TIME SERIES WITH NON-POSITIVE AUTOCORRELATIONS

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Abstract: We deal with time series models with non-positive autocorrelations. For a strictly stationary process with an autocorrelation function \( r_k \), such that \( \sum_{k=1}^{\infty} |r_k| < \infty \), we investigate the lower bound for the sum of autocorrelations \( \sum_{k=1}^{\infty} r_k \). The bound \(-1/2\) is reached if and only if \( f(0) = 0 \), where \( f \) is the continuous spectral density of the process \( X_t \). This gives a guideline how the "optimal" value \(-1/2\) can be reached by an ARMA process.

**Bernoulli variables**

The following problem was proposed in Bondesson (2003). It stays unsolved even in the simplest case of 1-dependent Bernoulli variables.

**Problem:** Let \( \{r_k\} \) be an autocorrelation sequence of a strictly stationary Bernoulli process and \( r_k \leq 0 \) for all \( k \geq 1 \). Which value of \( \alpha \) is the smallest possible such that \( \sum_{k=1}^{\infty} r_k \geq \alpha \)?

1-dependent variables

Consider two Bernoulli variables \( Y_1 \) and \( Y_2 \) with \( p = P(Y_1 = 1) \) and set \( q = 1 - p \). Then \( \text{corr}(Y_1, Y_2) = \frac{Pr(Y_1 = 1, Y_2 = 1) - pq}{\sqrt{P(Y_1 = 1)p} \sqrt{P(Y_2 = 1)q}} \).

Joe (1997) shows how the bound (2) can be attained for \( p = 0 \) and \( p = 1 \).

**Conclusion**

The lower bound for the sum of autocorrelations \( \sum_{k=1}^{\infty} r_k \) is investigated for stationary time series.

**Remarks**

- The inequality (4) holds for \( \mathbb{E}Z_1 \neq 0 \) as well. The proof of Theorem 2 can be found in Došlák (2008).
- The bound \(-1/3\) is reached for m-dependent variables if and only if \( \rho_m = -1/2 \) for some \( k \in \{1, \ldots, m\} \) and \( r_1 = 0 \) otherwise. For any other setting, the sum of correlations lies above \(-1/3\).

**Question:** Is \( \sum_{k=1}^{\infty} r_k \) always above \(-1/3\) for general dependent Bernoulli variables?

**Open problem:** Is \( \alpha = 1/e \) the lowest possible value for a general process with Bernoulli variables?

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**References**


