Ratio Type Statistics for Detection of Changes in Mean and the Bootstrap Method

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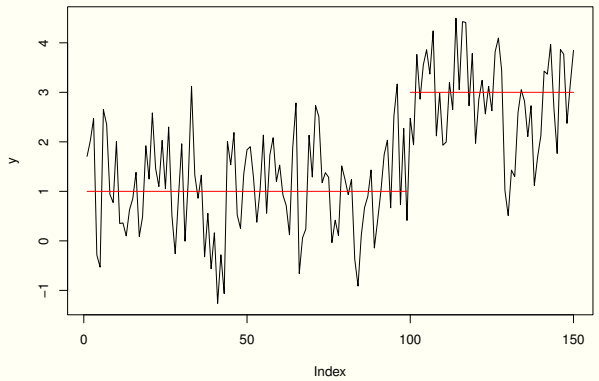
The studied problem

- Observations $Y_1, \ldots, Y_n$ obtained at n time-ordered points
- Location model with at most one abrupt change in the mean:

$$Y_k = \mu + \delta \mathbf{I}\{k > k^*\} + e_k, \qquad k = 1, \ldots, n,$$

  where μ, $\delta = \delta_n$ and $k^* = k_n^*$ are unknown parameters. k^* is called the change-point. $e_1, \ldots, e_n$ are random errors
- We are interested in studying null hypothesis of no change against the alternative that change has occurred at some time-point



Test statistic

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- A test statistic based on M-residuals is considered:

$$W_n = \max_{n\gamma \le k \le n-n\gamma} \frac{\max\limits_{1\le i\le k} \left| \sum\limits_{1\le j\le i} \psi(Y_j - \hat{\mu}_{1k}(\psi)) \right|}{\max\limits_{k\le i\le n} \left| \sum\limits_{i+1\le j\le n} \psi(Y_j - \hat{\mu}_{2k}(\psi)) \right|},$$

Motivation for studying ratio type statistics

- Ratio type statistics studied for the fact that variance estimation is not required for the construction of the test statistic
- May be an advantageous property in case of dependent random errors

Resampling methods

- The asymptotic distribution is a rather complex functional of Wiener processes
- One need to use simulation to approximate the critical values - may be problematic
- Resampling methods seem to be a better option
- In case of dependent random errors, it is more suitable to apply the *block* permutation principle

Summary

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- L_1 and L_2 procedures were studied
- Block bootstrap applied to non-i.i.d. data
- Reasonable results for AR(1) sequences with values of the autoregression coefficient between -0.5 and 0.5

Outlook

- Problem of finding the optimal block length
- Obtain theoretical results for non-i.i.d data