$\sqrt{n}$-CONSISTENCY OF THE LEAST WEIGHTED SQUARES UNDER HETEROSCEDASTICITY

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Obsah

The most frequent econometrical (statistical) framework is:

Regression model

$$Y_i = X_i' \beta^0 + \varepsilon_i = \sum_{j=1}^{p} X_{ij} \beta_j^0 + \varepsilon_i, \quad i = 1, 2, ..., n$$

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$$Y = X\beta^0 + \varepsilon$$

Data : (Y, X)

Response var, explanatory vars, error terms (?; disturbances !!), etc.

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Hence one of crucial task is:

Identification of regression model

$$\hat{\beta}^{(n)}(Y, X) \rightarrow R^p$$

$$\hat{\sigma}^2_{(n)}(Y, X) \rightarrow R$$

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Classical assumptions - several variants

Conditions: $\{(X_i', \varepsilon_i)'\}_{i=1}^{\infty}$ *is sequence*

of independent (?) $(p+1)$*-dimensional random variables.*

Explanatory variables are not correlated with disturbances

– to verify (?) or to reach (?) - if not .... .

Disturbances are normally distributed

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Durbin, J. (1960): Estimation of parameters in time-series regression models. *J. of Royal Statistical Society, Series B, 22, 139 - 153.*

Mizon, G. E. (1995): A simple message for autocorrelation correctors: Don't. *Journal of Econometrics 69, 267 - 288.*

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Equivariance - invariance of $\hat{\beta}^{(n)}$

$$\hat{\beta}(Y, X) : M(n, p+1) \to R^p$$

scale-equivariant : $\forall c \in R^+ \quad \hat{\beta}(cY, X) = c\hat{\beta}(Y, X)$

regression-equivariant : $\forall b \in R^p \quad \hat{\beta}(Y + Xb, X) = \hat{\beta}(Y, X) + b$

Examples : $\hat{\beta}^{(OLS,n)} = (X'X)^{-1} X'Y$

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$\hat{\beta}^{(L_1,n)} = \ \ldots$

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$\hat{\sigma}_{(L_1,n)} = 1.483 \cdot MAD \qquad \mathcal{L}(e) = \mathcal{N}(\mu, \sigma^2)$

$$MAD = \underset{1 \le i \le n}{\text{med}} \left| r_i(\hat{\beta}^{(L_1,n)}) - \underset{1 \le i \le n}{\text{med}} r_i(\hat{\beta}^{(L_1,n)}) \right|,$$

$$\mathbb{E}_{\mathcal{N}(0,1)} MAD = (1.2533)^{-1}$$

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Bickel, P. J. (1975): One-step Huber estimates in the linear model. *J. Amer. Statist. Assoc. 70, 428–433.*

To reach *scale-* and *regression-equivariance* of an *M*-estimator, say

$$\hat{\beta}^{(M,\rho,n)} = \arg\min_{\beta \in R^p} \sum_{i=1}^{n} \rho\left(\frac{Y_i - X_i'\beta}{\hat{\sigma}_{(n)}}\right)$$

$\hat{\sigma}_{(n)}$ is to be *scale-equivariant* and *regression-invariant*.

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An advantage of *M*-estimator = technically tractable.

Significant disappointment = low breakdown point equal to $\frac{1}{p+1}$.

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Examples Trimmed mean

Trimmed empirical variance

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Definition: Breakdown point

The break down point of the sequence of estimators $\{T_n\}_{n=1}^{\infty}$ at the d. f. F is defined by

$$\varepsilon^* = \sup \Big\{ \varepsilon \le 1; \exists \text{ a compact } K_\varepsilon \subsetneqq \Theta : \pi(F, G) < \varepsilon \;\Rightarrow\; G(\{T_n \in K_\varepsilon)\} \xrightarrow[n \to \infty]{} 1 \Big\}$$

Hampel, F. R. et al. (1986): *Robust Statistics – The Approach Based on Influence Functions.* New York: J.Wiley & Son.

$$\hat{\beta}^{(LMS,n,h)} = \arg\min_{\beta \in R^p} \; r^2_{(h)}(\beta)$$

Rousseeuw, P.J. (1984): Least median of square regression. *Journal of Amer. Statist. Association 79, pp. 871-880.*

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Notice that both are *scale-* and *regression-equivariance* and can be considered to be *L*-estimators (although it need not be clear at the first glance).

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Statistical folklore about high breakdown point

High sensitivity to the change of data

- This is an academic example - explaning “why”,
- there are also real data, exhibiting the same phenomenon.

"Decreasing" model

"Increasing" model

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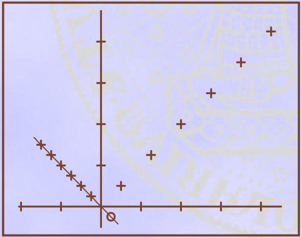

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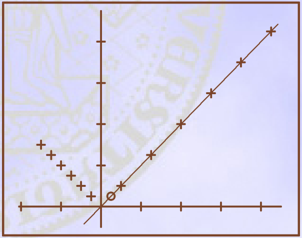

The first estimate of scale of disturbances which is consistent, scale-equivariant and regression-invariant:

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$i = 1, 2, \ldots, n$

The least weighted squares

Residuals $\quad \forall \beta \in R \quad \rightarrow \quad r_i(\beta) = Y_i - X_i'\beta$

Order statistics of squared residuals, i. e.

$$r^2_{(1)}(\beta) \le r^2_{(2)}(\beta) \le \ldots \le r^2_{(n)}(\beta)$$

Definition

Let $w(u): [0,1] \rightarrow [0,1], w(0) = 1$, nonincreasing. Then

$$\hat{\beta}^{(LWS,n,w)} = \arg\min_{\beta \in R^p} \sum_{i=1}^n w\left(\frac{i-1}{n}\right) r^2_{(i)}(\beta)$$

will be called *the least weighted squares* *(LWS)*.

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Víšek, J. Á. (2000): Regression with high breakdown point.
Robust 2000 (eds. Antoch, J. Dohnal, G.), 324 - 356.

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What about to employ it for estimating scale of disturbances?

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Conditions

Conditions $\mathcal{S}1$: (conditions on r.v.'s)

1. $\{(X_i', \varepsilon_i)'\}_{i=1}^{\infty}$ i.i.d., $F_{X,e}(x, v) = F_X(x) \cdot F_e(v)$, $\quad F_e(v) = F(v \cdot \sigma^{-1})$.

Conditions $\mathcal{S}2$: (conditions on weight function)

1. $w(u): \ [0, 1] \leftarrow [0, 1], \ w(0) = 1$ continuous, nonincreasing.
2. Lipschitz, i. e. $|w(u_1) - w(u_2)| \leq L \cdot |u_1 - u_2|$.

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2. $F(v), \quad \sup_{-\infty < v < \infty} f(v) < U, \quad E_F V = 0, \; E_F V^2 = 1$.
3. $E_X \|X_1\|^2 < \infty$.

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Proposal of estimator of scale of disturbances

Put

$$\gamma = \int w(F(|u|)) \cdot u^2 \cdot f(u)\mathrm{d}u$$

Definition: Scale estimate

Let $\hat{\beta}^{(n)}$ be an estimator of regression coefficients. Then put

$$\hat{\sigma}^2_{(n)} = \gamma^{-1} \cdot \frac{1}{n} \sum_{i=1}^{n} w\left(\frac{i-1}{n}\right) r^2_{(i)}(\hat{\beta}^{(n)}).$$

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$$r_i(\beta) = Y_i - X_i'\beta$$

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Consistency of estimator of scale of disturbances

Conditions $\mathcal{S}3$: (conditions on estimator of regression coefficients)

1. $\hat{\beta}^{(n)}$ is consistent.

Assertion: Consistency

Under Conditions $\mathcal{S}1$, $\mathcal{S}2$ and $\mathcal{S}3$ $\hat{\sigma}^2_{(n)}$ is consistent.

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Consistency of estimator of scale of disturbances

Conditions $\mathcal{S}4$: (conditions on estimator of regression coefficients)

1. $\hat{\beta}^{(n)}$ is $\sqrt{n}$-consistent.

Assertion: Consistency

Under Conditions $\mathcal{S}1$, $\mathcal{S}2$ and $\mathcal{S}4$ $\hat{\sigma}^2_{(n)}$ is $\sqrt{n}$-consistent.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

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$i = 1, 2, \ldots, n$

Consistency of the least weighted squares under heteroscedasticity

Can we meet heteroscedasticity frequently ?

1. Data in question represent the aggregates over some regions.
2. Explanatory vars are measured with random errors.
3. Models with randomly varying coeffs.
4. ARCH models.
5. Probit, logit or counting models.
6. Limited and censored reponse variable.
7. Error component (random effects) models.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

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Heteroscedasticity is implied by character of assumed model.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Consistency of the least weighted squares under heteroscedasticity

Can we meet heteroscedasticity frequently ?

1. Expenditure of households.
2. Demands for electricity.
3. Wages of employed married women.
4. Technical analysis of capital markets.
5. Models of export, import and FDI.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Consistency of the least weighted squares under heteroscedasticity

Can we meet heteroscedasticity frequently ? *(continued)*

1. Expenditure of households.
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Consistency of the least weighted squares under heteroscedasticity

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Consistency of the least weighted squares under heteroscedasticity

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Consistency of the least weighted squares under heteroscedasticity

Can we meet heteroscedasticity frequently ?(*continued*)

1. Expenditure of households.
2. Demands for electricity.
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4. Technical analysis of capital markets.
5. Models of export, import and FDI.

Heteroscedasticity was not assumed but “empirically found” for given dat.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Conditions for consistency

Conditions $\mathcal{C}1$: (conditions on r.v.'s)

1. $\{(X_i', \varepsilon_i)'\}_{i=1}^{\infty}$ i.d., $F_{X,e_i}(x,v) = F_X(x) \cdot F_{e_i}(v)$, $\quad F_{e_i}(v) = F(v \cdot \sigma_i^{-1})$.

Conditions $\mathcal{C}2$: (conditions on weight function)

1. $w(u): \ [0,1] \leftarrow [0,1]$, $w(0) = 1$ continuous, nonincreasing.
2. Lipschitz, i. e. $|w(u_1) - w(u_2)| \leq L \cdot |u_1 - u_2|$.

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2. $F(v)$, $\sup_{-\infty<v<\infty} f(v) < U$, $E_F V = 0$, $E_F V^2 = 1$.

Conditions $\mathcal{C}2$: (conditions on weight function)

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2. $F(v)$, $\sup_{-\infty < v < \infty} f(v) < U$, $E_F V = 0$, $E_F V^2 = 1$.
3. $\lim\limits_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sigma_i = 1$.
4. $1 : E_{F_X} \|X\|^{2q} < \infty$.
5. There is only one solution of $E\,[w(F_\beta($ … $)) = 0$.

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2. $F(v)$, $\sup_{-\infty<v<\infty} f(v) < U$, $E_F V = 0$, $E_F V^2 = 1$.
3. $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sigma_i = 1$.
4. $\exists q > 1 \ : \ E_{F_X} \|X\|^{2q} < \infty$.
5. There is only one solution of $E[w(F_\beta(r(\beta)))$ [illegible] $= 0$.

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4. $\exists q > 1 \; : \; E_{F_X} \|X\|^{2q} < \infty$.
5. There is only one solution of $E\left[w\left(F_\beta(|r(\beta)|)\right).X_1\left(e - X_1'\beta\right)\right] = 0$.

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Consistency of the least weighted squares

Assertion: Consistency

Under Conditions $\mathcal{C}1$ and $\mathcal{C}2$ $\hat{\beta}^{(LWS,n,w)}$ is consistent.

Motto: $Y_i = X_i' \beta^0 + \varepsilon_i$

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$\sqrt{n}$-consistency of the least weighted squares

Conditions $\mathcal{NC}1$

1. $\exists \; f'(v), \; \sup_{-\infty < v < \infty} |f'(v)| < \infty.$
2. $\exists \; w'(u)$ and is Lipschitz of the first order.

Assertion: $\sqrt{n}$-consistency

Under Conditions $\mathcal{C}1$, $\mathcal{C}2$ and $\mathcal{NC}1$ $\hat{\beta}^{(LWS,n,w)}$ is $\sqrt{n}$-consistent.

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

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Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Basic result for proving consistency

Put

$$F_\beta^{(n)}(v) = \frac{1}{n}\sum_{i=1}^{n} I\{|r_i(\beta)| < v\} = \frac{1}{n}\sum_{i=1}^{n} I\{|Y_i - X_i'\beta| < v\}$$

and

$$\overline{F}_{n,\beta}(v) = \frac{1}{n}\sum_{i=1}^{n} F_{i,\beta}(v) \text{ with } F_{i,\beta}(v) = P\left(\left|Y_i - X_i'\beta\right| < v\right).$$

Motto: $Y_i = X_i'\beta^0 + \varepsilon_i$

$i = 1, 2, \ldots, n$

Basic result for proving consistency

Put

$$F_\beta^{(n)}(v) = \frac{1}{n}\sum_{i=1}^{n} I\{|r_i(\beta)| < v\} = \frac{1}{n}\sum_{i=1}^{n} I\{|Y_i - X_i'\beta| < v\}$$

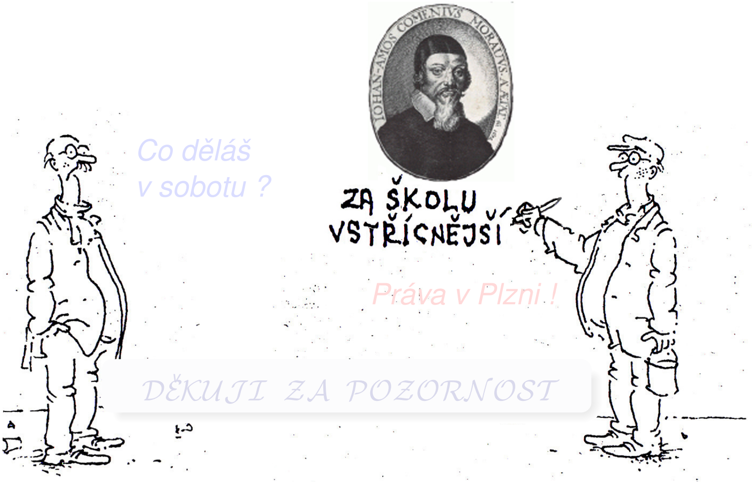
and

$$\overline{F}_{n,\beta}(v) = \frac{1}{n}\sum_{i=1}^{n} F_{i,\beta}(v) \ \text{ with } \ F_{i,\beta}(v) = P\left(\left|Y_i - X_i'\beta\right| < v\right).$$

Assertion: $\sqrt{n}$-consistency of d. f. under heteroscedasticity

Let Conditions $\mathcal{C}1$ hold. For any $\varepsilon > 0$ there is a constant K_ε and $n_\varepsilon \in \mathcal{N}$ so that for all $n > n_\varepsilon$

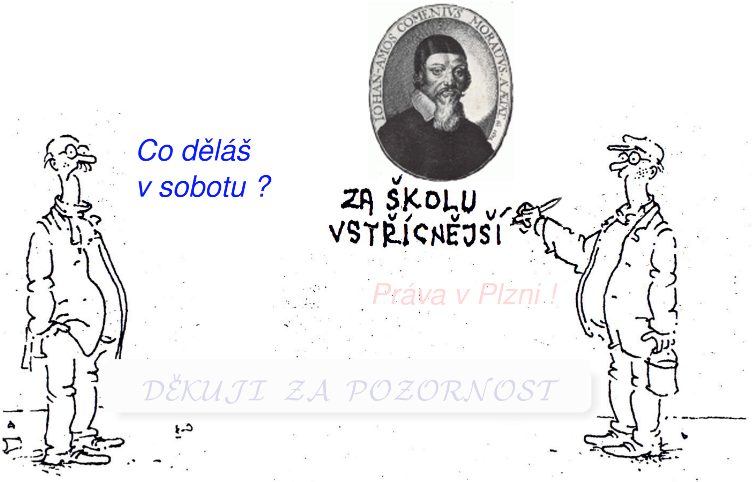
$$P\left(\left\{\omega \in \Omega : \sup_{v \in R^+} \sup_{\beta \in R^p} \sqrt{n}\left|F_\beta^{(n)}(v) - \overline{F}_{n,\beta}(v)\right| < K_\varepsilon\right\}\right) > 1 - \varepsilon.$$



Co děláš v sobotu ?

Práva v Plzni !

DĚKUJI ZA POZORNOST



Co děláš v sobotu ?

Práva v Plzni !

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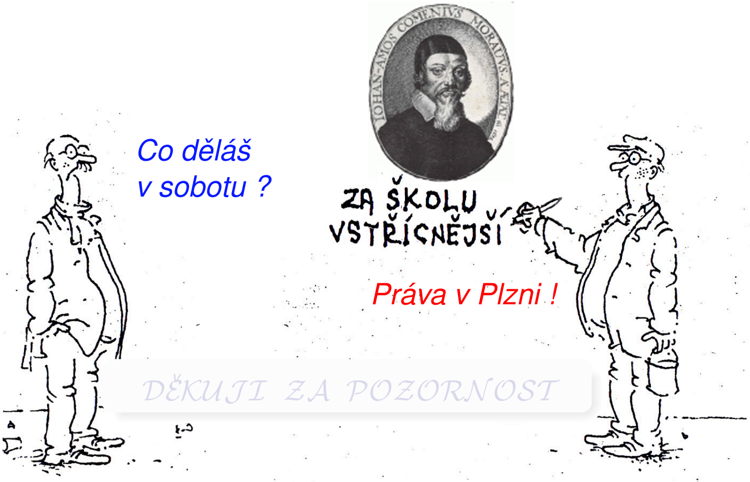
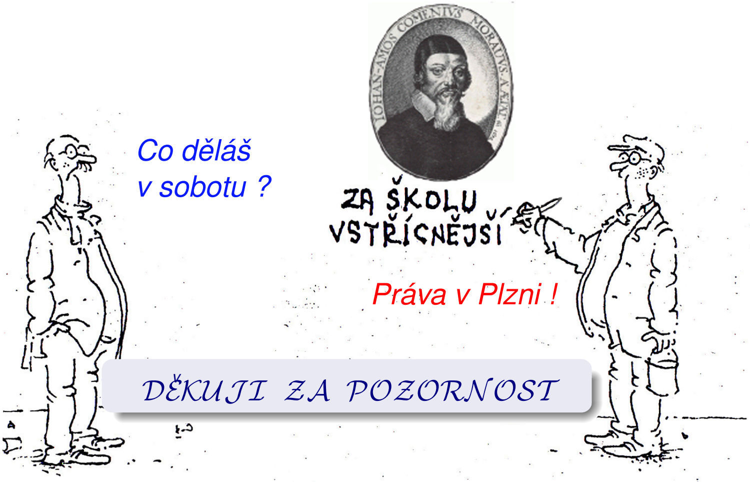

Co děláš
v sobotu ?
IOHAN-AMOS COMENIVS MORAVVS
ZA ŠKOLU
VSTŘÍCNĚJŠÍ
Práva v Plzni !
DĚKUJI ZA POZORNOST



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