On Testing Changes in Parameters of an Autoregressive Model

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Approach

- modification of likelihood ratio test in univariate AR(p) process for detecting a single change in variance of white noise residuals
- analytical approach

Advantages

- detect change only in variance
- better power (based on simulations)

Main result

Given a sample $Y_1, \ldots, Y_T$ drawn from stationary AR(p),

$$Y_t = \varphi_0 + \varphi_1 Y_{t-1} + \ldots + \varphi_p Y_{t-p} + \varepsilon_t,$$

$\{\varepsilon_t\}$ is iid sequence, $\mathrm{E}\,\varepsilon_t = 0$, $\mathrm{var}\,\varepsilon_t = \sigma^2 \mathbb{I}_{[t \leq k]} + \widetilde{\sigma}^2 \mathbb{I}_{[t>k]}$. We wish to test $H: \; k = T$ (no change occurred) against $K: \; k < T$ (a single change occurred).
The test for detecting a change in variance is based on the loglikelihood ratio test statistic. Denote

$$\Lambda_T(k) = (T-p)\log(\widehat{\sigma}^2) - (k-p)\log\left(\widehat{\sigma}_1^2\right) - (T-k)\log\left(\widehat{\sigma}_2^2\right)$$

and where

$$\widehat{\sigma}^2 = \frac{1}{T-p} \sum_{t=p+1}^{T} \left(Y_t - \mathbf{x}_t^\top \widehat{\beta} \right)^2,$$

$$\widehat{\sigma}_1^2 = \frac{1}{k-p} \sum_{t=p+1}^{k} \left(Y_t - \mathbf{x}_t^\top \widehat{\beta} \right)^2,$$

$$\widehat{\sigma}_2^2 = \frac{1}{T-k} \sum_{t=k+1}^{T} \left(Y_t - \mathbf{x}_t^\top \widehat{\beta} \right)^2,$$

$\widehat{\beta} = (\widehat{\varphi}_0, \ldots, \widehat{\varphi}_p)^\top$ is OLS estimate from the whole sample and $\mathbf{x}_t = (1, Y_{t-1}, \ldots, Y_{t-p})^\top$.

Denote

$$\Lambda_T := \max_{\{2p+2 \leq k \leq T-p-2\}} \{\Lambda_T(k)\},$$

then under H and certain moment and stationary conditions, for $T \to \infty$

$$\mathrm{P}\left[\frac{\Lambda_T - b_T(1)}{a_T(1)} \leq x\right] \to \exp\left\{-2e^{-\frac{x}{2}}\right\},$$

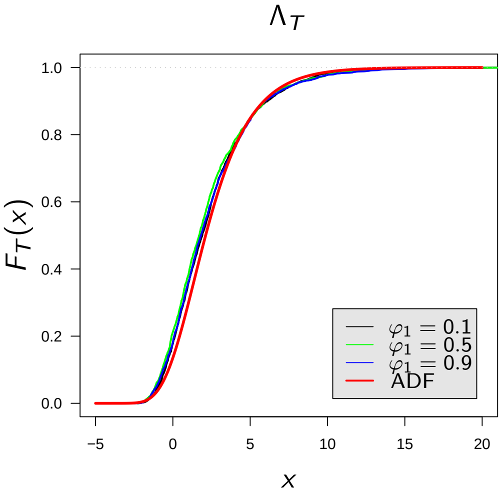
where

$$b_T(d) = \frac{\left(2\log\log T + \frac{d}{2}\log\log\log T - \log\Gamma\left(\frac{d}{2}\right)\right)^2}{2\log\log T},$$

$$a_T(d) = \sqrt{\frac{b_T(d)}{2\log\log T}}.$$

Simulation study - asymptotics:

Empirical distribution function for $(\Lambda_T - b_T(1))/a_T(1)$ for $T = 200$, $p = 1$, $\sigma^2 = 1$, $\varphi_0 = 0$, together asymptotic distribution function (=ADF) (red), $\texttt{rep} = 2\,000$



Further study

- detection of changes in VAR models (change in the parameters and variance)

Some articles

- Davis R. A. et al. (1995):
  Testing for a Change in the Parameter Values and Order of an Autoregressive Model.
  The Annals of Statistics, Vol. 23, No. 1, pp. 282-304
- Gombay E. (2008):
  Change Detection in Autoregressive Time Series.
  Journal of Multivariate Analysis Vol. 99, No. 3, 451-464.
- Hušková M., Prášková Z., Steinebach J. (2006):
  On the Detection of Changes in Autoregressive Time Series I..
  J. of Stat. Planning and Inference, Vol. 137, pp. 1243-1259.