Stochastic reconstruction for inhomogeneous point processes

K. Koňasová, J. Dvořák

Charles University
Faculty of Mathematics and Physics
Department of Probability and Mathematical Statistics

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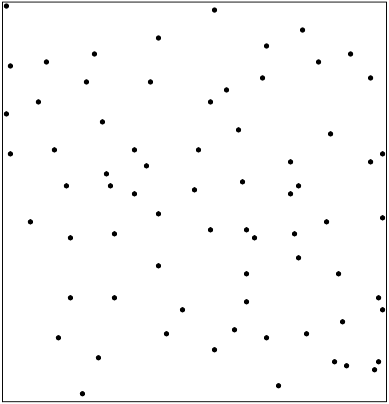
Overview

- Planar point processes
  - Definition
  - Intensity measure
  - Summary characteristics
- Importance of simulations
- Stochastic reconstruction
  - Algorithm
  - Applications
- Inhomogeneous case

Planar point processes

- A random countable subset of the two-dimensional Euclidean space.

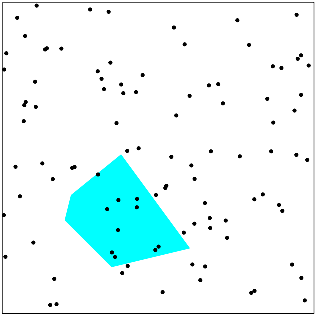


Planar point processes

- Planar point process $X \to$ random variable $X(B),\ B \in \mathfrak{B}^2$.
- Intensity measure:

$$\boxed{\Lambda(B) = \mathbb{E}\left[X(B)\right] = \int_B \lambda(u)\,\mathrm{d}u,\ B \in \mathfrak{B}^2}$$

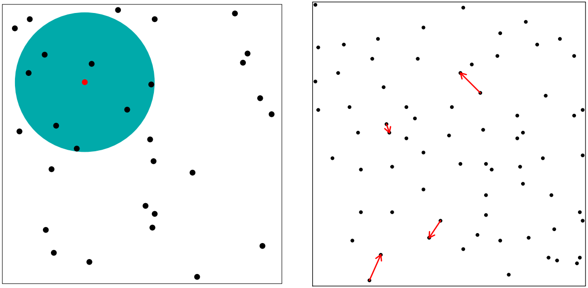
- Constant λ is called **intensity**.



Planar point processes

Summary characteristics:

- **numerical**
  - intensity,
- **functional**
  - K-function,
  - nearest neighbour distance distribution function.



Importance of simulations

- Simulations may be used to:
  - test statistical hypotheses,
  - assess estimation variance,
  - ...
- Simulation methods:
  - generate samples from explicit point process model,
  - parameter estimation.

Stochastic reconstruction (Tscheschel and Stoyan 2006)

- Algorithmic procedure.
- Useful tool for generating point patterns with prescribed summary characteristic.
- No need to specify any explicit model assumptions.

Notation:

- X, Y ... point patterns
- n_i ... i-th numerical summary characteristic
- f_j ... j-th functional summary characteristic
- R_j ... constant, depends on the observation window
- $\widehat{n}_i$ ... empirical estimator of the i-th numerical s. c.
- $\widehat{f}_j$ ... empirical estimator of the j-th functional s. c.

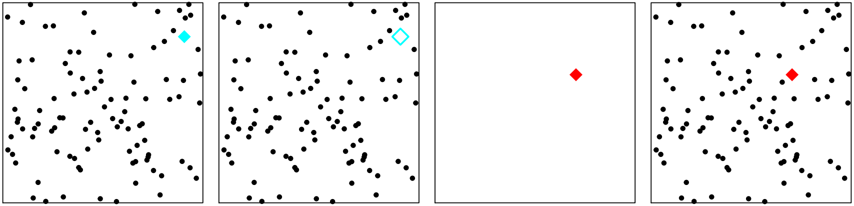
Stochastic reconstruction

$$
\begin{aligned}
E_{n_i}(X, Y) &= \left[\widehat{n}_i(X) - \widehat{n}_i(Y)\right]^2 \\
E_{f_j}(X, Y) &= \int_0^{R_j} \left[\widehat{f}_j(r, X) - \widehat{f}_j(r, Y)\right]^2 \, \mathrm{d}r
\end{aligned}
$$

- Energy functional:

$$
\boxed{E(X, Y) = \sum_{i=1}^{I} E_{n_i}(X, Y) + \sum_{j=1}^{J} E_{f_j}(X, Y)}
$$

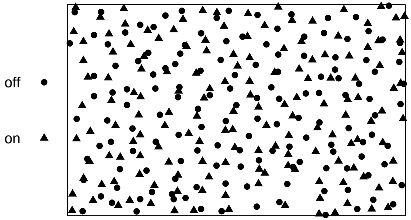
Stochastic reconstruction

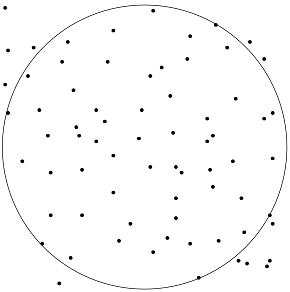


Stochastic reconstruction

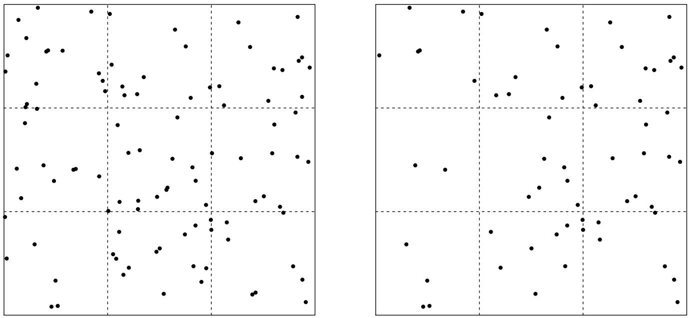
Applications:

- test of independece of two different patterns
  - Getzin, Wiegand and Hubbell (2014)
- conditional reconstruction
  - Pommerening (2006)
  - Tscheschel and Chiu (2008)





Inhomogeneous case



Modifications of the algorithm:

- initial configuration + generating new points,
- quality of the reconstruction,
- energy functional.

References

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A. Tscheschel and D. Stoyan, “Statistical reconstruction of random point patterns”, *Computational Statistics & Data Analysis*, vol. 51, pp. 859-871, 2006.

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