Markov Decision Process in Dynamic Optimization of Fare Price

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Decision-dependent Newsboy Problem

- Number of demanded newspapers D
- Fixed inventory level n
- Price p as a decision variable

$$\pi_k(p) = \mathbb{P}[D = k | p]$$

- Objective: maximize expected return wrt. price

$$\mathbb{E}[pD] = p\left(\sum_{k=0}^{n} k\pi_k(p) + n\sum_{k=n+1}^{\infty}\pi_k(p)\right)$$

- E.g. Poisson demand with log-linear model

$$\pi_k(p) = \frac{\lambda^k(p)}{k!}\mathrm{e}^{-\lambda(p)}, \qquad \lambda^k(p) = \exp(\beta_0 + \beta_1 \log(p))$$

Optimal control

- Selling over time interval $D_t, t \in [0, 1]$
- Markov chain with state space representing number of sold newspapers

$$\mathcal{S} = \{0, ..., n\}, \qquad X_0 = 0$$

- Objective: find optimal policy φ^* for price

$$\varphi : \mathcal{S} \times [0, 1] \to \mathbb{R}^+$$

- Piecewise deterministic function

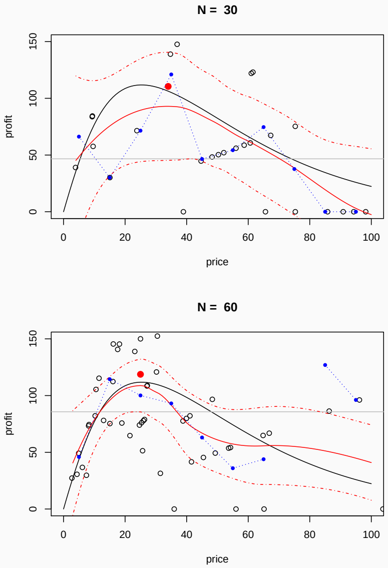
Random inventory level

- Model for train fare price with K stations
- Single passenger type, single class
- $\binom{K}{2}$ routes indexed by boarding and exiting stations (k, l), $1 \leq k < l \leq K$
- Bounded sums of inventory levels

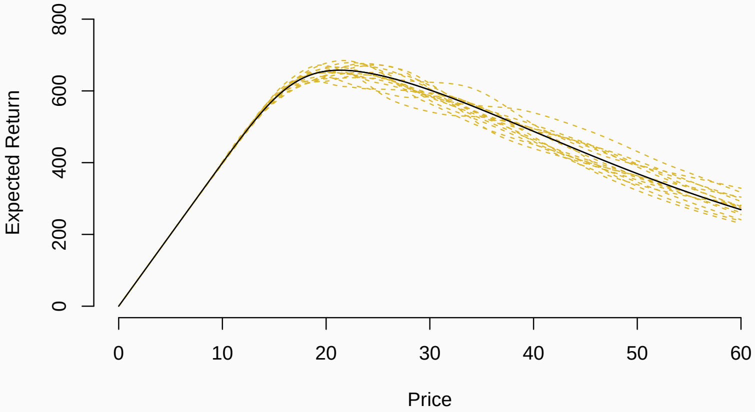
$$\mathcal{S} = \left\{ \boldsymbol{s} \in \mathbb{N}_0^{\binom{K}{2}} : \sum_{k=1}^{h} \sum_{l=h+1}^{K} s_{k,l} \leq n, 1 \leq h \leq K-1 \right\}$$

- Non-computable transition probabilities

Simulated optimization I



Simulated optimization II



Further generalizations

- Multiple passenger types
- Unlimited number of passenger per ticket
- Multiple seat classes (substitutes)
- Distinct seats, passenger can choose a seat

Content of the Poster

- Formal notation of objects and optimization problem
- Model for transition intensities (demand)
- Algorithms for simulated optimization
- Way how to transform the problem to endogenous
- Numerical results for fictional train

Thank You for Your Attention!