ZONE CONTROL CHART

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SUMMARY

A special type of control chart, called a zone control chart, has been proposed recently as a simple alternative to the $\bar{X}$ and R chart which respects supplementary run rules. The average run length (ARL) for this new type of zone control chart can be calculated using phase-type distribution. The chart limits are computed to optimize statistical properties of the chart. As optimization algorithm was used Nelder-Mead simplex method.

INTRODUCTION

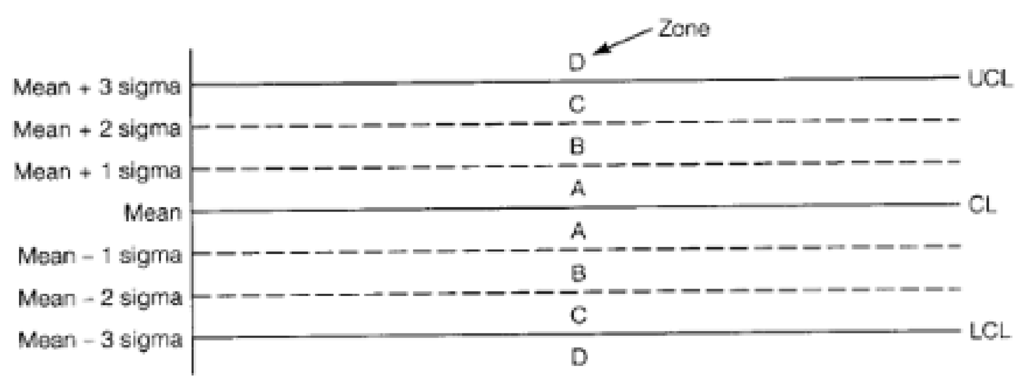
The basic Shewhart control chart is a common tool used in monitoring the mean of a process to ensure that it remains in control. This chart has a center line at the in-control mean value and three-sigma limits on either side of the center line. The chart signals an out-of-control condition if any observed sample mean falls beyond the three-sigma limits.

Jaehn (1987) has developed a chart which he implies will signal at roughly the same time as a Shewhart chart with the common runs rules. Like the similar chart proposed by Reynolds (1971), this chart is meant to be simplex for personnel to apply.

ZONE CONTROL CHARTS

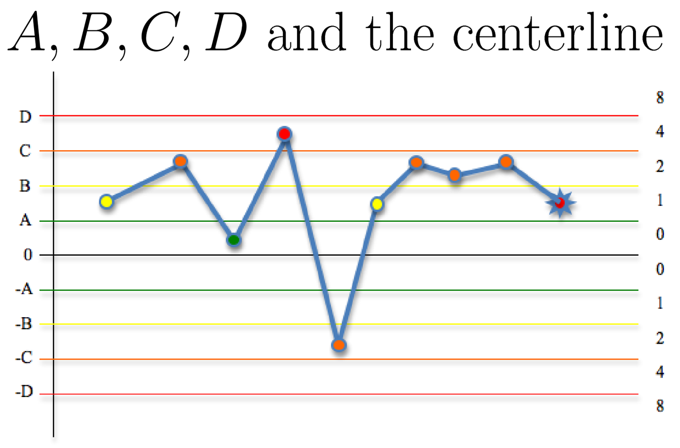
The concept behind the zone control chart is to allow for automatic signaling of the following out-of-control indicators in the Shewart chart:

– a point falling outside the 3σ limits,
– two of three successive points falling outside the 2σ limits on the same side of the center line,
– four of five successive points falling outside the 1σ limits on one side of the center line, and
– eight consecutive points falling on the same side of the center line.



THE PROPOSAL OF NEW CONTROL CHARTS

Consider improved zone chart with eigth limits $-D, -C, -B, -A, A, B, C, D$ and the centerline dividing chart into 10 zones:



Scores are assigned according to the right column in the picture. The significant change consists in enlargement of "zero-zone", i.e. the zone between $-A$ and A. Moreover, the limits are given as a result of an optimization algorithm which minimize the probability of Error type II for some shift value δ_u (undistinguish the shift) preserving the given probability of Error type I (false alarm).The MATLAB program for application of the Nelder - Mead simplex algorithm was used to evaluate statistically optimal limits.

AVERAGE RUN LENGTH

Average run length (ARL) – average number of samples until an alert signal occurs. This is a good measure of chart suitability.
$ARL(0)$ = average run length when process is under control
$ARL(\delta)$ = average run length when the process mean shift is equal to δ

The objective is the following: maximal $ARL(0)$ and minimal $ARL(\delta_u)$ for some given δ_u.

MARKOV MODEL

Consider a Markov chain with the states $\{0, 1, 2, 3, 4, 5, 6, 7, 8+\}$. All the states are transient, but the $\{8+\}$ which is absorbing.

The number of transition until absorption X can be described using discrete PH-distribution with representation $(\vec{\pi}, \mathbf{P})$, where $\vec{\pi}$ is an initial probability distribution and $\mathbf{P}$ is the transient probability matrix of transient states (8×8).

The mean value of X is given by the formula $E(X) = \vec{\pi}(\mathbf{I} - \mathbf{P})^{-1}\vec{1}$, where $\mathbf{I}$ is unity matrix, $\vec{1}$ is a vector of ones.

Let us denote Z the observed characteristic. For arbitrarily shift s we are able to evaluate the following probabilities (using a cumulative distribution function $F(z)$ of Z):

$$\begin{aligned}
p_{-0} &= \mathrm{P}(-A - s < Z < -s), & p_{+0} &= \mathrm{P}(-s < Z < A - s),\\
p_{-A} &= \mathrm{P}(-B - s < Z < -A - s), & p_{+A} &= \mathrm{P}(A - s < Z < B - s),\\
p_{-B} &= \mathrm{P}(-C - s < Z < -B - s), & p_{+B} &= \mathrm{P}(B - s < Z < C - s),\\
p_{-C} &= \mathrm{P}(-D - s < Z < -C - s), & p_{+C} &= \mathrm{P}(C - s < Z < D - s),\\
p_{-D} &= \mathrm{P}(Z < -D - s), & p_{+D} &= \mathrm{P}(D - s < Z),
\end{aligned}$$

The matrix $\mathbf{P}$ can be evaluated by means of these probabilities.

EXAMPLE

We compare ARL for the following control charts:

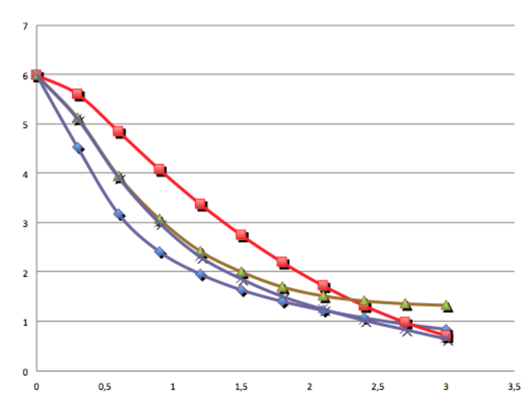
CUSUM – CUSUM control chart

Z_1 – "Shewhart" type control chart with one UCL and LCL (not necessary in $\pm 3\sigma$)

Z_3 – the control chart which signals assignable cause after "four succeeding observations of Z fall above the limit B"

Z_{1234}= proposed zone control chart

shift	CUSUM	Z_1	Z_3	Z_{1234}
0	400,96	400	400,75	400,44
0,3	92,87	272,04	168,4	164,36
0,6	24,09	127,64	51,77	50,4
0,9	11,19	59,15	21,67	20,02
1,2	7,1	29,29	11,3	10,08
1,5	5,16	15,66	7,46	6,45
1,8	4,09	9,04	5,5	4,53
2,1	3,41	5,62	4,58	3,49
2,4	2,93	3,75	4,14	2,79
2,7	2,58	2,68	3,91	2,31
3	2,32	2,04	3,79	1,92



$ARL(shift)$ were evaluated under conditions:
$ARL(0) = 400$, sample size $n = 1$.
Results of optimization:
CUSUM $(k = 0.5, h = 5)$, $Z_1(C = 3.03)$, $Z_3(B = 1.06)$,
$Z_{1234}(A = 0.34, B = 1.24, C = 2.24, D = 3.37)$

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References.

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