namerané hodnoty obidvomi meradlami sú realizáciami nezávislých, normálne rozdelených náhodných veličín $X_{i,j}$ je $j-$te meranie objektu V_i prístrojom $\mathcal{X}$, $X_{i,j} \sim N(\mu_i, \sigma_x^2)$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, pričom μ_i je skutočná (bezchybná) hodnota objektu V_i v jednotkách meracieho prístroja $\mathcal{X}$, σ_x^2 je neznáme. $Y_{i,j}$ je $j-$te meranie objektu V_i prístrojom $\mathcal{Y}$ a $Y_{i,j} \sim N(\nu_i, \sigma_y^2)$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$, pričom ν_i je skutočná (bezchybná) hodnota objektu V_i v jednotkách meracieho prístroja $\mathcal{Y}$, σ_y^2 je neznáme.

Kalibračná funkcia $\nu = f(\mu)$ vyjadruje vzťah medzi bezchybnými (skutočnými) hodnotami merania toho istého objektu pomocou dvoch meracích prístrojov $\mathcal{X}$ a $\mathcal{Y}$.

Predpokladáme, že kalibračná fumkcia je polynóm $k-$teho stupňa, teda $\nu = a_0 + a_1\mu + ... + a_k\mu^k$.

1. Model kalibrácie, jeho linearizácia a NNLO parametrov

Označme

$$\boldsymbol{X}_i = \begin{pmatrix} \boldsymbol{X}_{1,i} \\ \boldsymbol{X}_{2,i} \\ \vdots \\ \boldsymbol{X}_{m,i} \end{pmatrix}, \quad i = 1, 2, ..., n, \qquad \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_m \end{pmatrix}$$

and

$$\boldsymbol{Y}_i = \begin{pmatrix} \boldsymbol{Y}_{1,i} \\ \boldsymbol{Y}_{2,i} \\ \vdots \\ \boldsymbol{Y}_{m,i} \end{pmatrix}, \quad i = 1, 2, ..., n, \qquad \boldsymbol{\nu} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_m \end{pmatrix}.$$

Model kalibrácie je

$$\begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{Y}_1 \\ \boldsymbol{X}_2 \\ \boldsymbol{Y}_2 \\ \vdots \\ \boldsymbol{X}_n \\ \boldsymbol{Y}_n \end{pmatrix} \sim N\left[(\mathbf{1}_n \otimes \boldsymbol{I}_{2m,2m}) \begin{pmatrix} \boldsymbol{\mu} \\ \boldsymbol{\nu} \end{pmatrix}, \boldsymbol{I}_n \otimes \begin{pmatrix} \sigma_x^2 \boldsymbol{I}_m & \mathbf{0} \\ \mathbf{0} & \sigma_y^2 \boldsymbol{I}_m \end{pmatrix} \right]$$

($\otimes$ je Kronekerov súčin) s nelineárnymi ohraničeniami (väzbami) parametrov

$$\boldsymbol{\nu} = a_0 \mathbf{1}_m + a_1 \boldsymbol{\mu} + ... + a_k \boldsymbol{\mu}^k,$$

$\mathbf{1}_m$ je $m \times 1$ vektor $(1, 1, \ldots, 1)'$, $\boldsymbol{\mu}^b = (\mu_1^b, ..., \mu_m^b)'$

Tento model sa volá **model s chybami v premenných**, alebo **errors-in-variables model**

Pomocou Taylorovho rozvoja okolo vhodných hodnôt $\boldsymbol{a}_0 = (a_{00}, a_{10}, a_{20}, ..., a_{k0})', \boldsymbol{\mu}_0^b = (\mu_{10}^b, ..., \mu_{m0}^b)'$, zanedbaním členov druhého a vyšších rádov ako aj položením $\delta\mu_i = \mu_i - \mu_{i0}$, $i = 1, 2, ..., m,\ \boldsymbol{\delta}_\mu = (\delta\mu_1, ..., \delta\mu_m)'$ dostávame lineárny regresný model s lineárnymi podmienkami na parametre

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{X}_1 - \boldsymbol{\mu}_0 \\ \boldsymbol{Y}_1 \\ \boldsymbol{X}_2 - \boldsymbol{\mu}_0 \\ \boldsymbol{Y}_2 \\ \vdots \\ \boldsymbol{X}_n - \boldsymbol{\mu}_0 \\ \boldsymbol{Y}_n \end{pmatrix} \sim N\left[(\mathbf{1}_n \otimes \boldsymbol{I}_{2m,2m}) \begin{pmatrix} \boldsymbol{\delta}_\mu \\ \boldsymbol{\nu} \end{pmatrix}, \boldsymbol{I}_n \otimes \begin{pmatrix} \sigma_x^2 \boldsymbol{I}_m & \mathbf{0} \\ \mathbf{0} & \sigma_y^2 \boldsymbol{I}_m \end{pmatrix}\right] \quad (1)$$

$$\left(diag(a_{00}\mathbf{1}_m + ... + ka_{k0}\boldsymbol{\mu}_0^{k-1}), -\boldsymbol{I}_m\right) \begin{pmatrix} \boldsymbol{\delta}_\mu \\ \boldsymbol{\nu} \end{pmatrix} + \left(\mathbf{1}_m, \boldsymbol{\mu}_0, ..., \boldsymbol{\mu}_0^k\right) \boldsymbol{a} = \mathbf{0} \quad (2)$$

($diag(a_{00}\mathbf{1}_m + ... + ka_{k0}\boldsymbol{\mu}_0^{k-1})$ je diagonálne matica s prvkami vektora $(a_{00}\mathbf{1}_m + ... + ka_{k0}\boldsymbol{\mu}_0^{k-1})$ na diagonále).

Pri označení $\boldsymbol{\beta} = (\boldsymbol{\delta}_\mu', \boldsymbol{\nu}')'$, $\boldsymbol{a} = (a_0, a_1, ..., a_k)'$, μ_2 $\boldsymbol{S} = diag(a_{00}\mathbf{1}_m + ... + ka_{k0}\boldsymbol{\mu}_0^{k-1})$, $\boldsymbol{B}_1 = (\boldsymbol{S}, -\boldsymbol{I})$, $\boldsymbol{B}_2 = \left(\mathbf{1}_m, \boldsymbol{\mu}_0, ..., \boldsymbol{\mu}_0^k\right)$, $\boldsymbol{\Sigma} = \boldsymbol{I}_n \otimes \begin{pmatrix} \sigma_x^2 \boldsymbol{I}_m & \mathbf{0} \\ \mathbf{0} & \sigma_y^2 \boldsymbol{I}_m \end{pmatrix}$, $\boldsymbol{C}^{-1} = \frac{1}{n}\begin{pmatrix} \sigma_x^2 \boldsymbol{I}_m & \mathbf{0} \\ \mathbf{0} & \sigma_y^2 \boldsymbol{I}_m \end{pmatrix}$, $\boldsymbol{A}_1 = \boldsymbol{B}_1 \boldsymbol{C}^{-1} \boldsymbol{B}_1' = \frac{1}{n}(\sigma_x^2 \boldsymbol{SS} + \sigma_y^2 \boldsymbol{I})$, $\overline{\boldsymbol{X}} = \frac{1}{n}\left(\sum_{i=1}^n X_{1,i}, ..., \sum_{i=1}^n X_{m,i}\right)'$, $\overline{\boldsymbol{Y}} = \frac{1}{n}\left(\sum_{i=1}^n Y_{1,i}, ..., \sum_{i=1}^n Y_{m,i}\right)'$
lineárne (linearizované) podmienky (2) sú

$$(\boldsymbol{S}, -\boldsymbol{I}) \begin{pmatrix} \boldsymbol{\delta}_\mu \\ \boldsymbol{\nu} \end{pmatrix} + \boldsymbol{B}_2 \boldsymbol{a} = \mathbf{0}, \quad \text{resp.} \quad \boldsymbol{B}_1 \begin{pmatrix} \boldsymbol{\delta}_\mu \\ \boldsymbol{\nu} \end{pmatrix} + \boldsymbol{B}_2 \boldsymbol{a} = \mathbf{0} \qquad (3)$$

NNLO (najlepší nevychýlený lineárny odhad) vektora parametrov $(\beta', \boldsymbol{a}')'$ je

$$\begin{pmatrix} \widehat{\beta} \\ \widehat{\boldsymbol{a}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{I}_{2m} - \boldsymbol{C}^{-1}\boldsymbol{B}_1'\boldsymbol{Q}_{11}\boldsymbol{B}_1 \\ -\boldsymbol{Q}_{21}\boldsymbol{B}_1 \end{pmatrix} \begin{pmatrix} \overline{\boldsymbol{X}} - \mu_0 \\ \overline{\boldsymbol{Y}} \end{pmatrix},$$

a kovariančná matica odhadu $\widehat{\boldsymbol{a}}$ je

$$cov(\widehat{\boldsymbol{a}}) = -\boldsymbol{Q}_{22},$$

kde

$$\begin{pmatrix} \boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1' & \boldsymbol{B}_2 \\ \boldsymbol{B}_2' & \boldsymbol{0} \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{21} & \boldsymbol{Q}_{22} \end{pmatrix}.$$

Po výpočtoch dostávame

$$\widehat{\mu} = \overline{\boldsymbol{X}} - \frac{\sigma_x^2}{n}\boldsymbol{S}\boldsymbol{Q}_{11}(\boldsymbol{S}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0) - \overline{\boldsymbol{Y}}), \quad \widehat{\boldsymbol{\nu}} = \overline{\boldsymbol{Y}} + \frac{\sigma_y^2}{n}\boldsymbol{Q}_{11}(\boldsymbol{S}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0) - \overline{\boldsymbol{Y}}),$$

$$\widehat{\boldsymbol{a}} = \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \vdots \\ \hat{a}_k \end{pmatrix} = -\boldsymbol{Q}_{21}\boldsymbol{B}_1 \begin{pmatrix} \overline{\boldsymbol{X}} - \boldsymbol{\mu}_0 \\ \overline{\boldsymbol{Y}} \end{pmatrix}$$

a

$$cov(\widehat{\boldsymbol{a}}) = -\boldsymbol{Q}_{22} = (\boldsymbol{B}_2'\boldsymbol{A}_1^{-1}\boldsymbol{B}_2)^{-1}. \tag{4}$$

Iba poznamenávame, že

$$\begin{pmatrix} \overline{\boldsymbol{X}} - \boldsymbol{\mu}_0 \\ \overline{\boldsymbol{Y}} \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{\delta}_\mu \\ \boldsymbol{\nu} \end{pmatrix}, \boldsymbol{C}^{-1}\right). \tag{5}$$

2. MINQUE odhady σ_x^2 a σ_y^2

MINQUE (minimum norm quadratic unbiased estimator) variančných koeficientov σ_x^2 and σ_y^2 (lokálny pri vhodných hodnotách $\sigma_{x0}^2,\ \sigma_{y0}^2$) je

$$\begin{pmatrix} \widehat{\sigma_x^2} \\ \widehat{\sigma_y^2} \end{pmatrix} = \boldsymbol{S}^{-1}_{(M_L \Sigma_0 M_L)^+} \boldsymbol{F}$$

kde

$$\boldsymbol{F} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix},$$

$$F_1 = \frac{1}{\sigma_{x0}^4} \left[\sum_{j=1}^{n} (\boldsymbol{X}_j - \overline{\boldsymbol{X}})'(\boldsymbol{X}_j - \overline{\boldsymbol{X}}) - n(\overline{\boldsymbol{X}} - \widehat{\boldsymbol{\mu}})'(\overline{\boldsymbol{X}} - \widehat{\boldsymbol{\mu}}) \right],$$

$$F_2 = \frac{1}{\sigma_{y0}^4} \left[\sum_{j=1}^{n} (\boldsymbol{Y}_j - \overline{\boldsymbol{Y}})'(\boldsymbol{Y}_j - \overline{\boldsymbol{Y}}) - n(\overline{\boldsymbol{Y}} - \widehat{\boldsymbol{\nu}})'(\overline{\boldsymbol{Y}} - \widehat{\boldsymbol{\nu}}) \right],$$

$$\boldsymbol{S}_{(M_L \Sigma_0 M_L)^+} = \begin{pmatrix} \frac{(n-1)m}{\sigma_{x0}^4} + \frac{1}{n^2} Tr(\boldsymbol{S}\boldsymbol{Q}_{11}\boldsymbol{S}\boldsymbol{S}\boldsymbol{Q}_{11}\boldsymbol{S}) & \frac{1}{n^2} Tr(\boldsymbol{Q}_{11}\boldsymbol{S}\boldsymbol{S}\boldsymbol{Q}_{11}) \\ \frac{1}{n^2} Tr(\boldsymbol{Q}_{11}\boldsymbol{S}\boldsymbol{S}\boldsymbol{Q}_{11}) & \frac{(n-1)m}{\sigma_{y0}^4} + \frac{1}{n^2} Tr(\boldsymbol{Q}_{11}\boldsymbol{Q}_{11}) \end{pmatrix}.$$

Kovariančná matica $\begin{pmatrix} \widehat{\sigma_x^2} \\ \widehat{\sigma_y^2} \end{pmatrix}$ je

$$\boldsymbol{W}(\sigma_{x0}^2, \sigma_{y0}^2) = 2\boldsymbol{S}^{-1}_{(M_L \Sigma_0 M_L)^+}. \tag{6}$$

3. Iteratívna procedúra pre odhad $\boldsymbol{a}, \mu, \nu, \sigma_x^2, \sigma_y^2$

1. Spočítame počiatočné (približné) hodnoty parametrov σ_x^2 and σ_y^2 ako realizácie náhodných premenných

$$\sigma_{x0}^2 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{i,j} - \frac{1}{n}\sum_{s=1}^{n} X_{i,s}\right)^2, \sigma_{y0}^2 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(Y_{i,j} - \frac{1}{n}\sum_{s=1}^{n} Y_{i,s}\right)^2,$$

ďalej spočítame počiatočné hodnoty $\boldsymbol{\mu}_0$ ako realizácie náhodného vektora

$$\boldsymbol{\mu}_0 = \overline{\boldsymbol{X}},$$

a vektor $\boldsymbol{a}_0$ ako realizáciu náhodného vektora

$$\boldsymbol{a}_0 = (\boldsymbol{B}_2'\boldsymbol{B}_2)^{-1}\boldsymbol{B}_2'\overline{\boldsymbol{Y}}$$

$(\boldsymbol{B}_2 = \left(\boldsymbol{1}_m, \boldsymbol{\mu}_0, \dots, \boldsymbol{\mu}_0^k\right))$.

2. Získame odhady

$$\widehat{\mu} = \overline{\boldsymbol{X}} - \frac{\sigma_{x0}^2}{n}\boldsymbol{S}\boldsymbol{Q}_{11}(\boldsymbol{S}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0) - \overline{\boldsymbol{Y}}, \quad \widehat{\boldsymbol{\nu}} = \overline{\boldsymbol{Y}} + \frac{\sigma_{y0}^2}{n}\boldsymbol{Q}_{11}(\boldsymbol{S}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0) - \overline{\boldsymbol{Y}},$$

$$\widehat{\boldsymbol{a}} = \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \vdots \\ \hat{a}_k \end{pmatrix} = -\boldsymbol{Q}_{21}\boldsymbol{B}_1 \begin{pmatrix} \overline{\boldsymbol{X}} - \boldsymbol{\mu}_0 \\ \overline{\boldsymbol{Y}} \end{pmatrix} \tag{7}$$

where $\boldsymbol{S} = diag(a_{00}\mathbf{1}_m + ... + ka_{k0}\boldsymbol{\mu}_0^{k-1}), \ \ \boldsymbol{B}_1 = (\boldsymbol{S}, -\boldsymbol{I}), \ \ \boldsymbol{C}^{-1} = \frac{1}{n}\begin{pmatrix} \sigma_{x0}^2\boldsymbol{I}_m & \mathbf{0} \\ \mathbf{0} & \sigma_{y0}^2\boldsymbol{I}_m \end{pmatrix},$

$$\boldsymbol{Q}_{11} = (\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1} - (\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1}\boldsymbol{B}_2\Big(\boldsymbol{B}_2'(\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1}\boldsymbol{B}_2\Big)^{-1}\boldsymbol{B}_2'(\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1},$$

$$\boldsymbol{Q}_{12} = (\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1}\boldsymbol{B}_2\left(\boldsymbol{B}_2'(\boldsymbol{B}_1\boldsymbol{C}^{-1}\boldsymbol{B}_1)^{-1}\boldsymbol{B}_2\right)^{-1}, \quad \boldsymbol{Q}_{21} = \boldsymbol{Q}_{12}'.$$

3. Položíme realizácie vektora $\widehat{\boldsymbol{a}}$ ako počiatočné hodnoty $\boldsymbol{a}_0 = (a_{00}, a_{10}, ..., a_{k0})'$ a spočítame MINQUE odhad $\widehat{\sigma_x^2},\ \widehat{\sigma_y^2}$. Takto dostaneme (približný) NNLO vektora $\boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{a}$ aj s kovariančnou maticou $cov(\widehat{\boldsymbol{a}})$.

4. Položíme realizácie odhadov $\widehat{\sigma_x^2},\ \widehat{\sigma_y^2}$ ako hodnoty σ_{x0}^2 a σ_{x0}^2 a súčasne položíme realizácie vektora $\widehat{\boldsymbol{\mu}}$ ako počiatočné hodnoty $\boldsymbol{\mu}_0$; vrátime sa na bod 2. "Zlepšili" sme odhady.

Pokračujeme v tomto iteračnom procese (body 2,3 a 4) až budú odhady (nasledujúci a predchádzajúci) "dostatočne blízko seba". Podľa našich skúseností je potrebných 4-7 iterácií.

Matlabový kód pre túto iteratívnu procedúru (program.m, beta_12.m, poc_odhad.m, sx_sy.m) nájdete na adrese http://www.math.muni.cz/xirucko/.

4. Inferencie pre pevné efekty v malých výberoch (podľa Kenwarda a Rogera)

Uvažujme všeobecný gaussovský lineárny model s s observáciami $\boldsymbol{\xi}_{s,1}$,

$$\boldsymbol{\xi} \sim N(\boldsymbol{X}\boldsymbol{a}; \boldsymbol{V}),$$

kde $\boldsymbol{X}$ $(s \times (k+1)$, hodnosti $k+1)$ a $\boldsymbol{V}$ sú známe matice,
$\boldsymbol{a}$ $((k+1) \times 1)$ je vektor neznámych parametrov a prvky kovariančnej matice $\boldsymbol{V}(\sigma_1^2, \sigma_2^2)$ sú funkciami dvoch parametrov σ_1^2 and σ_2^2. REML (restricted maximum likelihood) odhad vektora of $\boldsymbol{a}$ je odhadom získaným zovšeobecnenou metódou najmenších štvorcov

$$\widehat{\boldsymbol{a}} = \boldsymbol{\Phi}(\sigma_1^2, \sigma_2^2)\boldsymbol{X}'\boldsymbol{V}^{-1}(\sigma_1^2, \sigma_2^2)\boldsymbol{\xi}$$

kde

$$\boldsymbol{\Phi}(\sigma_1^2, \sigma_2^2) = \{\boldsymbol{X}'\boldsymbol{V}^{-1}(\sigma_1^2, \sigma_2^2)\boldsymbol{X}\}^{-1}.$$

Kenward a Roger odporúčajú použiť upravený odhad kovariančnej matice vektora $\widehat{\boldsymbol{a}}$ v prípade malých výberov

$$\widehat{\boldsymbol{\Phi}}_A = \boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}) + 2\boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}) \left\{ \sum_{i=1}^{2} \sum_{j=1}^{2} \{\boldsymbol{W}\}_{i,j} (\boldsymbol{U}_{i,j} - \boldsymbol{P}_i \boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}) \boldsymbol{P}_j - \frac{1}{4} \boldsymbol{R}_{i,j}) \right\} \boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}),$$

kde

$$\boldsymbol{P}_i = \boldsymbol{X}' \frac{\partial \boldsymbol{V}^{-1}(\sigma_1^2, \sigma_2^2)}{\partial \sigma_i^2} \bigg|_{\substack{\sigma_1^2 = \widehat{\sigma_1^2} \\ \sigma_2^2 = \widehat{\sigma_2^2}}} \boldsymbol{X}, \quad i = 1, 2,$$

$$\boldsymbol{U}_{i,j} = \boldsymbol{X}' \frac{\partial \boldsymbol{V}^{-1}(\sigma_1^2, \sigma_2^2)}{\partial \sigma_i^2} \bigg|_{\substack{\sigma_1^2 = \widehat{\sigma_1^2} \\ \sigma_2^2 = \widehat{\sigma_2^2}}} \boldsymbol{V}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}) \frac{\partial \boldsymbol{V}^{-1}(\sigma_1^2, \sigma_2^2)}{\partial \sigma_j^2} \bigg|_{\substack{\sigma_1^2 = \widehat{\sigma_1^2} \\ \sigma_2^2 = \widehat{\sigma_2^2}}} \boldsymbol{X}, \quad i = 1, 2,$$

$$\boldsymbol{R}_{i,j} = \boldsymbol{X}' \boldsymbol{V}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})^{-1} \frac{\partial^2 \boldsymbol{V}(\sigma_1^2, \sigma_2^2)}{\partial \sigma_i^2 \partial \sigma_j^2} \bigg|_{\substack{\sigma_1^2 = \widehat{\sigma_1^2} \\ \sigma_2^2 = \widehat{\sigma_2^2}}} \boldsymbol{V}^{-1}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2}) \boldsymbol{X}$$

a $\{\boldsymbol{W}\}_{i,j}$ je $(i, j)-$ty prvok matice $\boldsymbol{W}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})$ danej v (6).

Robíme inferencie simultánne pre celý vektor $\boldsymbol{a}$. Kenward a Rogers ukázali, že štatistika

$$\frac{1}{k+1}(\widehat{\boldsymbol{a}}-\boldsymbol{a})'\widehat{\boldsymbol{\Phi}}_A^{-1}(\widehat{\boldsymbol{a}}-\boldsymbol{a}) \sim \frac{1}{\lambda}F_{k+1,u}$$

t.j. má približne $\frac{1}{\lambda}F_{k+1,u}$ rozdelenie.

$(1-\alpha)$ konfidenčná oblasť pre vektor $\boldsymbol{a}$ (pri použití tejto metódy) je

$$\mathcal{C}_{(1-\alpha)} = \left\{\boldsymbol{a}: \ (\widehat{\boldsymbol{a}}-\boldsymbol{a})'\widehat{\boldsymbol{\Phi}}_A^{-1}(\widehat{\boldsymbol{a}}-\boldsymbol{a}) \leq \frac{k+1}{\lambda}F_{k+1,u}(1-\alpha)\right\}.$$

λ a u sa počítajú nasledovne

$$A_1 = \sum_{i=1}^{2} \sum_{j=1}^{2} \{\boldsymbol{W}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})\}_{i,j} tr(\boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})\boldsymbol{P}_i) tr(\boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})\boldsymbol{P}_j),$$

$$A_2 = \sum_{i=1}^{2} \sum_{j=1}^{2} \{\boldsymbol{W}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})\}_{i,j} tr(\boldsymbol{P}_i \boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})\boldsymbol{P}_j \boldsymbol{\Phi}(\widehat{\sigma_1^2}, \widehat{\sigma_2^2})),$$

$$g = \frac{(k+2)A_1 - (k+5)A_2}{(k+3)A_2}, \qquad B = \frac{1}{2(k+1)}(A_1 + 6A_2),$$

$$c_1 = \frac{g}{3(k+1) + 2(1-g)}, \quad c_2 = \frac{k+1-g}{3(k+1) + 2(1-g)}, \quad c_3 = \frac{k+3-g}{3(k+1) + 2(1-g)},$$

$$\varrho = \frac{(1 + c_1 B)(1 - \frac{1}{k+1} A_2)^2}{(k+1)(1 - c_2 B)^2 (1 - c_3 B)},$$

a konečne

$$u = 4 + \frac{k+3}{(k+1)\varrho - 1}, \qquad \lambda = \frac{u}{u-2}\left(1 - \frac{1}{k+1} A_2\right)$$

5. Konfidenčné oblasti typu 1 a typu 2 pre vektor $\boldsymbol{a}$

Pre odhad vektora $\boldsymbol{a}$, ktorý získame podľa (7) platí

$$\widehat{\boldsymbol{a}} \approx N\left(\boldsymbol{a}, (\boldsymbol{B}_2' \boldsymbol{A}_1^{-1} \boldsymbol{B}_2)^{-1}\right)$$

V značení kapitolky 4 $\sigma_1^2 = \sigma_x^2$, $\sigma_2^2 = \sigma_y^2$, $\widehat{\sigma_1^2} = \sigma_{x0}^2$, $\widehat{\sigma_2^2} = \sigma_{y0}^2$ (σ_{x0}^2 a σ_{y0}^2 dostaneme podľa kapitolky 3, bod 4) podľa postupu Kenwarda a Rogera (kapitolka 4) dostávame $(1-\alpha)$ konfidenčnú oblasť typu 1 pre vektor $\boldsymbol{a}$

$$ {}^{(1)}\mathcal{C}_{(1-\alpha)} = \left\{ \boldsymbol{a} : \quad (\widehat{\boldsymbol{a}} - \boldsymbol{a})' \, {}^{(1)}\widehat{\boldsymbol{\Phi}}_A^{-1} (\widehat{\boldsymbol{a}} - \boldsymbol{a}) \leq \frac{k+1}{{}^{(1)}\lambda} F_{k+1,\, {}^{(1)}u}(1-\alpha) \right\}. \tag{8}$$

kde $F_{k+1,\, {}^{(1)}u}(1-\alpha)$ je $(1-\alpha)$ kvantil F rozdelenia s $k+1$ a ${}^{(1)}u$ stupňami voľnosti.

Teraz uvažujme náhodný vektor

$$\boldsymbol{\eta} = -\boldsymbol{S}(\overline{\boldsymbol{X}} - \boldsymbol{\mu}_0) + \overline{\boldsymbol{Y}} = -\boldsymbol{B}_1 \begin{pmatrix} \overline{\boldsymbol{X}} - \boldsymbol{\mu}_0 \\ \overline{\boldsymbol{Y}} \end{pmatrix}. \tag{9}$$

Ľahko ukážeme, že

$$\boldsymbol{\eta} \sim N(\boldsymbol{B}_2\boldsymbol{a}, \boldsymbol{A}_1),$$

pričom $\boldsymbol{B}_2 = \left(\mathbf{1}_m, \boldsymbol{\mu}_0, ..., \boldsymbol{\mu}_0^k\right)$, $\boldsymbol{A}_1 = \frac{1}{n}(\sigma_x^2 \boldsymbol{SS} + \sigma_y^2 \boldsymbol{I})$. V značení kapitolky 4 $\sigma_1^2 = \sigma_x^2,\ \ \sigma_2^2 = \sigma_y^2,\ \ \widehat{\sigma_1^2} = \sigma_{x0}^2,\ \ \widehat{\sigma_2^2} = \sigma_{y0}^2$ (σ_{x0}^2 a σ_{y0}^2 dostaneme podľa kapitolky 3, bod 4) podľa postupu Kenwarda a Rogera (kapitolka 4) dostávame $(1-\alpha)$ konfidenčnú oblasť typu 2 pre vektor $\boldsymbol{a}$

$${}^{(2)}\mathcal{C}_{(1-\alpha)} = \left\{\boldsymbol{a} :\ \ (\widehat{\boldsymbol{a}} - \boldsymbol{a})' \, {}^{(2)}\widehat{\boldsymbol{\Phi}}_A^{-1} (\widehat{\boldsymbol{a}} - \boldsymbol{a}) \leq \frac{k+1}{{}^{(2)}\lambda} F_{k+1,\, {}^{(2)}u}(1-\alpha)\right\}. \tag{10}$$

6. Porovnanie konfidenčných oblastí typu 1 a typu 2 pomocou simulácií

Pre porovnanie empirických vlastností (správania sa) $(1-\alpha)$ konfidenčných oblastí (8) a (10) sa realizovala (menšia) simulačná štúdia. Pre rôzne sady parametrov bolo realizovaných vždy 1000 opakovaných meraní kalibračnej funkcie (polynómu stupňa 2,3 a 4) a vyšetrovalo sa empirické percentuálne pokrytie skutočného parametra $(1-\alpha)$ konfidenčnou oblasťou (8) (označené KR1) a konfidenčnou oblasťou (10) (označené KR2), α bolo vždy 0,05. Výsledky simulácií sú v nasledujúcich tabuľkách.

Polynóm stupňa 2

$$f_2(x) = 0.25 + 0.5x + 0.05x^2$$

$\mu = (0; 2.5; 5)'$	KR1	KR2
$\sigma_X = 0.125,\ \sigma_Y = 0.0625$		
n=2	0.8763	0.8763
n=3	0.9246	0.9246
n=4	0.9361	0.9361
n=5	0.9409	0.9409
n=10	0,9466	0.9466
n=20	0.9501	0.9501
$\sigma_X = 0.25,\ \sigma_Y = 0.125$		
n=2	0.8925	0.8925
n=3	0.9209	0.9209
n=4	0.9279	0.9279
n=5	0.9365	0.9365
n=10	0.9432	0.9432
n=20	0.9518	0.9518
$\sigma_X = 0.5,\ \sigma_Y = 0.25$		
n=2	0.9412	0.9412
n=3	0.9306	0.9306
n=4	0.9272	0.9272
n=5	0.9283	0.9283
n=10	0.9416	0.9416
n=20	0.9447	0.9447
$\sigma_X = 1,\ \sigma_Y = 0.5$		
n=2	0.9481	0.9481
n=3	0.9328	0.9328
n=4	0.9268	0.9268
n=5	0.9302	0.9302
n=10	0.9293	0.9293
n=20	0.9353	0.9353

$\mu = (0; 1; \ldots; 9)'$	KR1	KR2
$\sigma_X = 0.125,\ \sigma_Y = 0.0625$		
n=2	0.9263	0.9264
n=3	0.9376	0.9376
n=4	0.9407	0.9409
n=5	0.9470	0.9470
n=10	0.9499	0.9500
n=20	0.9511	0.9511
$\sigma_X = 0.25,\ \sigma_Y = 0.125$		
n=2	0.9268	0.9268
n=3	0.9406	0.9407
n=4	0.9429	0.9430
n=5	0.9434	0.9435
n=10	0.9480	0.9481
n=20	0.9456	0.9456
$\sigma_X = 0.5,\ \sigma_Y = 0.25$		
n=2	0.9174	0.9178
n=3	0.9288	0.9288
n=4	0.9332	0.9333
n=5	0.9353	0.9354
n=10	0.9458	0.9458
n=20	0.9461	0.9461
$\sigma_X = 1,\ \sigma_Y = 0.5$		
n=2	0.9066	0.9072
n=3	0.9093	0.9096
n=4	0.9200	0.9202
n=5	0.9219	0.9220
n=10	0.9364	0.9364
n=20	0.9409	0.9409

$g_2(x) = 2 + 0.3x + 0.01x^2$

$\mu = (0; 25; 50)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.8692	0.8692
n=3	0.9212	0.9212
n=4	0.9340	0.9340
n=5	0.9434	0.9434
n=10	0.9488	0.9488
n=20	0.9516	0.9516
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.8865	0.8865
n=3	0.9257	0.9257
n=4	0.9332	0.9332
n=5	0.9361	0.9361
n=10	0.9464	0.9464
n=20	0.9491	0.9491
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9473	0.9473
n=3	0.9376	0.9376
n=4	0.9345	0.9345
n=5	0.9401	0.9401
n=10	0.9462	0.9462
n=20	0.9505	0.9505
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.9508	0.9508
n=3	0.9363	0.9363
n=4	0.9381	0.9381
n=5	0.9362	0.9362
n=10	0.9366	0.9366
n=20	0.9466	0.9466

$\mu = (0; 10; \ldots; 90)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.9249	0.9250
n=3	0.9364	0.9364
n=4	0.9417	0.9418
n=5	0.9454	0.9455
n=10	0.9482	0.9484
n=20	0.9531	0.9531
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9210	0.9211
n=3	0.9386	0.9388
n=4	0.9445	0.9447
n=5	0.9489	0.9490
n=10	0.9468	0.9469
n=20	0.9476	0.9476
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9230	0.9236
n=3	0.9349	0.9353
n=4	0.9430	0.9431
n=5	0.9407	0.9411
n=10	0.9466	0.9468
n=20	0.9475	0.9475
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.9253	0.9260
n=3	0.9302	0.9370
n=4	0.9328	0.9329
n=5	0.9368	0.9368
n=10	0.9437	0.9437
n=20	0.9468	0.9469

$\mu = (10; 20; \ldots; 60)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.9108	0.9108
n=3	0.9320	0.9322
n=4	0.9388	0.9389
n=5	0.9466	0.9466
n=10	0.9472	0.9474
n=20	0.9492	0.9492
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9042	0.9042
n=3	0.9276	0.9276
n=4	0.9351	0.9352
n=5	0.9421	0.9423
n=10	0.9451	0.9452
n=20	0.9478	0.9478
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9077	0.9080
n=3	0.9138	0.9141
n=4	0.9212	0.9212
n=5	0.9258	0.9258
n=10	0.9347	0.9347
n=20	0.9437	0.9437
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.8827	0.8834
n=3	0.8899	0.8901
n=4	0.8962	0.8966
n=5	0.9024	0.9026
n=10	0.9136	0.9136
n=20	0.9278	0.9278
$\sigma_X = 15$, $\sigma_Y = 7.5$		
n=2	0.8569	0.8571
n=3	0.8535	0.8541
n=4	0.8631	0.8631
n=5	0.8651	0.8652
n=10	0.8976	0.8976
n=20	0.9173	0.9173

$\mu = (10; 20; \ldots; 50)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.9055	0.9057
n=3	0.9313	0.9315
n=4	0.9382	0.9385
n=5	0.9411	0.9412
n=10	0.9453	0.9453
n=20	0.9514	0.9514
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9015	0.9019
n=3	0.9187	0.9187
n=4	0.9261	0.9262
n=5	0.9283	0.9283
n=10	0.9380	0.9380
n=20	0.9432	0.9432
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9124	0.9130
n=3	0.9141	0.9143
n=4	0.9135	0.9135
n=5	0.9157	0.9157
n=10	0.9322	0.9323
n=20	0.9430	0.9430
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.8906	0.8911
n=3	0.8867	0.8872
n=4	0.8871	0.8871
n=5	0.8941	0.8942
n=10	0.9080	0.9080
n=20	0.9234	0.9234
$\sigma_X = 15$, $\sigma_Y = 7.5$		
n=2	0.8871	0.8875
n=3	0.8741	0.8744
n=4	0.8663	0.8665
n=5	0.8736	0.8738
n=10	0.8903	0.8903
n=20	0.9034	0.9035

$\mu = (-10; 0; \ldots; 40)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0., 625$		
n=2	0.9203	0.9209
n=3	0.9300	0.9303
n=4	0.9430	0.9431
n=5	0.9469	0.9474
n=10	0.9501	0.9502
n=20	0.9541	0.9542
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9319	0.9322
n=3	0.9370	0.9372
n=4	0.9395	0.9395
n=5	0.9415	0.9418
n=10	0.9446	0.9446
n=20	0.9489	0.9489
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9424	0.9431
n=3	0.9447	0.9447
n=4	0.9417	0.9419
n=5	0.9425	0.9427
n=10	0.9476	0.9477
n=20	0.9490	0.9491
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.9049	0.9055
n=3	0.9129	0.9136
n=4	0.9240	0.9242
n=5	0.9284	0.9285
n=10	0.9416	0.9417
n=20	0.9435	0.9435
$\sigma_X = 15$, $\sigma_Y = 7.5$		
n=2	0.8872	0.8876
n=3	0.8868	0.8872
n=4	0.8957	0.8958
n=5	0.9051	0.9052
n=10	0.9273	0.9275
n=20	0.9316	0.9317

$\mu = (50; 60; \ldots; 100)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.8986	0.8986
n=3	0.9258	0.9258
n=4	0.9384	0.9384
n=5	0.9407	0.9408
n=10	0.9484	0.9484
n=20	0.9488	0.9488
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9006	0.9007
n=3	0.9174	0.9174
n=4	0.9281	0.9281
n=5	0.9333	0.9333
n=10	0.9376	0.9376
n=20	0.9477	0.9477
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.8696	0.8697
n=3	0.8907	0.8907
n=4	0.8969	0.8971
n=5	0.9094	0.9094
n=10	0.9335	0.9335
n=20	0.9393	0.9393
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.7895	0.7901
n=3	0.8066	0.8067
n=4	0.8199	0.8201
n=5	0.8428	0.8430
n=10	0.8824	0.8824
n=20	0.9109	0.9109
$\sigma_X = 15$, $\sigma_Y = 7.5$		
n=2	0.7787	0.7791
n=3	0.7607	0.7608
n=4	0.7606	0.7606
n=5	0.7692	0.7693
n=10	0.8266	0.8266
n=20	0.8742	0.8743

Polynóm stupňa 3
$f_3(x) = -0.8 + 2.46x - 0.38x^2 + 0.025x^3$

$\mu = (1; 3.5; 6; 8.5)'$	KR1	KR2
$\sigma_X = 0.125,\ \sigma_Y = 0.0625$		
n=2	0.8731	0.8731
n=3	0.9153	0.9153
n=4	0.9342	0.9342
n=5	0.9420	0.9420
n=10	0.9431	0.9431
n=20	0.9509	0.9509
$\sigma_X = 0.25,\ \sigma_Y = 0.125$		
n=2	0.8684	0.8684
n=3	0.9070	0.9070
n=4	0.9321	0.9321
n=5	0.9382	0.9382
n=10	0.9422	0.9422
n=20	0.9463	0.9463
$\sigma_X = 0.5,\ \sigma_Y = 0.25$		
n=2	0.8583	0.8583
n=3	0.8979	0.8979
n=4	0.9195	0.9195
n=5	0.9231	0.9231
n=10	0.9388	0.9388
n=20	0.9460	0.9460
$\sigma_X = 1,\ \sigma_Y = 0.5$		
n=2	0.8997	0.8997
n=3	0.8822	0.8822
n=4	0.8717	0.8717
n=5	0.8759	0.8759
n=10	0.9073	0.9073
n=20	0.9322	0.9322

$\mu = (0; 1; \ldots; 10)'$	KR1	KR2
$\sigma_X = 0.125,\ \sigma_Y = 0.0625$		
n=2	0.9228	0.9230
n=3	0.9368	0.9370
n=4	0.9429	0.9429
n=5	0.9439	0.9441
n=10	0.9447	0.9447
n=20	0.9475	0.9475
$\sigma_X = 0.25,\ \sigma_Y = 0.125$		
n=2	0.9145	0.9146
n=3	0.9298	0.9299
n=4	0.9352	0.9353
n=5	0.9418	0.9418
n=10	0.9480	0.9480
n=20	0.9462	0.9462
$\sigma_X = 0.5,\ \sigma_Y = 0.25$		
n=2	0.9074	0.9075
n=3	0.9203	0.9205
n=4	0.9243	0.9244
n=5	0.9311	0.9311
n=10	0.9441	0.9442
n=20	0.9465	0.9466
$\sigma_X = 1,\ \sigma_Y = 0.5$		
n=2	0.8610	0.8619
n=3	0.8776	0.8784
n=4	0.8943	0.8948
n=5	0.9032	0.9036
n=10	0.9219	0,9219
n=20	0.9294	0.9295

$g_3(x) = 0.00023x^3 - 0.035x^2 + 2.2x + 1$

$\mu = (10; 35; 60; 85)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.8778	0.8778
n=3	0.9219	0.9219
n=4	0.9376	0.9376
n=5	0.9412	0.9412
n=10	0.9433	0.9433
n=20	0.9520	0.9520
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.8776	0.8776
n=3	0.9136	0.9136
n=4	0.9310	0.9310
n=5	0.9375	0.9375
n=10	0.9458	0.9458
n=20	0.9479	0.9479
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.8683	0.8683
n=3	0.8973	0.8973
n=4	0.9235	0.9235
n=5	0.9218	0.9218
n=10	0.9382	0.9382
n=20	0.9442	0.9442
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.9058	0.9058
n=3	0.8856	0.8856
n=4	0.8796	0.8796
n=5	0.8900	0.8900
n=10	0.9098	0.9098
n=20	0.9306	0.9306

$\mu = (0; 10; \ldots; 100)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.9242	0.9243
n=3	0.9415	0.9415
n=4	0.9414	0.9414
n=5	0.9461	0.9461
n=10	0.9481	0.9482
n=20	0.9457	0.9457
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9147	0.9150
n=3	0.9354	0.9354
n=4	0.9354	0.9356
n=5	0.9393	0.9359
n=10	0.9452	0.9454
n=20	0.9489	0.9489
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9010	0.9011
n=3	0.9155	0.9159
n=4	0.9200	0.9204
n=5	0.9297	0.9298
n=10	0.9457	0.9457
n=20	0.9413	0.9414
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.8633	0.8639
n=3	0.8773	0.8777
n=4	0.8911	0.8915
n=5	0.9045	0.9047
n=10	0.9215	0.9215
n=20	0.9346	0.9347

Polynóm stupňa 4

$f_4(x) = -0.45 + 0.8x + 0.35x^2 - 0.07x^3 + 0.0037x^4$

$\mu = (0; 2.5; \ldots; 10)'$	KR1	KR2
$\sigma_X = 0.125$, $\sigma_Y = 0.0625$		
n=2	0.8658	0.8658
n=3	0.9157	0.9157
n=4	0.9295	0.9295
n=5	0.9345	0.9345
n=10	0.9466	0.9466
n=20	0.9475	0.9475
$\sigma_X = 0.25$, $\sigma_Y = 0.125$		
n=2	0.8500	0.8500
n=3	0.9108	0.9108
n=4	0.9242	0.9242
n=5	0.9241	0.9241
n=10	0.9333	0.9333
n=20	0.9412	0.9412
$\sigma_X = 0.5$, $\sigma_Y = 0.25$		
n=2	0.8658	0.8658
n=3	0.9025	0.9025
n=4	0.9134	0.9134
n=5	0.9169	0.9169
n=10	0.9211	0.9211
n=20	0.9296	0.9296
$\sigma_X = 1$, $\sigma_Y = 0.5$		
n=2	0.9334	0.9334
n=3	0.9121	0.9121
n=4	0.9086	0.9086
n=5	0.9114	0.9114
n=10	0.9172	0.9172
n=20	0.9247	0.9247

$\mu = (0; 1; \ldots; 11)'$	KR1	KR2
$\sigma_X = 0.125$, $\sigma_Y = 0.0625$		
n=2	0.9204	0.9206
n=3	0.9342	0.9344
n=4	0.9358	0.9358
n=5	0.9381	0.9384
n=10	0.9445	0.9446
n=20	0.9496	0.9496
$\sigma_X = 0.25$, $\sigma_Y = 0.125$		
n=2	0.8995	0.8996
n=3	0.9210	0.9211
n=4	0.9253	0.9254
n=5	0.9332	0.9333
n=10	0.9423	0.9423
n=20	0.9457	0.9457
$\sigma_X = 0.5$, $\sigma_Y = 0.25$		
n=2	0.8859	0.8859
n=3	0.8991	0.8992
n=4	0.9084	0.9086
n=5	0.9176	0.9177
n=10	0.9268	0.9268
n=20	0.9376	0.9376
$\sigma_X = 1$, $\sigma_Y = 0.5$		
n=2	0.8297	0.8306
n=3	0.8628	0.8634
n=4	0.8782	0.8790
n=5	0.8951	0.8955
n=10	0.9068	0.9068
n=20	0.9146	0.9146

$g_4 = 5 - 2.47x + 0.175x^2 - 0.0027x^3 + 0.000013x^4$

$\mu = (0; 25; \ldots; 100)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.8605	0.8605
n=3	0.9157	0.9157
n=4	0.9316	0.9316
n=5	0.9356	0.9356
n=10	0.9482	0.9482
n=20	0.9494	0.9494
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.8527	0.8527
n=3	0.9090	0.9090
n=4	0.9254	0.9254
n=5	0.9399	0.9399
n=10	0.9423	0.9423
n=20	0.9482	0.9482
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.8356	0.8356
n=3	0.8823	0.8823
n=4	0.9079	0.9079
n=5	0.9158	0.9158
n=10	0.9372	0.9372
n=20	0.9431	0.9431
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.8794	0.8794
n=3	0.8529	0.8529
n=4	0.8592	0.8592
n=5	0.8606	0.8606
n=10	0.8962	0.8962
n=20	0.9214	0.9214

$\mu = (0; 10; \ldots; 110)'$	KR1	KR2
$\sigma_X = 1.25$, $\sigma_Y = 0.625$		
n=2	0.9193	0.9200
n=3	0.9388	0.9392
n=4	0.9464	0.9469
n=5	0.9443	0.9446
n=10	0.9537	0.9537
n=20	0.9479	0.9479
$\sigma_X = 2.5$, $\sigma_Y = 1.25$		
n=2	0.9159	0.9162
n=3	0.9341	0.9348
n=4	0.9443	0.9447
n=5	0.9397	0.9403
n=10	0.9501	0.9503
n=20	0.9495	0.9495
$\sigma_X = 5$, $\sigma_Y = 2.5$		
n=2	0.9092	0.9105
n=3	0.9256	0.9259
n=4	0.9296	0.9303
n=5	0.9392	0.9394
n=10	0.9442	0.9442
n=20	0.9494	0.9494
$\sigma_X = 10$, $\sigma_Y = 5$		
n=2	0.8549	0.8565
n=3	0.8895	0.8900
n=4	0.8976	0.8978
n=5	0.9105	0.9106
n=10	0.9285	0.9289
n=20	0.9432	0.9433

Zo simulácií je očividné, že správanie sa oboch konfidenčných oblastí (8) and (10) je prakticky rovnaké.

Záver

V príspevku sme analyzovali prvú fázu kalibračného procesu, a síce odhad parametrov kalibračnej funkcie a určenie konfidenčnej oblasti pre vektor parametrov (dvomi prakticky rovnocennými postupmi). Táto fáza sa môže nazvať kalibrovanie meradla. V nasledujúcom treba ešte určiť konfidenčný pás okolo odhadu kalibračnej funkcie a definovať postup, ako merať s nakalibrovaným meradlom. O tom snáď niekedy nabudúce.

Ďakujeme za pozornosť