Robustní monitorování stability v modelu CAPM

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Contents

- Introduction
- Robust sequential monitoring in CAPM
- Asymptotic results
- Numerical results
**CAPM (Capital Assets Pricing Model)**

\[ r_{ tj} - r_{ t,f} = \beta_j (r_{ t,M} - r_{ t,f}) + \varepsilon_{ tj}, \quad j = 1, \ldots, d, \quad t = 1, \ldots, N \]

- \( r_{ tj} \) - return of an asset \( j \) at time \( t \)
- \( r_{ t,M} \) - return of the market portfolio at time \( t \)
- \( r_{ t,f} \) - riskless security
- \( r_{ tj} - r_{ t,f} \) - excess return (risk premium)
- \( \beta_j \) - measure of risk of the asset \( j \) w.r.t. the market portfolio

reparametrized model

\[ r_{ tj} = \alpha_j + \beta_j r_{ t,M} + \varepsilon_{ tj} \]
Model with time varying betas:

$$r_{tj} = \alpha_{tj} + \beta_{tj} r_{t,M} + \varepsilon_{tj}, \quad j = 1, \ldots, d, \quad t = 1, \ldots, N$$

or more generally,

$$r_i = \alpha_i + \beta_i r_{i,M} + \varepsilon_i, \quad i = 1, \ldots,$$

$r_i, \alpha_i, \beta_i, \varepsilon_i$ are $d$-dimensional vectors
Model with time varying betas:

\[ r_{tj} = \alpha_{tj} + \beta_{tj} r_{t,M} + \varepsilon_{tj}, \quad j = 1, \ldots, d, \ t = 1, \ldots, N \]

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Sequential monitoring:

We assume stable historical data of size \( m \) such that

\[ \alpha_1 = \ldots = \alpha_m = \alpha_0, \quad \beta_1 = \ldots = \beta_m = \beta_0, \]

and with any new observation we want to decide whether the stability in parameters is violated or not.
Hypothesis testing problem:

\[ H_0 : \beta_1 = \ldots = \beta_0 \]
\[ H_1 : \beta_0 = \beta_1 = \ldots = \beta_{m+k^*} \neq \beta_{m+k^*+1} = \ldots \]

where \( k^* = k_m^* \) is an unknown change point.

The null hypothesis is rejected whenever for the first time

\[ \hat{Q}(k, m)/q_\gamma(k/m) \geq c_\alpha \]

where \( \hat{Q} \) is a test statistic, \( q_\gamma(t), \ t \in (0, \infty) \) is a boundary function and \( c_\alpha \) is an appropriately chosen critical value.
The stopping rule:

\[ \tau_m(\gamma) = \begin{cases} 
\inf\{1 \leq k < mT + 1 : \frac{\hat{Q}(k, m)}{q_\gamma(k/m)} \geq c_\alpha\}, \\
\infty, & \text{if } \frac{\hat{Q}(k, m)}{q_\gamma(k/m)} < c_\alpha \quad \forall 1 \leq k < mT + 1. 
\end{cases} \]

( closed-end procedure)
The stopping rule:

$$\tau_m(\gamma) = \begin{cases} \inf\{1 \leq k < mT + 1 : \hat{Q}(k, m)/q_\gamma(k/m) \geq c_\alpha\}, \\ \infty, \text{ if } \hat{Q}(k, m)/q_\gamma(k/m) < c_\alpha \quad \forall \ 1 \leq k < mT + 1. \end{cases}$$

( closed-end procedure)

The constant $c = c_\alpha$ is chosen such that

$$\lim_{m \to \infty} P(\tau_m < \infty | H_0) = \alpha,$$

$$\lim_{m \to \infty} P(\tau_m < \infty | H_1) = 1,$$
Sequential monitoring in linear models

- Chu, Stinchcombe and White, 1996
- Horváth, Hušková, Kokoszka, Steinebach, 2004
- Aue, Hörmann, Horváth, Hušková, and Steinebach, 2011 for CAPM
- Koubková 2006 (monitoring, L1-norm)
Robust procedures in linear models:

- Wu (2007) robust estimators, dependent variables
- Koenker, Portnoy, 1990 - multivariate, independent
- Bai et al. 1990, 1992 - multivariate, independent
- Chan, Lakonischok (1992), Genton and Ronchetti (2008) - empirical studies in CAPM
Sequential robust monitoring in CAPM

\[ r_{ij} = \alpha_j^0 + \beta_j^0 \tilde{r}_{iM} + (\alpha_j^1 + \beta_j^1 \tilde{r}_{iM}) I\{i > m + k^*\} + \varepsilon_{ij}, i = 1, 2, \ldots, \]

\( k^* - \) change point, \( \alpha_j^0, \beta_j^0, \alpha_j^1, \beta_j^1, j = 1, \ldots, d \) unknown parameters

\[
\tilde{r}_{iM} = r_{i,M} - \bar{r}_M, \quad \bar{r}_M = \frac{1}{m} \sum_{i=1}^{m} r_{i,M}.
\]

\( M\)– estimators \( \hat{\alpha}_{jm}, \hat{\beta}_{jm} \) of \( \alpha_j^0, \beta_j^0 \), based on the training sample:

\[
\min \sum_{i=1}^{m} \rho_j(r_{ij} - a_j - b_j \tilde{r}_{iM})
\]

\( \rho_j \) are convex loss functions with the derivatives \( \psi_j, j = 1, \ldots, d \)

\( M\)-residuals

\[
\psi(\hat{\varepsilon}_i) = (\psi_1(\hat{\varepsilon}_{i1}), \ldots, \psi_d(\hat{\varepsilon}_{id}))^T
\]

with

\[
\hat{\varepsilon}_{ij} = r_{ij} - \hat{\alpha}_{jm}(\psi) - \tilde{r}_{iM} \hat{\beta}_{jm}(\psi)
\]
A test statistic based on first \( m + k \) observations

\[
\hat{Q}(k, m) = \left( \frac{1}{\sqrt{m}} \sum_{i=m+1}^{m+k} \tilde{r}_{iM} \psi(\hat{\varepsilon}_i) \right)^T \Sigma_m^{-1} \left( \frac{1}{\sqrt{m}} \sum_{i=m+1}^{m+k} \tilde{r}_{iM} \psi(\hat{\varepsilon}_i) \right),
\]

where the matrix \( \Sigma_m \) is an estimator of the asymptotic variance

\[
\lim_{m \to \infty} \text{var}\left\{ \frac{1}{\sqrt{m}} \sum_{i=1}^{m} (r_{i,M} - Er_{i,M}) \psi(\varepsilon_i) \right\}
\]

based on the first \( m \) observations.
Typical score functions $\psi(x) = \rho'(x)$

- $\psi(x) = x, \ x \in R^1 \ (\rho(x) = x^2)$ - least squares estimators and $L_2$ residuals
- $\psi(x) = \text{sign} \ x, \ x \in R^1 \ (\rho(x) = |x|)$ - $L_1$ estimators and $L_1$ residuals
- Huber

$$\psi(x) = \begin{cases} |x| & |x| \leq K \\ K \text{sign} \ x & |x| > K \end{cases}$$

for $x \in R^1$ and some $K > 0$
Assumptions on score functions $\psi_j$’s

1. $\psi_j$ are monotone functions,

2. functions $\lambda_j(t) = -\int \psi_j(x - t) dF_j(x), \ t \in \mathbb{R}$, satisfy
   $\lambda_j(0) = 0$, $\lambda_j'(0) > 0$, $\lambda_j'(t)$ exists in a neighborhood of 0 and is Lipschitz in neighborhood of 0 for $|t| \leq c_o$ for some $c_o > 0$.

3. $\int |\psi_j(t)|^{2+\Delta} dF_j(t) < \infty$ for some $\Delta > 0$ and

\[
\int |\psi_j(x + t_2) - \psi_j(x + t_1)|^2 dF_j(x) \leq C_0 |t_2 - t_1|^\kappa,
\]

for some $1 \leq \kappa \leq 2$, $c_o > 0$, $C_0 > 0

$F_j$ distribution function of $\varepsilon_{ij}$
Assumptions on regressors

- For any \( i \in \mathbb{Z} \), \( r_{i,M} = h(\xi_i, \xi_{i-1}, \ldots) \), where \( h \) is measurable, \( \{\xi_i\}_i \) is a sequence of i.i.d. random vectors and \( E|r_{0M}|^{2+\Delta} < \infty \) for some \( \Delta > 0 \).

- For all \( i \in \mathbb{Z} \),
  \[
  \sum_{L=1}^{\infty} \|r_{iM} - r_{iM}^{(L)}\|_2 < \infty
  \]
where

\[
  r_{iM}^{(L)} = h(\xi_i, \xi_{i-1}, \ldots \xi_{i-L+1}, \xi_{i-L}^{(L)}, \xi_{i-L-1}^{(L)}, \ldots),
\]

\( \xi_{i-L}^{(L)}, \xi_{i-L-1}^{(L)}, \ldots \) are i.i.d. with the same distribution as \( \xi_i \) independent of \( \{\xi_i\}_i \); \( \Rightarrow \{r_{i,M}^{(L)}\} \) is \( L- \) dependent, \( r_{i,M}^{(L)} \overset{\mathcal{D}}{=} r_{iM} \forall i \in \mathbb{Z} \).
Assumptions on errors

- For any \( i \in \mathbb{Z} \), \( \varepsilon_i = g(\zeta_i, \zeta_{i-1}, \ldots) \), where \( g \) is measurable,
  \( \{\zeta_i\}_i \) is sequence of i.i.d. random vectors
- For all \( i \in \mathbb{Z} \),
  \[
  \sum_{L=1}^{\infty} \|\psi(\varepsilon_i) - \psi(\varepsilon_i^{(L)})\|_2 < \infty
  \]
  \[
  \sum_{L=1}^{\infty} \sup_{|a| \leq a_0} \|\psi(\varepsilon_i - a) - \psi(\varepsilon_i^{(L)} - a)\|_2 < \infty
  \]
  for some \( a_0 > 0 \), where
  \[
  \varepsilon_i^{(L)} = g(\zeta_i, \zeta_{i-1}, \ldots \zeta_{i-L+1}, \zeta_{i-L}, \zeta_{i-L-1}, \ldots)
  \]
  \( \zeta_{i-L}, \zeta_{i-L-1}, \ldots \) are i.i.d., independent of \( \{\zeta_i\}_i \), with the same distribution as \( \zeta_i \).
Asymptotic results

Model under the null hypothesis

**Theorem.** Let the above assumptions be satisfied and

\[ \Sigma_m - \Sigma = o_P(1) \text{ as } m \to \infty. \]

Then under the null hypothesis as \( m \to \infty \)

\[
\max_{1 \leq k \leq mT} \frac{\hat{Q}(k, m)}{q_{2\gamma}(k/m)} \xrightarrow{D} \sup_{0 < t < T/(T+1)} \frac{\sum_{j=1}^{d} W_j^2(t)}{t^{2\gamma}},
\]

where \( \{W_j(t), \ t \in (0, 1)\}, \ j = 1, \ldots, d \) are independent Brownian motions and

\[ q_{\gamma}(t) = (1 + t)(t/(t + 1))^{\gamma}, \ t \in (0, \infty), \ \gamma \in [0, 1/2) \]
Critical values $c_\alpha$ satisfies

$$P\left( \sup_{0 < t < T/(T+1)} \frac{\sum_{j=1}^{d} W_j^2(t)}{t^{2\gamma}} \geq c_\alpha \right) = \alpha.$$ 

The explicit form of the limit distribution is unknown.

Model under local alternatives:

$$r_{ij} = \alpha_j^0 + \beta_j^0 \tilde{r}_{iM} + (\alpha_j^1 + \beta_j^1 \tilde{r}_{iM}) \delta_m I\{i > m + k^*\} + \varepsilon_{ij}$$

$\delta_m \to 0$ and $k^* < Tm + 1$

**Theorem (consistency):** When $\delta_m \to 0$, $|\delta_m|m^{1/2} \to \infty$, $\beta_j^1 \neq 0$ for at least one $j$ and $k^* = \lfloor ms \rfloor$, $0 < s < T$, as $m \to \infty$

$$\max_{1 \leq k \leq mT} \frac{\hat{Q}(k, m)}{q_{\gamma}(k/m)} \to \infty, \text{ in probability.}$$
Estimator of asymptotic variance matrix $\Sigma$

$$\Sigma = \sum_{i=-\infty}^{\infty} E[(r_{0M} - Er_{0M})(r_{iM} - Er_{iM})\psi(\varepsilon_0)\psi(\varepsilon_i)^T]$$

Bartlett -type estimator

$$\Sigma_m = \sum_{|k| \leq q} \omega_q(k) \hat{\Gamma}_k$$

$$\hat{\Gamma}_k = \begin{cases} 
\frac{1}{m} \sum_{j=1}^{m-k} \tilde{r}_{jM} \tilde{r}_{j+k,M} \psi(\hat{\varepsilon}_j)\psi(\hat{\varepsilon}_{j+k})^T & k \geq 0 \\
\hat{\Gamma}_{|k|} & k < 0 
\end{cases}$$

$$\omega_q(x) = (1 - \frac{|x|}{q}) I\{|x| \leq q\}$$

$q(m) \to \infty$ as $m \to \infty$, $q(m)/m \to 0$
Numerical results

Simulations

- Critical values computed from simulated limit distribution
- Empirical quantiles of sample values of test statistic by Monte Carlo method
- Empirical quantiles of sample values of test statistic generated by pair bootstrap from historical period
- Empirical level of test
- Empirical power
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<tr>
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**Table:** Empirical sizes for 5% level, \( T = 10 \), dependent observations.
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**Table:** Empirical sizes for 5% level, \(T = 10\), different error distribution.
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**Table:** Medians of detection delays, \( k^* = 10 \).
Table: Empirical power of the test (in %) for $t_1$ errors

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</tr>
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</table>
Application to real data

Data: not traded indices computed from sector data of global world economy, serve as benchmarks for investors:

- World Consumer Discretionary (3)
- World Consumer Staples (4)
- World Energy (5)
- World Financials (6)
- World Health Care (7)
- Information Technology (9)
- Telecommunication Services (11)

- Market portfolio: MSCI World Daily Index (NDDUWI)
- Risk-free asset: S&P 3M US Treasury Bill
- Monitoring period: 1.12.2006-1.10. 2010
<table>
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<th></th>
<th>L2</th>
<th>L1</th>
<th>Huber</th>
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<td></td>
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<td>$\hat{\beta}$</td>
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<td>1.30334</td>
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**Table:** Estimators of $\alpha, \beta$, historical period 31.12. 2004-30.11. 2006:

4-Consumer Staples, 6-Financial, 7-Health Care
Financials (6), Health Care (7): red - $\gamma = 0$, blue - $\gamma = 0.25$, black - $\gamma = 0.45$

solid line - asymptotic critical values, dashed - closed end
Index: 4, 6  Type: 3

Test statistic

Consumer Staples (4), Financials (6)
Index: 4, 7   Type: 3

Consumer Staples (4), Health Care (7)
References


Thank you for your attention