

Covariate-dependent modeling of  
extreme events by non-stationary  
Peaks Over Threshold analysis  
A review and a case study

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Nemcicky, September 9th 2012

# Motivation, I

- The extreme value theory (EVT) provided an excellent framework for the analysis of climatic hazard: it's elegant, simple, and provides useful and understandable results in terms of magnitude / frequency curves.
- The stationarity assumption, though, is an important limitation of the EVT.
- Recent extensions of the EVT allow for non-stationary analysis (Coles, 2001), and an increasing number of authors are exploring their possibilities for the analysis of climatic hazard.

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## Motivation, II

- Most studies up to now focused on identifying temporal trends in the occurrence of extreme events, i.e. making time a covariate.
- In the last few years other covariates with an expected influence on the occurrence of extreme events are being used, too → high relevance for the statistical downscaling of reanalysis or model data, which typically cannot be used for local impact studies.

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# Summary

In this talk I will present ongoing research on the relationship between extreme precipitation and teleconnection indices in Spain, using non-stationary EVT techniques. The talk is organized as follows:

- A review of non-stationary Peaks Over Threshold analysis
- Case study: relationship between teleconnection indices and extreme rainfall events in Spain
- Conclusions and future work

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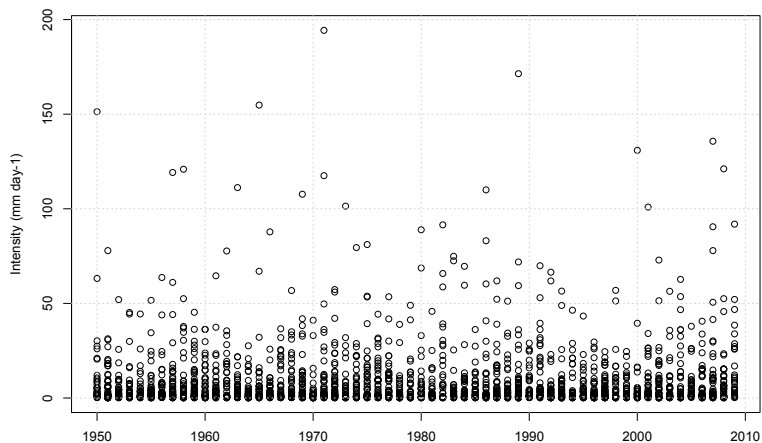
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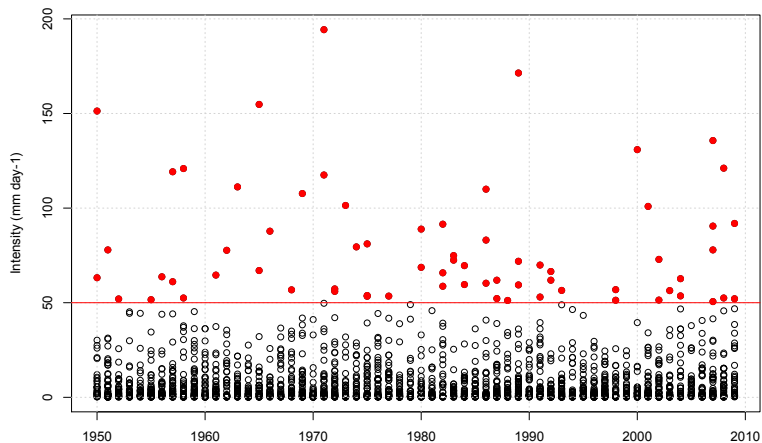
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# Short review of NSPOT analysis, I



Peaks-over-threshold (POT) sampling: take only exceedances over a threshold,  $X > x_0$

# Short review of NSPOT analysis, II



Peaks-over-threshold (POT) sampling: take only exceedances over a threshold,  $X > x_0$

## Short review of NSPOT analysis, III

Stationary POT: assuming independent inter-arrival times, the POT data follows a Generalized-Pareto distribution.

Probability of exceedance:

$$P(X > x | X > x_0) = 1 - \lambda \left( 1 + \kappa \frac{x - x_0}{\alpha} \right)^{-1/\kappa} \quad (1)$$

Quantile corresponding to a return period  $T$ :

$$X_T = x_0 + \frac{\alpha}{\kappa} \left[ 1 - \left( \frac{1}{\lambda T} \right)^\kappa \right] \quad (2)$$

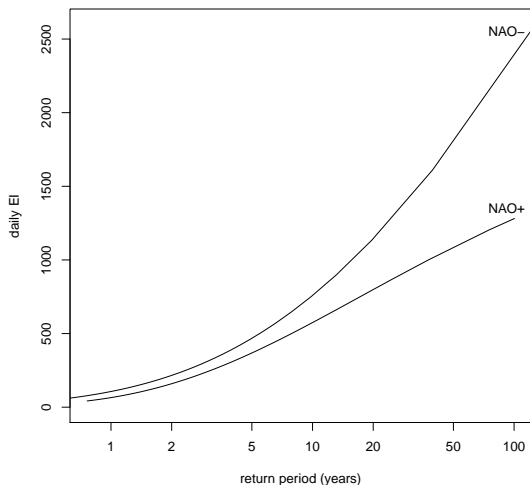
(beware of alternative conventions:  $x_0 = u$ ,  $\alpha = \sigma$ ,  $\kappa = \xi$ )

## Short review of NSPOT analysis, IV

Approaches for assessing non-stationarity in POT modeling:

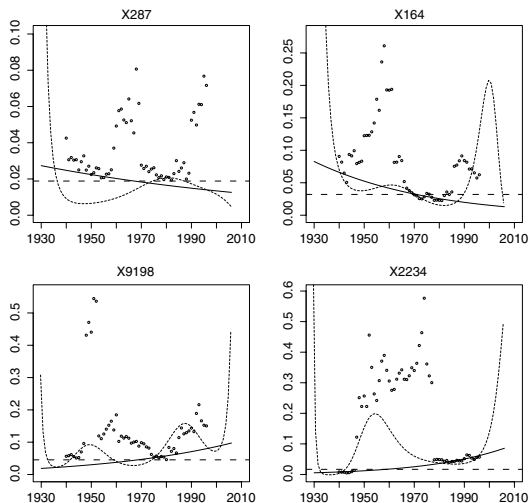
- Split-sample approach (Li et al., 2005)
- Moving kernel (Hall and Tajvidi, 2000)
- Non-stationary POT (NSPOT) modeling

# Short review of NSPOT analysis, V



Split-sample approach: independent models for positive and negative phases of NAO (Angulo et al., 2011).

# Short review of NSPOT analysis, VI



Moving kernel approach: time variability in the P10 quantile, based on a moving window of the previous 20 years of data (Beguería et al., 2011).



## Short review of NSPOT analysis, VII

Stationary POT:

$$P(X > x | X > x_0) = 1 - \lambda \left( 1 + \kappa \frac{x - x_0}{\alpha} \right)^{-1/\kappa}$$

Non-stationary POT:

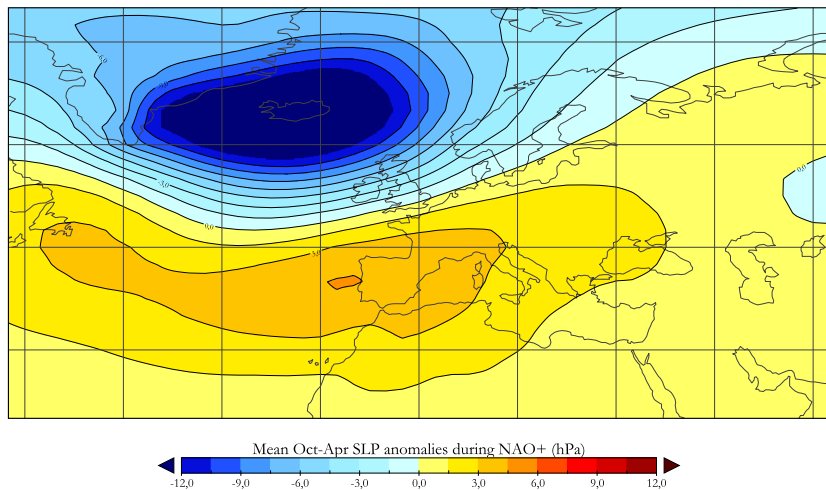
$$P(X > x | X > x_0, C) = 1 - \lambda(c) \left( 1 + \kappa(c) \frac{x - x_0(c)}{\alpha(c)} \right)^{-1/\kappa(c)} \quad (3)$$

## Short review of NSPOT analysis, VIII

Some examples of NSPOT analysis of climatic variables:

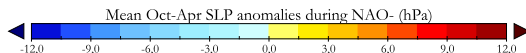
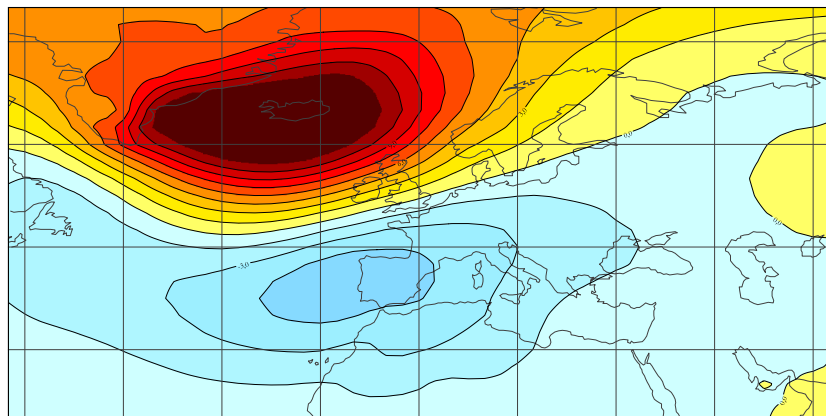
- Time dependence of T and P (Smith, 1999)
- Nogaj et al. (2006) time trends of T extremes over the NA region
- Laurent and Parey (2007), Parey et al. (2007), T extremes in France
- Méndez et al. (2006), trends and seasonality of POT wave height
- Yiou et al. (2006) trends of POT discharge in the Czech Republic
- Abaurrea et al. (2007) trends of POT T in the IP
- Acero et al. (2011), Beguería et al. (2011), trends in POT P, IP
  
- Friederichs (2010), Kallache et al. (2011), downscaling of POT P based on reanalysis / GCM data
- Tramblay et al. (2012), covariation between POT P extremes and atmospheric covariates, SE France

# Teleconnections affecting precipitation in the IP, I



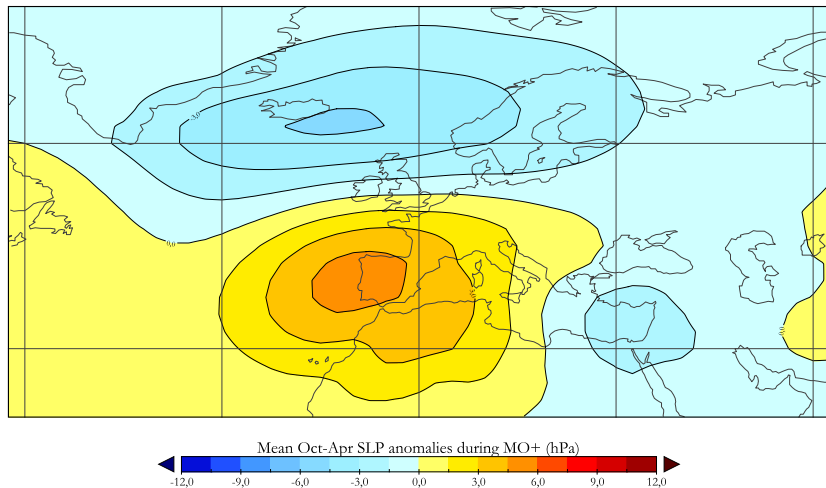
The North Atlantic Oscillation (NAO).

# Teleconnections affecting precipitation in the IP, II



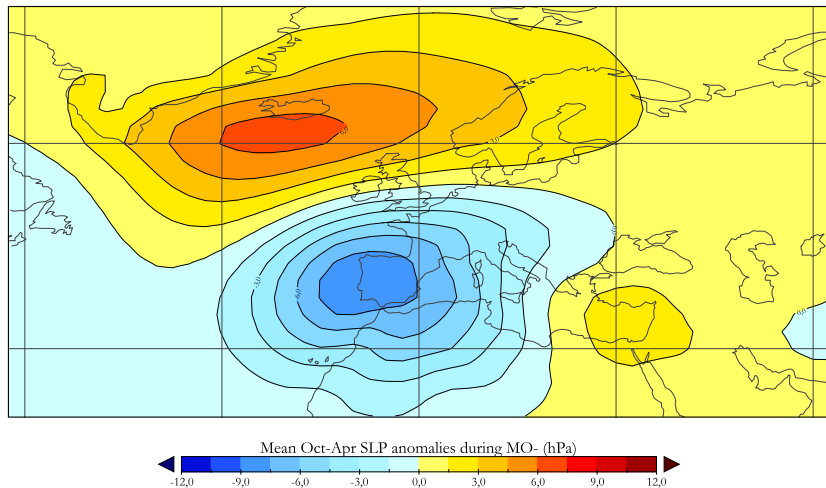
The North Atlantic Oscillation (NAO).

# Teleconnections affecting precipitation in the IP, III



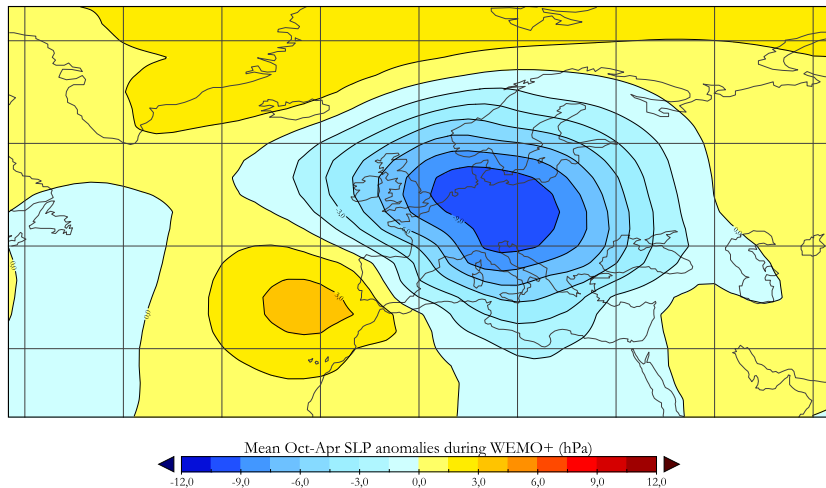
The Mediterranean Oscillation (MO, Palutikof 2003).

# Teleconnections affecting precipitation in the IP, IV



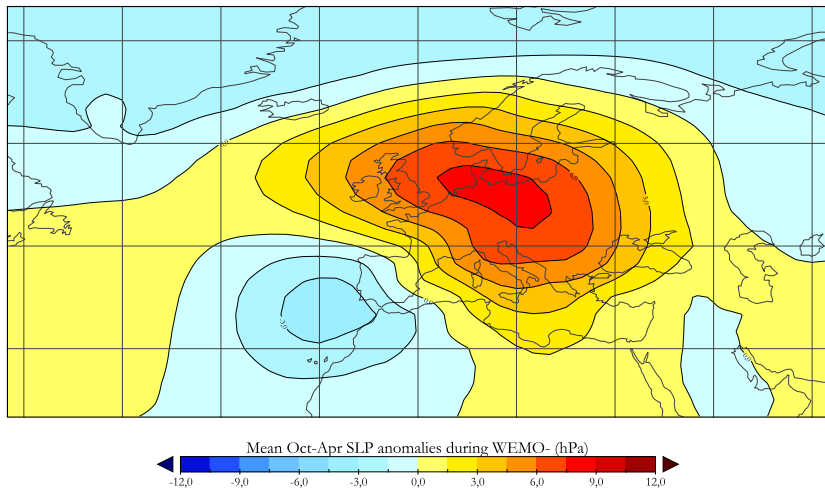
The Mediterranean Oscillation (MO, Palutikof 2003).

# Teleconnections affecting precipitation in the IP, V



The Western Mediterranean Oscillation (WEMO, Martín-Vide and López-Bustins 2006).

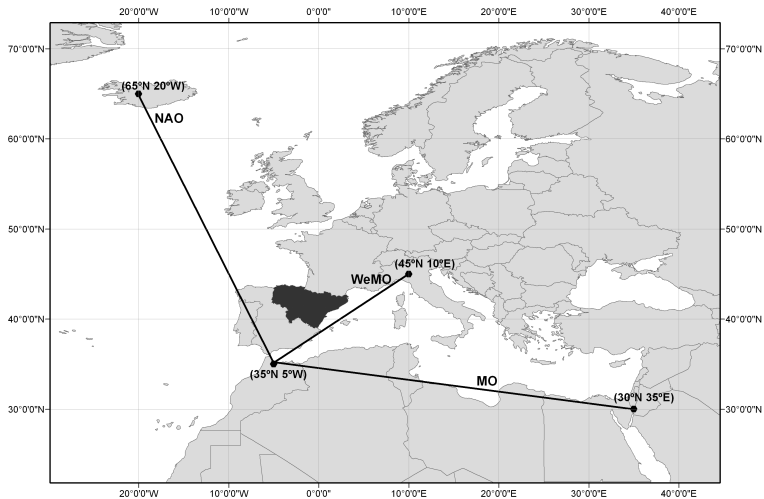
# Teleconnections affecting precipitation in the IP, VI



The Western Mediterranean Oscillation (WEMO, Martín-Vide and López-Bustins 2006).

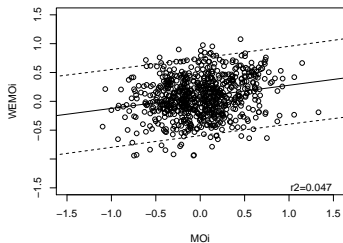
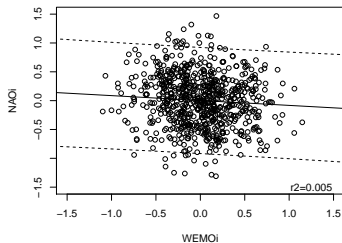
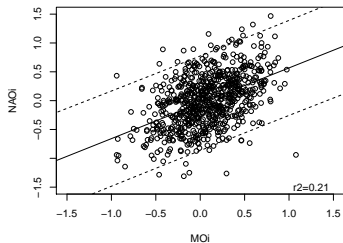


# Dataset, II



Teleconnection indices (Reykjavik, Padova, Lod and Gibraltar). Sources: <http://www.cru.uea.ac.uk>, <http://www.ub.es>.

# Teleconnections affecting precipitation in the IP, VII



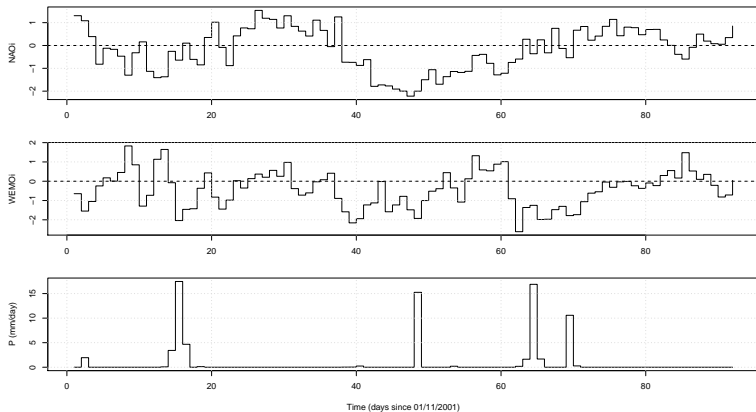
Correlations between teleconnection indices.

# Dataset, I



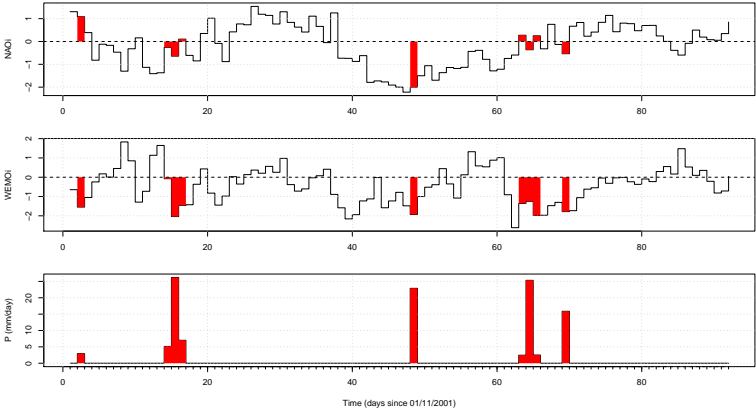
106 stations, 58 daily precipitation series reconstructed for the period 1950-2009 (source: AEMET).

# Dataset, III



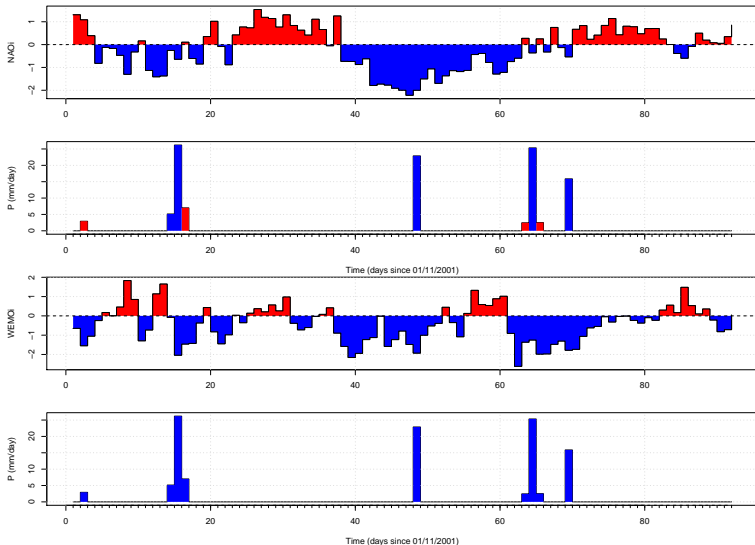
**Declustering: intensity and magnitude series and associated teleconnection indices.**

# Dataset, IV



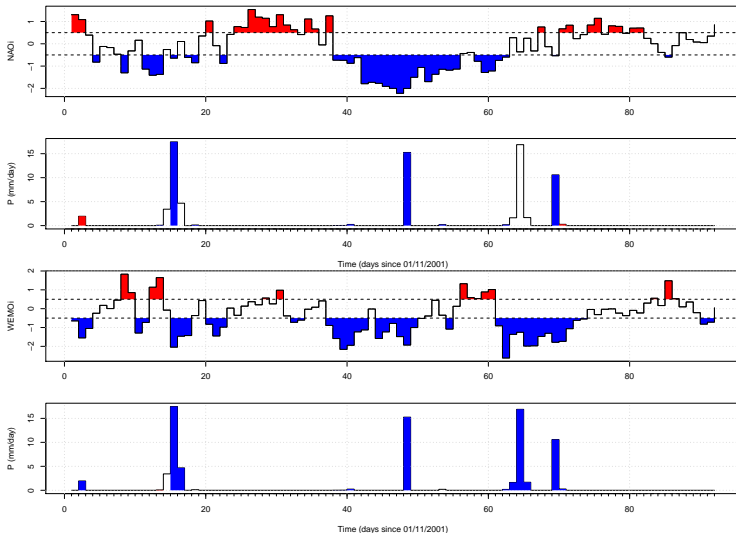
Declustering: intensity and magnitude series and associated teleconnection indices.

# Dataset, V



Declustering: intensity and magnitude series and associated teleconnection indices.

# Dataset, VI



Decustering: intensity and magnitude series and associated teleconnection indices.

## Analysis, I

$$M0: P(X > x | X > x_0) = 1 - \lambda \left( 1 + \kappa \frac{x - x_0}{\alpha} \right)^{-1/\kappa}$$

$$M1: P(X > x | X > x_0, C) = 1 - \lambda \left( 1 + \kappa \frac{x - x_0(c)}{\alpha(c)} \right)^{-1/\kappa}$$

$$x_0 = \beta_0 + \beta_i c \quad (4)$$

$$\alpha = \gamma_0 \gamma_i^c \quad (5)$$

$$\kappa = \delta \quad (6)$$

$$\lambda = \varepsilon \quad (7)$$

Likelihood ratio test:

$$D = -2 (\ell_1(M_1) - \ell_0(M_0)) \quad (8)$$

distributed according to  $\chi_k^2$  (with d.f.  $k = 4$ ).

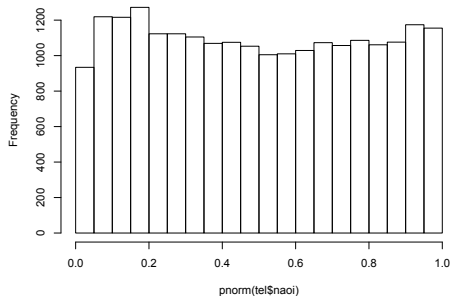
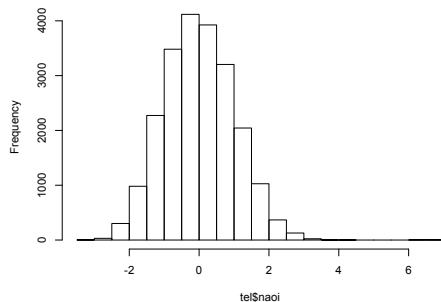


## Analysis, II

R, package `ismev` (Stuart Coles, ported to R by Alec Stephenson).

```
...  
m0 <- gpd.fit(xdat=dat, threshold=u, npy=rate)  
...  
uu <- predict(lm(v~cdat))  
m1 <- gpd.fit(xdat=dat, threshold=uu, npy=rate, ydat=cdat,  
sigl=1, siglink=exp)
```

# Analysis, III



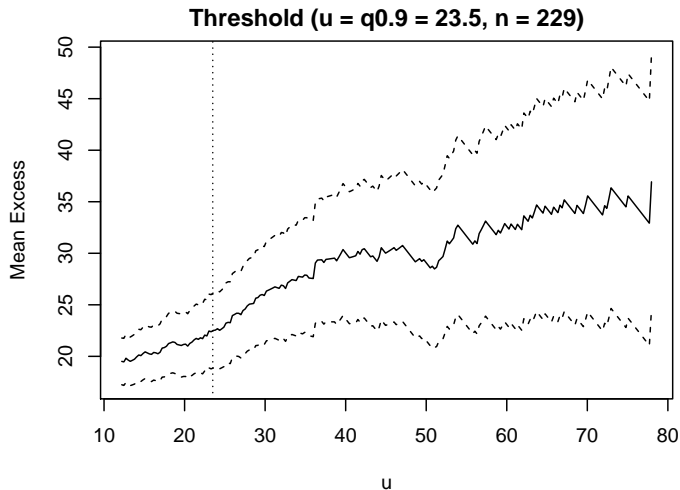
Covariates: NAOi and `pnorm(NAOi)`

## Example: Valencia, I



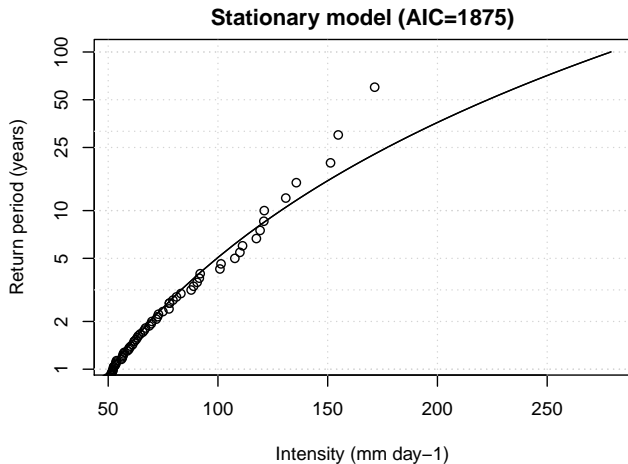
Spatial location

## Example: Valencia, II



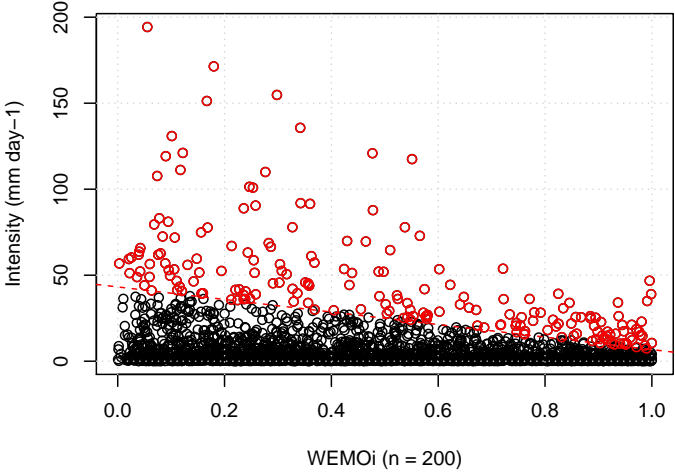
Stationary model: fixed threshold

## Example: Valencia, III



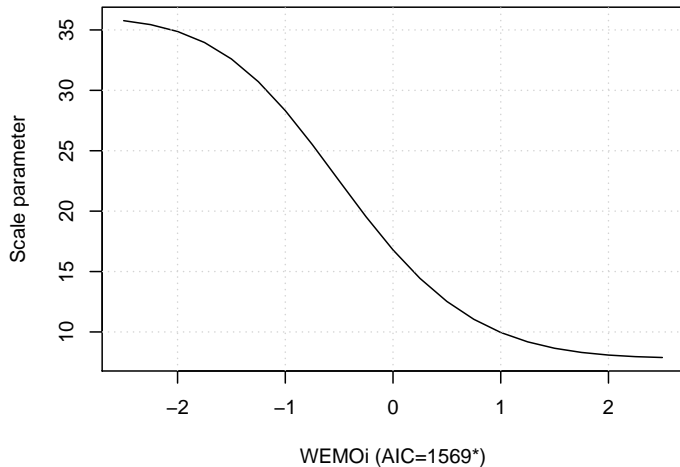
Stationary model: quantile plot

# Example: Valencia, IV



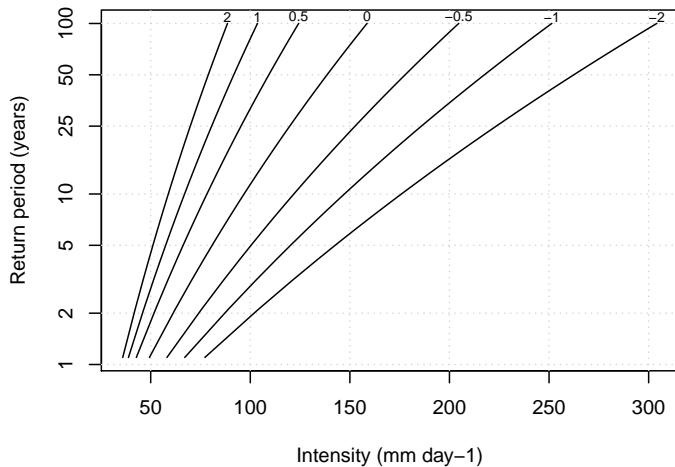
Non-stationary model: threshold model

## Example: Valencia, V



Non-stationary model: scale parameter model

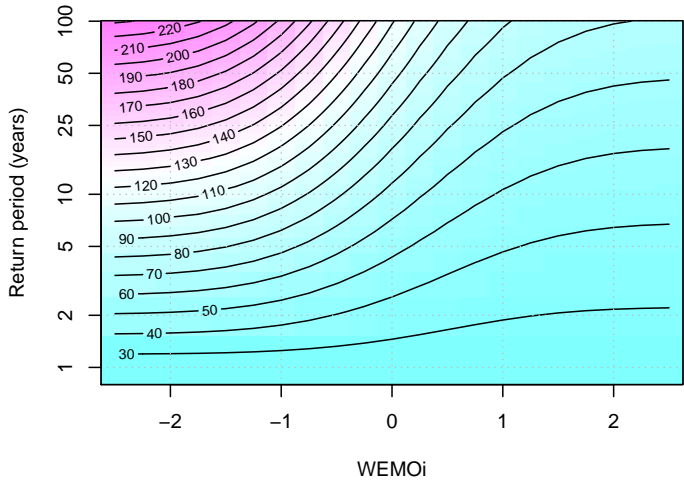
## Example: Valencia, VI



Non-stationary model: quantile plot

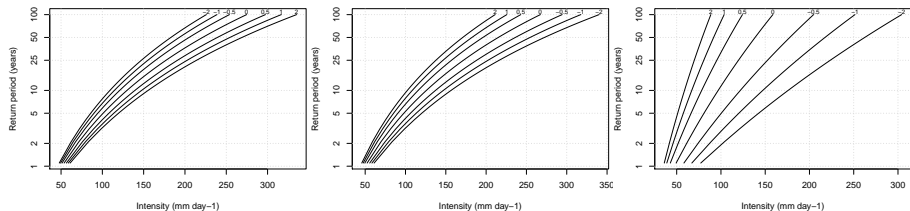


# Example: Valencia, VII



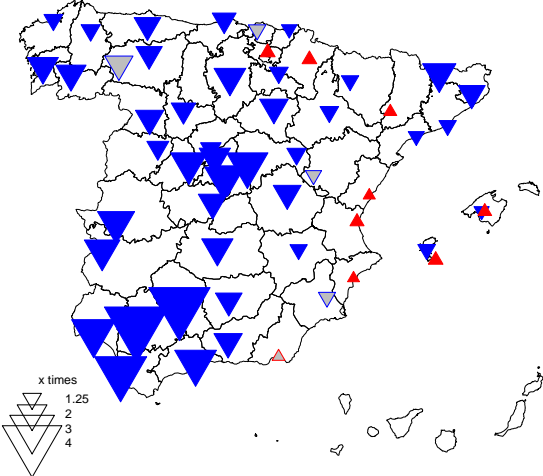
Non-stationary model: quantile plot

# Example: Valencia, VIII



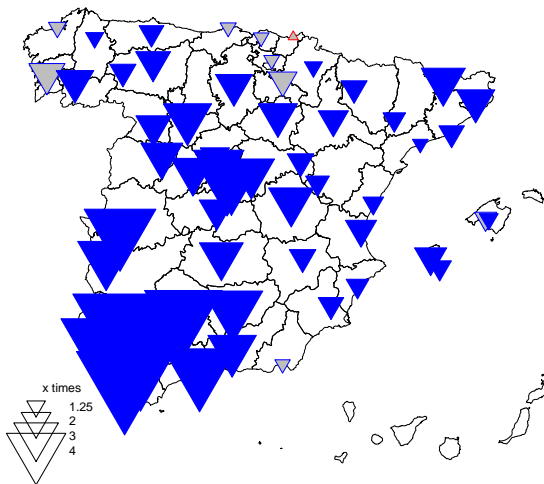
Non-stationary model: NAO (left), MO (center), WEMO (right)

# Results: event's magnitude, I



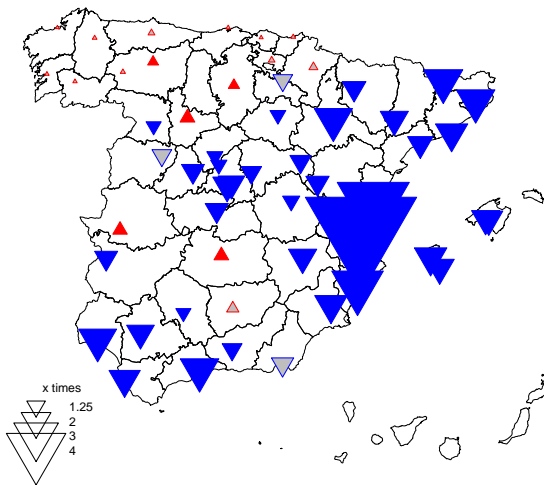
Effect of NAO on the 100-years return period event:

# Results: event's magnitude, II



Effect of MO on the 100-years return period event

# Results: event's magnitude, III



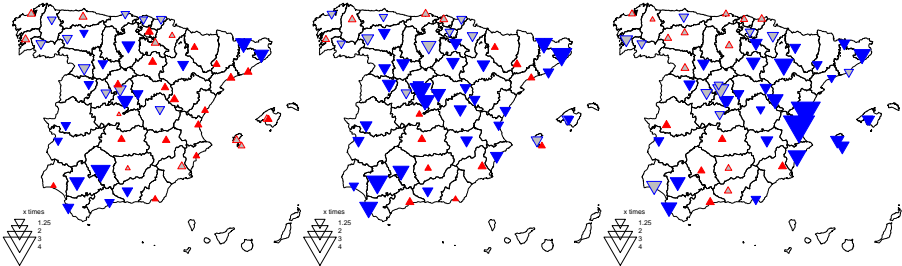
Effect of WEMO on the 100-years return period event

# Results: event's intensity

NAO effect on q100

MO effect on q100

WEMO effect on q100



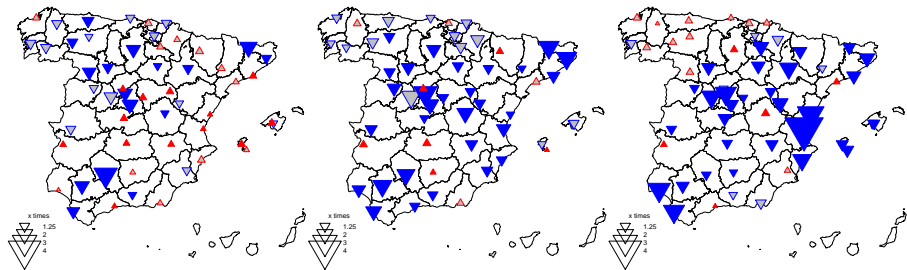
Effect of NAO, MO and WEMO on the 100-years return period event

# Results: event's magnitude, winter

NAO effect on q100

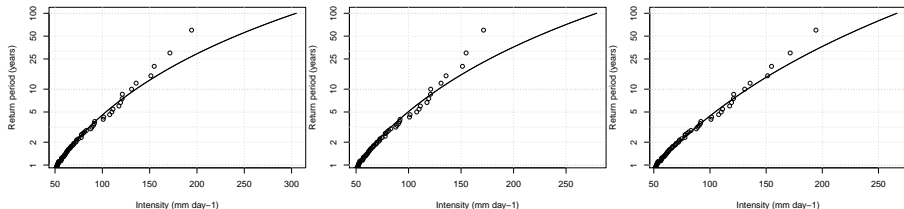
MO effect on q100

WEMO effect on q100



Effect of NAO, MO and WEMO on the 100-years return period event

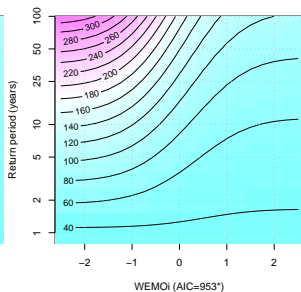
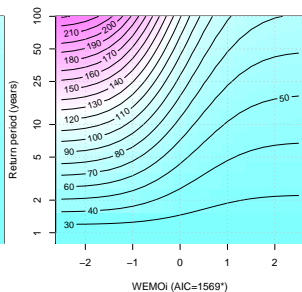
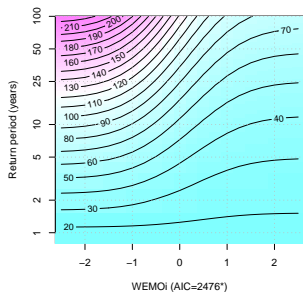
# Results: threshold independence, I



Quantile plots for rainfall intensity in Valencia, thresholds at  $u=q85$ ,  $u=q90$  and  $u=q95$



# Results: threshold independence, II



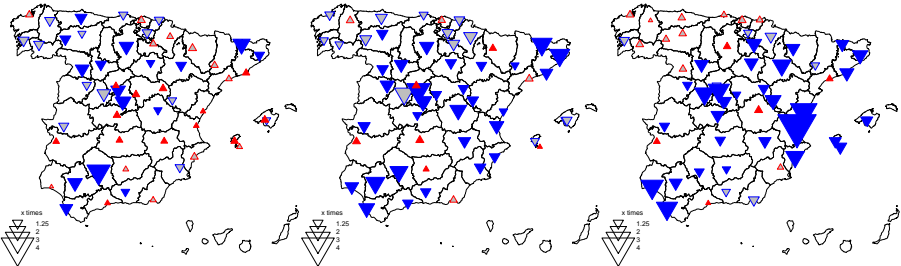
Quantile plots for rainfall intensity in Valencia, thresholds at  $u=q85$ ,  $u=q90$  and  $u=q95$

# Results: threshold independence, III

NAO effect on q100

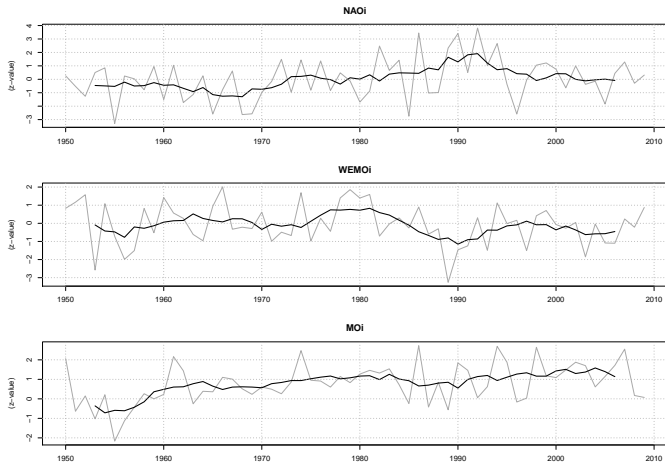
MO effect on q100

WEMO effect on q100



Effect of WEMO in rainfall magnitude, thresholds at  $u=q85$ ,  $u=q90$  and  $u=q95$

# Projected evolution of NAOi, MOi and WEMOi



Time variation of NAOi, MOi and WEMOi in the 21th Century, INMCM3.0 model output, 48-months convolution  
(envelope of SRES A1b, A2 and B1 scenarios)

# Conclusions and future work I

- NSPOT analysis is good at capturing the relationship between extreme precipitation processes and atmospheric circulation indices.
- The results are promising for a variety of applications, including short-term warning systems and the statistical downscaling of GCM/RCM outputs.

## Conclusions and future work II

- Clustering methods based on the series of covariates (and not on P).
- Other covariates: synoptic scale airflow parameters (direction, strength, vorticity), specific humidity, etc.
- Multi-covariate analysis.
- Spatial model: take advantage of spatial dependence to reduce uncertainty.

# References I

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# References II

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Thank you!

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