



Universitas Carolina  
**Charles University in Prague**



# **Matrix Visualization:** *Approaching Statistics and Statistical Approach*

## **Lecture 1: General Introduction**

**Chun-houh Chen**

**Institute of Statistical Science Academia Sinica**



**April 10, 2013**





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**Matematicko-fyzikální fakulta**

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# Matrix Visualization:

## *Approaching Statistics and Statistical Approach*

### Lecture 1: General Introduction

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**April 10, 2013**







Jaromír ANTOCH



Jaromír ANTOCH

# COMPSTAT 2004, 16th Symposium of IASC PRAGUE, August 23-27, 2004

## Matrix Visualization and Information Mining

Chun-houh Chen



Institute of Statistical Science  
Academia Sinica  
Taipei, Taiwan







# 1

## Goals of ESDA

Exploratory spatial data analysis (ESDA) aims to discover the following information from data

- Variable Structure:** such as variable-grouping
- Subject Structure:** such as subject-clustering
- Spatial Locations:** such as cartography
- Variable-Subject Interactions**
- Variable-Location Relations**
- Subject-Location Relations**

Variables	Subjects	Locations
Variable-Subject Interaction	Subject Structure	
Variable-Location Relations	Subject-Location Relations	Spatial Locations

All the goals above can be compactly represented in the six cells of the left table.

To explore the spatial locations of subjects, a cartograph is essential. The following map contains the information about the spatial locations of 43 European countries. We can add some attributes in data onto the map, e.g., maps in two right cells, to explore the relations between data structure and locations.

Using the order of a hierarchical tree (see the following cell) to construct a color spectrum, closeness of subjects can be seen from color-similarity. This method is discussed by White and Sufness (2002) based on a regression tree.

However, branches at each node of the tree can be flipped, and to color-similarity may not really coincide with closeness of subjects.

To understand spatial distribution of a variable, every variable needs a map. There will be 32 different maps, and six of them are shown below, where grey regions indicate that resource occurs.

It may be difficult to take an overview when variables become more and more. Besides, human eyes tend to compare white regions and grey regions separately, and are likely to ignore that natural gas and hydropower have a almost reverse relationship.

# 2

## Conventional Visualization

Based on some proximity measure and clustering method, subjects are reorder to reveal the relativity and grouping structure. San Marino, Vatican city, and Monaco are closest (having no resource).

The dataset is two-way sorted so that similar subjects and similar variables are put nearby. It is easier to explore the variable-subject interaction. Why variables / subjects are put together can be investigated in detail.

Based on a certain proximity measure and a clustering method, the relativity and grouping of variables can be revealed. For example, peat and dolomite are the closest pair, but fish and lignite have few subjects in common.



Mr. Kao, Chiun-How  
(Poster, Mon. Tue.)

# 3

## Proposed Integrated Visualization

The proximity used for variables coincides with color-similarity. Similar variables will have alike distributions of colors for categories.

Instead of white-black coding, categories with similar colors are almost occurring on the same subjects. Some "reverse" relations, e.g., hydropower and natural gas, are more easily to identify.

The proximity used for subjects take into account that "distances" among categories are not uniform. If two subjects have the same same categories, they will be regarded as similar.

When representing data for the whole world, a 3D spinning globe is more faithful to the relative positions than a 2D map projection. But the latter provides a convenient overview. Each has its merits.

Six orders of the (R,G,B) triplet.

◆ The proposed representation for variable-location relations is equal to figures in the most-right cell.

◆ Variable colors indicate their spatial distribution. For example, green variables are highly likely to appear in green regions.

◆ Of course, one map per variable can still be made. With these colors, more information is presented. As the following two maps show, we can quickly see that clay has very different information from the other two.

◆ Natural gas and hydropower almost do not occur at the same place, but they provide almost the same information about subjects. Therefore, their colors for categories are reverse and the two map look alike.

◆ The proposed representation for subject-location relations is equal to figures in the right cell.

◆ Unlike color spectrum for subjects based on a tree, this approach assign colors consistent with their 3D proximity, which will not suffer a flipping problem.

◆ Subject colors can tell more than the subjects presenting. For example, green subjects are highly likely to have green categories. It is more easier to interpret why those subjects are similar.

◆ Subject colors on the map also show some principal features of how variables changes across spatial locations.

◆ Actually, these figures summarize three-way relations, i.e., locations, subjects, and variables, on a single map.

◆ Colors in all the figures are compatible, so colors play a key role in this approach as guidance to link related things together.

# 4

## MCA Coordinates

Chen (2002) develop a method, generalized association plots, to visualize quantitative data through coloring and reordering the data. Color-coding for general qualitative data is more difficult. Chang et al.(2002) suggest a solution for coloring and a categorical version of generalized association plots, which is briefly described here.

Using multiple correspondence analysis(MCA), the table of data can be described as coordinates for variable categories and subjects in a three-dimensional space. Each coordinate is then transformed into red, green, and blue color spectrums (R,G,B), respectively.

The 3D coordinates are used for calculating proximity among subjects and among variables. Moreover, syntheses of (R,G,B) give suitable colors to categories and subjects.

The match of (R,G,B) triplet and three MCA coordinates is not unique. There are six different matches, as shown in the left.

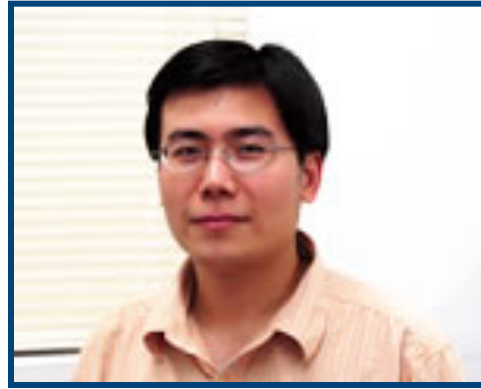
# 5

## Reference

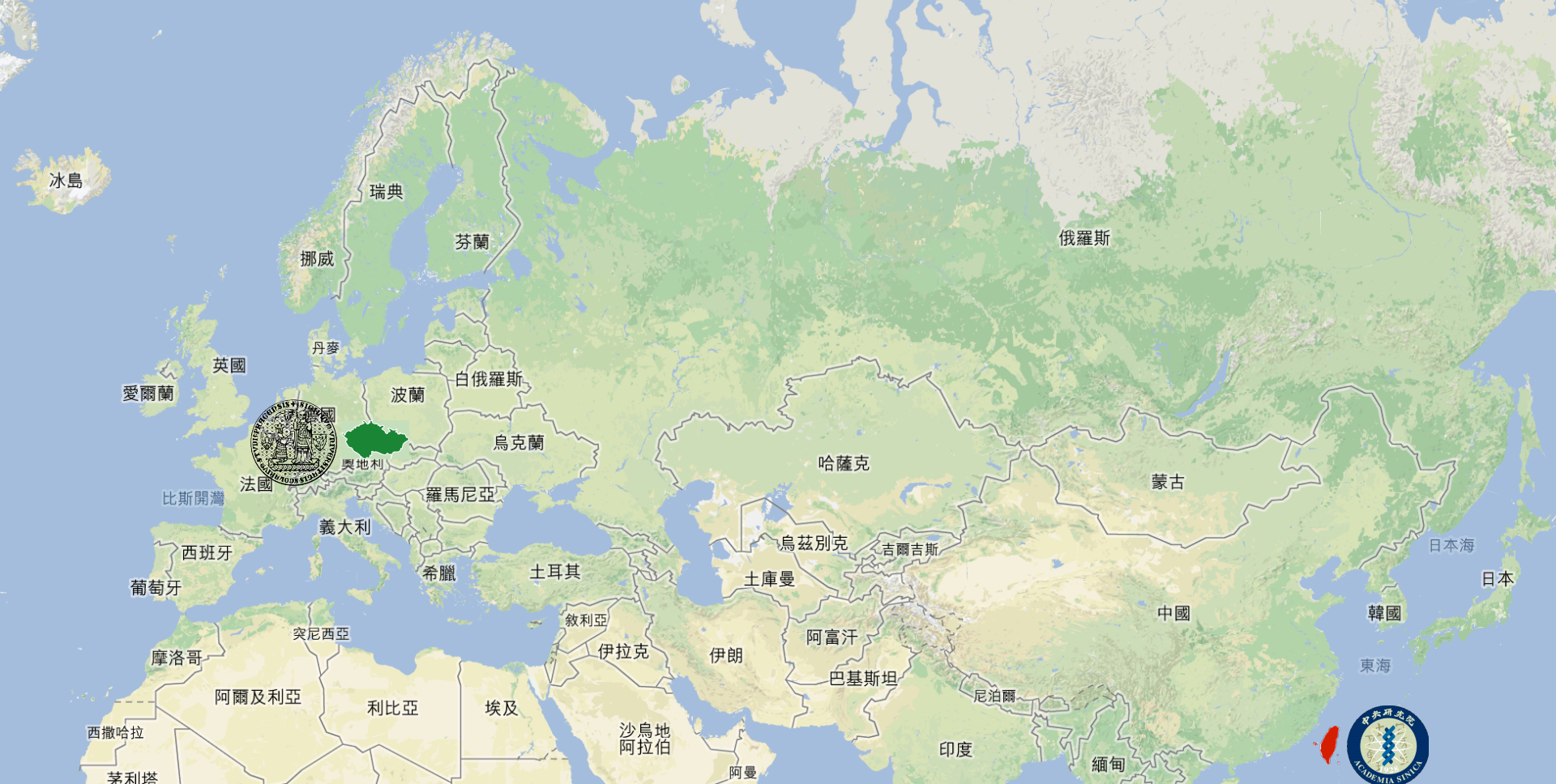
[1] Chen, C. B. (2002). Generalized association plots: information visualization on three-wayly generated correlation matrices.

[2] Chang, Y. C., Chen, C. H., Chen, Y. Y., and Chen, C. W. (2002). Relativity and reordering for high dimensional data with generalized association plots (GAP). Proceedings of International Database Workshop (IDW'02), Berlin, Germany, 19-21, 1997.

[3] White, D., and J. Sufness. (2002). Algorithms for cartography. Journal of Computational and Graphical Statistics, 11.



Mr. Tzeng, ShengLi



## Czech Republic

Area: 78,866 km<sup>2</sup>

Pop: 10,513,209

Den: 134/km<sup>2</sup>

## Taiwan (ROC)

Area: 36,193 km<sup>2</sup>

Pop: 23,315,822

Den: 643/km<sup>2</sup>



# Campus of Academia Sinica



- 1 Main Entrance
- 2 Institute of Biomedical Sciences
- 3 Environment, Health, and Safety Management Division
- 4 Institute of Cellular and Organismic Biology
- 4 Biodiversity Research Center
- 5 Institute of Molecular Biology
- 6 Institute of Biological Chemistry
- 6 Life Science Library
- 7 National Laboratory Animal Center, NLAC
- 8 Interdisciplinary Research Building for Science and Technology (under construction)
- 9 Greenhouse
- 10 Central Office of Administration
- 11 Biodiversity Research Center
- 11 Biodiversity Research Museum
- 12 Institute of Plant and Microbial Biology
- 13 Research Center for Information Technology Innovation
- 14 Tsai Yuan-Pei Memorial Hall

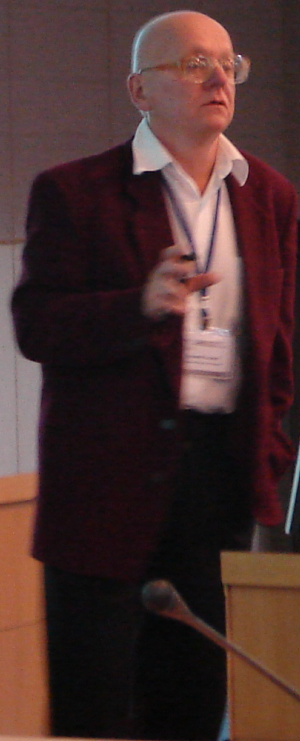


\* The Institute of Mathematics, Institute of Atomic and Molecular Sciences, Institute of Astronomy and Astrophysics and some bio-science institutes are located outside the main campus.

## 15 Institute of Statistical Science



LINGUISTIC DATA







**Institute of**

**Statistical Science**





Strengthened by your nurturing and participation since 1982, the Institute of Statistical Science, Academia Sinica now enters its **age of standing firm**.

子曰：“吾十有五而志於學，三十而立，四十而不惑，五十而知天命，六十而耳順，七十而從心所欲，不踰矩。”

Confucius's own account of his gradual progress and attainments. The Master said, "At 15, I had my mind bent on learning. At **30**, I stood firm. At 40, I had no doubts. At 50, I knew the decrees of Heaven. At 60, my ear was an obedient organ for the reception of truth. At 70, I could follow what my heart desired, without transgressing what was right."



Chi-Huey Wong,  
President of Academia Sinica  
2006 ~ Present

Ph.D. in Chemistry,  
Massachusetts Institute of Technology in 1982.  
Postdoctoral fellow: Harvard University  
Assistant Professor: Texas A&M University in  
1983, Professor and Ernest W. Hahn Chair:  
Scripps Research Institute (1989-2006)  
Director of the Genomics Research Center at  
Academia Sinica, Taipei (2003-2006).



Yuan T. Lee,  
President of Academia Sinica  
1994 ~ 2006

Ph.D. in Chemistry,  
University of California, Berkeley in 1965  
Postdoctoral Fellow: Berkeley (1965~1967)  
Assistant Professor: University of Chicago in 1968  
Professor: University of California, Berkeley ('74~'94)

**Nobel Prize laureate**, Chemistry in 1986  
(with John C. Polanyi and Dudley R. Herschbach)  
**President, International Council for Science Units  
(ICSU)** (2011 ~ 2014)





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We currently offer nine interdisciplinary Ph.D. programs:

1. Chemical Biology and Molecular Biophysics
2. Molecular Science and Technology
3. Molecular and Biological Agricultural Sciences
4. Bioinformatics
5. Molecular and Cell Biology
6. Nano Science and Technology
7. Molecular Medicine
8. Computational Linguistics and Chinese Language Processing
9. Earth System Science
10. Biodiversity



# Matrix Visualization: *Approaching Statistics* and *Statistical Approach*

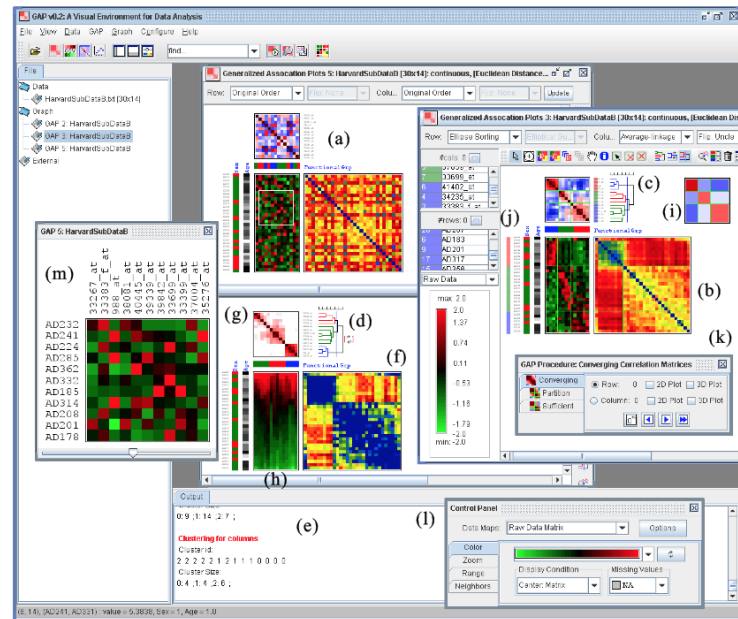
**Chun-houh Chen**

**Institute of Statistical Science, Academia Sinica, Taiwan**

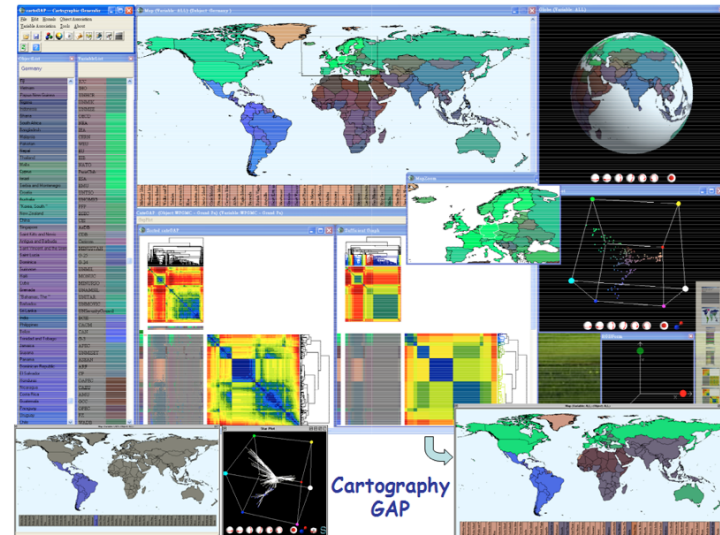
*“It is important to understand what you CAN DO before you learn to measure how WELL you seem to have DONE it”* (John Tukey, 1977). Data analysts and statistics practitioners nowadays are facing difficulties in understanding higher and higher dimensional data with more and more complex nature while conventional graphics/visualization tools do not answer the needs. It is statisticians' responsibility for coming up with graphics/visualization environment that can help users really understand what one CAN DO for complex data generated from modern techniques and sophisticated experiments.

In this lecture I'll summarize our works on matrix visualization for interpreting statistics and statistical approach for implementing matrix visualization. We create matrix visualization environment (GAP: Generalized Association Plots) for conducting statistical analyses, from descriptive statistics, model fitting, inference, to diagnosing. On the other end, we also introduce statistical concepts into matrix visualization environment for visualizing more versatile and complex data structure. With these two matrix-visualization procedures interact with each other we hope a good statistics solution can be achieved.

<http://gap.stat.sinica.edu.tw/Software/>



## 2. GAPsoftware



#### 4. GAP for cartography data

# Matrix Visualization by Chun-houh Chen at Charles University, Spring 2013

Week 1 (April 10, 2013)

Introduction and Overview of MV  
GAP software

Week 2 (April 17, 2013)

MV for continuous data  
MV for binary data

Week 3 (April 24, 2013)

MV for nominal data  
MV for cartography data

Week 4 (May 1, 2013)

Interactive Diagnostic Plots for Multidimensional Scaling  
MV for proximity matrix modeling

Week 5 (May 8, 2013)

MV with covariate-adjustment  
MV for ANOVA data

Week 6 (May 15, 2013)

MV for SDA data  
Miscellaneous issues: MV with missing values,

Week 7 (May 15, 2013)

Student presentations

# Lab 309 (???) for Information Visualization



**Mr. C.H. Kao**  
Ph.D. student



**Dr. Gary Tien**  
Postdoc. Fellow

張勝傑  
張文宗  
陳柏旭  
鐘雅齡  
黃建勳  
林香誼  
劉勝宗  
曾聖澧  
葉紫君  
吳怡真  
林倩如  
歐陽智聞

...



**Prof. H.M. Wu**  
Dept. Math.  
Tamkang U.



**Prof. S.Y. Shiu**  
Dept. Stat.  
Nat'l Taipei U.

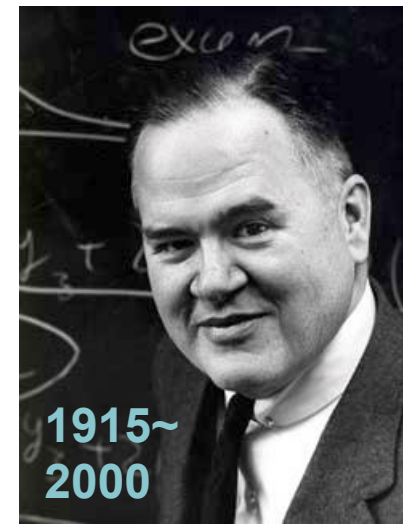


**Dr. Mirrian Ho**  
Postdoc. Fellow



# Exploratory Data Analysis

## EDA, John Tukey (1977)



It is important to understand what you **CAN DO** before you learn to measure how **WELL** you seem to have **DONE** it.

allow the **data to speak** for themselves  
before standard assumptions or formal modeling

**The greatest value of a picture** is when it **forces** us to notice what we **never expected to see**.

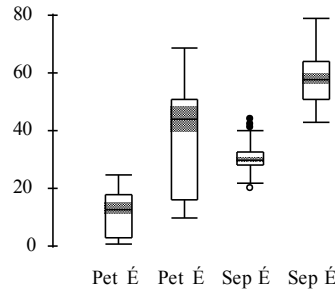
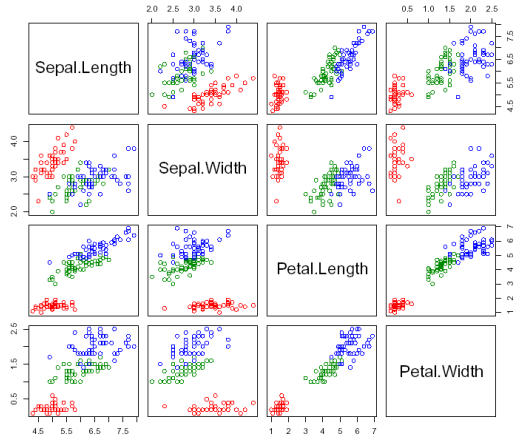
**Matrix Visualization as an EDA tool for  
assisting formal mathematical modeling**



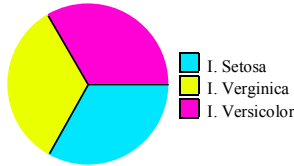
**The Iris Data**

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
5.1	3.5	1.4	0.2
4.9	3.0	1.4	0.2
4.7	3.2	1.3	0.2
4.6	3.1	1.5	0.2
5.0	3.6	1.4	0.2
...	...	...	...

Scatterplot Matrices

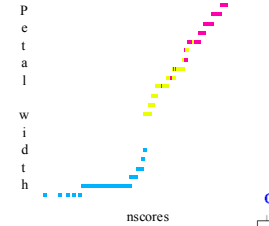
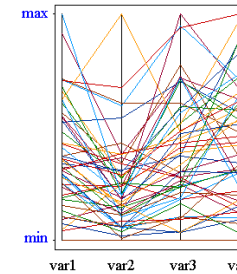
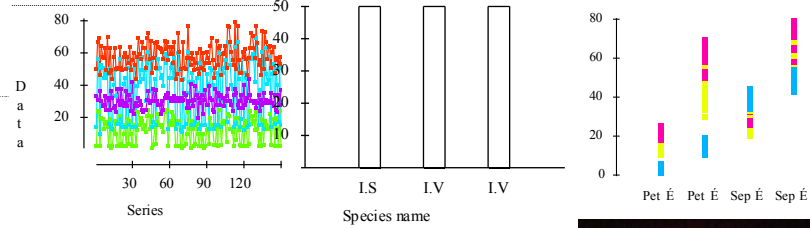


Species name

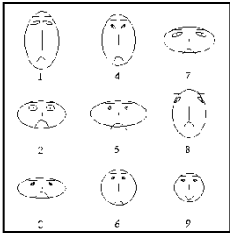
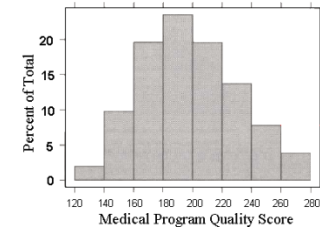


# Graphics/Visualization for high dimensional data?

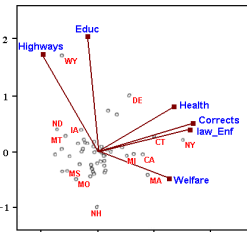
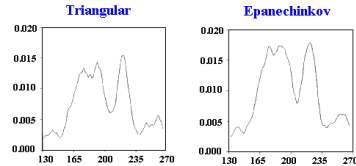
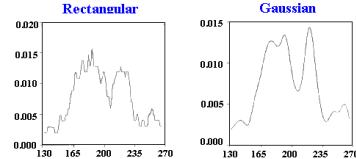
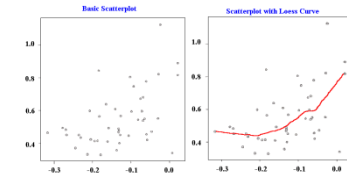
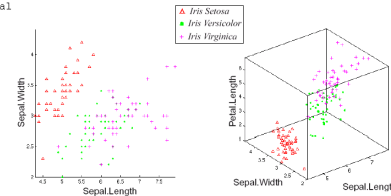
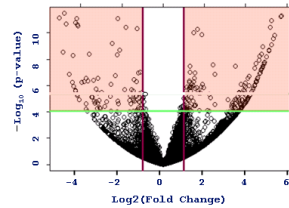
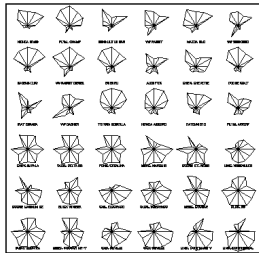
$P > 5$     $p > 10$     $p > 100$     $p > 10000$



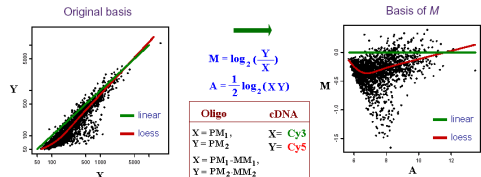
O. Bin origin at 120, bin widths of 20.



Star plot of Automobile Data

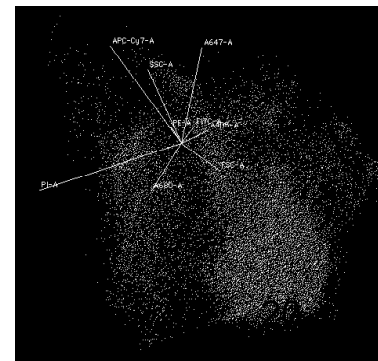
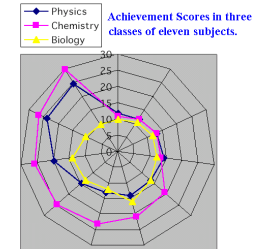


Biplot of 1992 State Policy Spending



$$M = \log_2 \left( \frac{Y}{X} \right)$$

$$A = \frac{1}{2} \log_2 (XY)$$



# What can we (**statisticians**) do for data/information visualization

1. Same as information scientist:  
to create **effective** graphical/visualization tools/  
environments
2. To bring in more **statistical sense/concept** into  
graphical/visualization tools/environments

Our **approach**:

**MV** (Matrix Visualization)

Our **tools/environment**:

**GAP** (Generalized Association Plots)

# *Approaching Statistics*

We create matrix visualization environment for conducting statistical analyses:

✓ **descriptive statistics:**

continuous data: proximity measure, color coding

binary data: proximity measure, black-white (for monary data)

nominal data: Homogeneity Analysis (Dual Scaling, Multiple Corresp. Analysis)

clustering:

nonlinear data structure

symbolic data analysis

Huge Data Sets

✓ **model fitting:**

data with cartography link (here or descriptive)

statistical genetics

✓ **inference:**

missing value

missing mechanism identification, estimation

EDA for identifying/formulating better hypotheses

MANCOVA

Covariate-Adjusted MV

✓ **diagnosing:**

Interactive Diagnosing System for Statistical methods modeling proximity matrices:

HCT (Hierarchical Clustering Tree), MDS (Multidimensional Scaling)

FA (Factor Analysis)



# *Statistical Approach*

We also introduce statistical concepts into matrix visualization environment for visualizing more versatile and complex data structure.

Homogeneity Analysis (Dual Scaling, Multiple Correspondence Analysis)

Nonlinear data structure: use isomap proximity measure

MANCOVA

Covariate-Adjusted MV

Symbolic Data Analysis for handling data with dependent structure:

- Clustered (non-independent) Data

- Repeated Measures Analysis –

- Genetic Familial Data

- Huge Data Sets

  - Large  $n$

  - Large  $p$

  - Large  $n$  &  $p$

Other Types of Symbolic Data

# Recent Review Articles for MV

## The History of the Cluster Heat Map Leland WILKINSON and Michael FRIENDLY

The American Statistician,  
May 2009, Vol. 63, No. 2 179

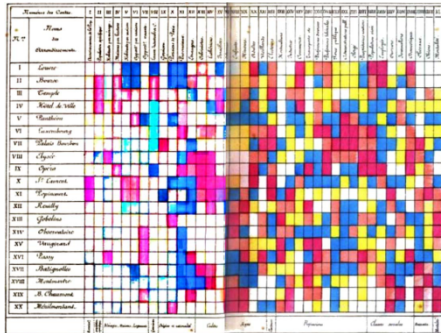


Figure 2. Shaded matrix display from Loua (1873), available online at <http://books.google.com/books/>. This was designed as a summary of 40 separate maps of Paris, showing the characteristics (e.g., national origin, professions, age, social classes) of 20 districts, using a color scale ranging from white (low) through yellow and blue to red (high).

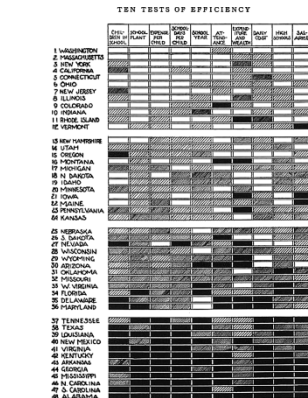


Figure 3. Sorted shaded display from Brinton (1914). The data are ranks of U.S. states on each of 10 educational features assessed in 1910. The matrix has been sorted by the row-marginal ranks.

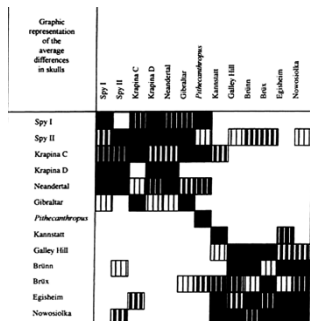


Figure 5. Sorted shaded display from Czekanowski (1909), reproduced in Hage and Harary (1995).

Permuted Data Matrix

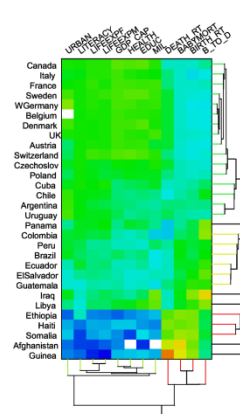
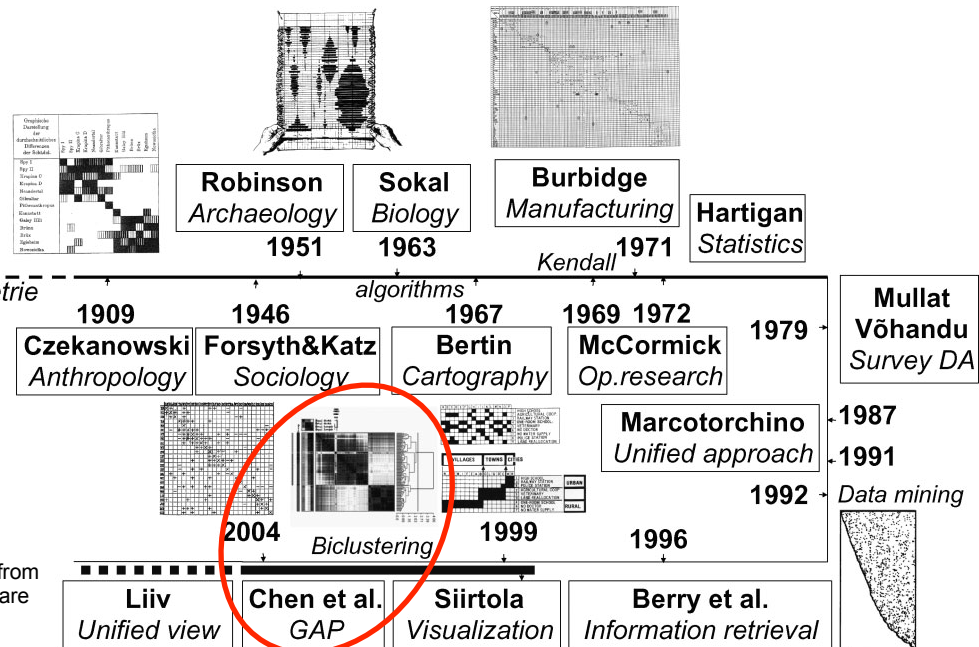


Figure 9. Cluster heat map from Wilkinson (1994). The data are social statistics (i.e., urbanization, literacy, life expectancy for females, GDP, health expenditures, educational expenditures, military expenditures, death rate, infant mortality, birth rate, and ratio of birth to death rate) from a United Nations survey of world countries. The variables were standardized before the hierarchical clustering was performed.

## REVIEW

## Seriation and Matrix Reordering Methods: An Historical Overview by Innar Liiv

Statistical Analysis and Data Mining  
3: 70–91, 2010



**Matrix Visualization (MV):**  
reorderable matrix, heatmap,  
color histogram, data image

# Data

Taiwan **Multidimensional Psychopathological**  
Group Research Project, (**MPGRP**) Part I: Schizophrenia

Project period: July 1, 1993 to June 30, 1998

Patients (**95 subjects**):

**95 First-Admission Psychosis Patients**

Schizophrenia (69) + Bipolar Disorder(26)

Rating scales (**50 variables**):

**SAPS**: Scale for Assessment of **Positive Symptom (30)**

**SANS**: Scale for Assessment of **Negative Symptom (20)**

### SAPS (Scale for Assessment of Positive Symptom)

AH1	Auditory Hallucinations
AH2	Voices Commenting
AH3	Voices Conversing
AH4	Somatic or Tactile Hallucinations
AH5	Olfactory Hallucinations
AH6	Visual Hallucinations
DL1	Persecutory Delusions
DL2	Delusion of Jealousy
DL3	Delusion of Sin or Guilt
DL4	Grandiose Delusions
DL5	Religious Delusions
DL6	Somatic Delusions
DL7	Ideas and Delusions of Reference
DL8	Delusions of Being Controlled
DL9	Delusions of Mind Reading
DL10	Thought Broadcasting
DL11	Thought Insertion
DL12	Thought Withdrawal
BEH1	Clothing and Appearance
BEH2	Social and Sexual Behavior
BEH3	Aggressive and Agitated Behavior
BEH4	Repetitive or Stereotyped Behavior
TH1	Derailment
TH2	Tangentiality
TH3	Incoherence
TH4	Illogicality
TH5	Circumstantiality
TH6	Pressure of Speech
TH7	Distractible Speech
TH8	Clanging

### SANS (Scale for Assessment of Negative Symptom)

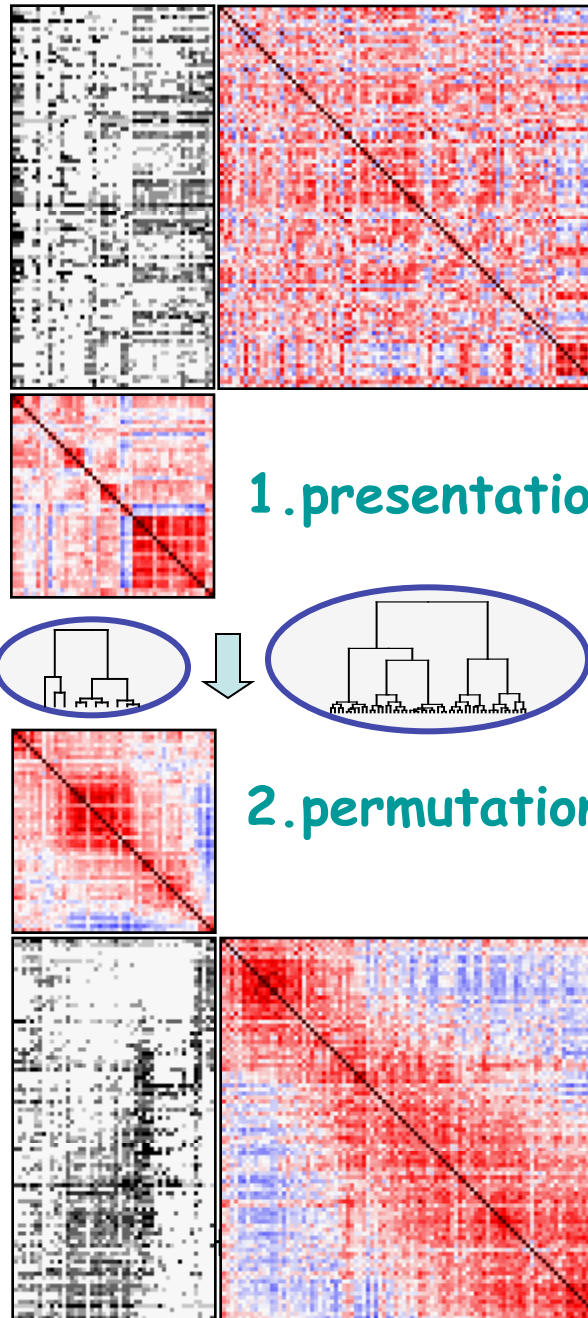
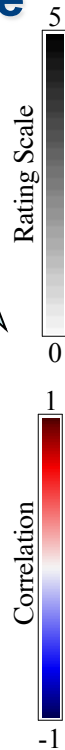
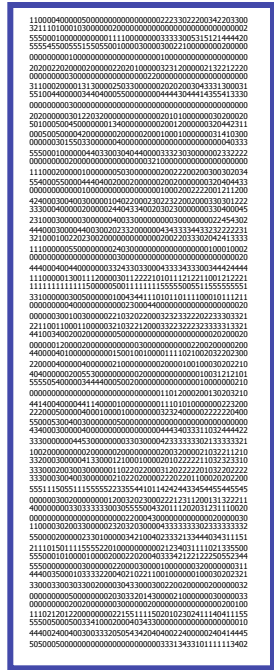
NA1	Unchanging Facial Expression
NA2	Decreased Spontaneous Movements
NA3	Paucity of Expressive Gestures
NA4	Poor Eye Contact
NA5	Affective Nonresponsivity
NA6	Inappropriate Affect
NA7	Lack of Vocal Inflections
NB1	Poverty of Speech
NB2	Poverty of Content of Speech
NB3	Blocking
NB4	Increased Latency of Response
NC1	Grooming and Hygiene
NC2	Impersistence at Work or School
NC3	Physical Anergia
ND1	Recreational Interest and Activities
ND2	Sexual Interest and Activity
ND3	Ability to Feel Intimacy and Closeness
ND4	Relation With Friends and Peers
NE1	Social Inattentiveness
NE2	Inattentiveness During MSE



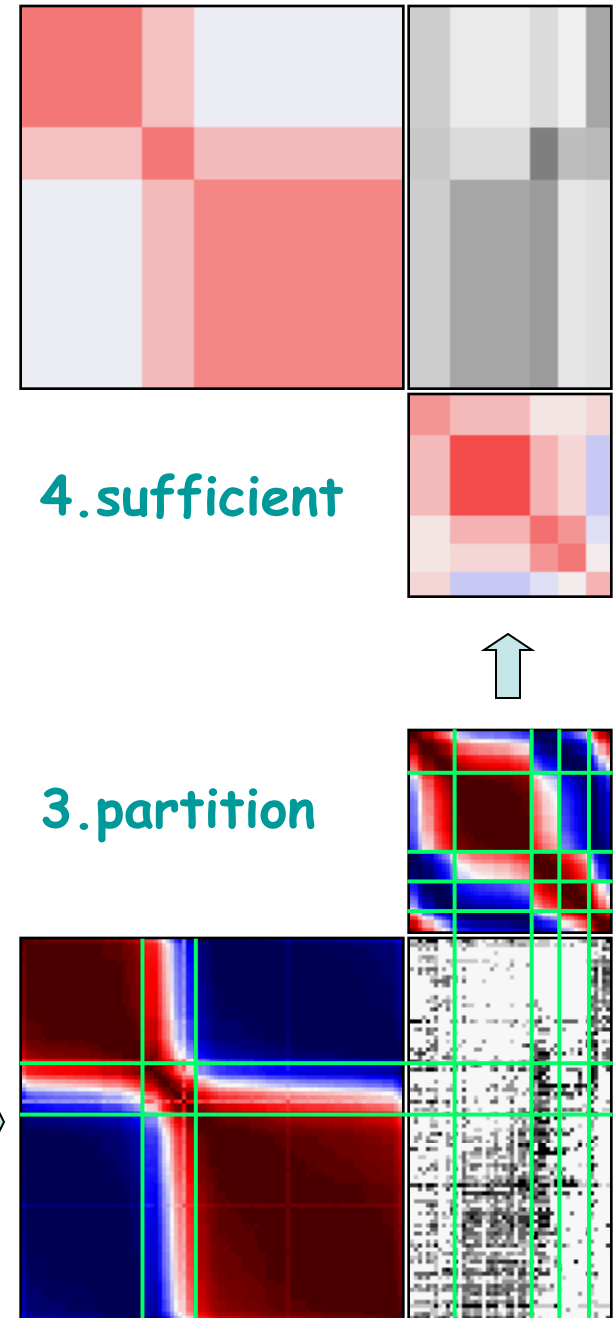
# A Standard GAP Procedure

## 0. Data Matrix

4/15/13



## 2.permutation



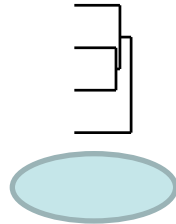
### 3.partition

# Some essential elements in a GAP MV procedure

## 3. Proximity (Variable $p * p$ )

Continuous  
Ordinal  
Binary  
Nominal

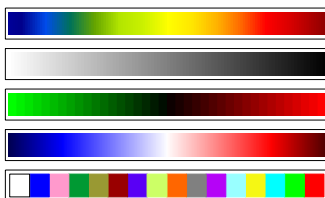
## 4. Permutation (variable)



## 1. Data Matrix ( $n * p$ )

(w/ Color coding)

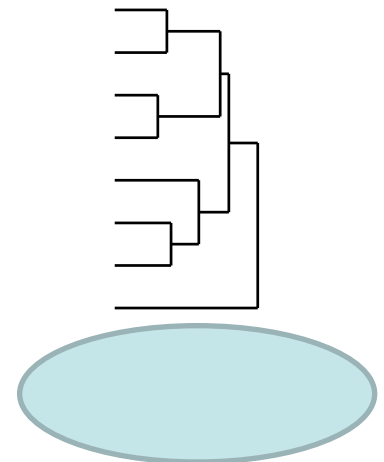
Continuous  
Ordinal  
Binary  
Nominal



## 2. Proximity Matrix for Subject ( $n * n$ )

Continuous  
Ordinal  
Binary  
Nominal

## 4. Permutation (subject)



## Continuous GAP Example



Are there **major** and **minor** institutes  
in Academia Sinica?

an example of **matrix visualization**

**Knowledge World**

陳君厚副研究員(統計科學研究所)

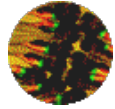
(大或小、人員多或少；如果你蒐集

了許多變數(variable)，則需要多變量統計方法去分析資料。筆者在此介紹一套"看"高維度資料的方法: 矩陣視覺化(matrix visualization: MV)。為了介紹MV，我們以本院31個所(處)中心為樣本蒐集20個變數(表一：17數值變數、3共變數(covariate))；資料之蒐集以公開及方便性為主。讀者對這20個變數的選擇當然有所疑慮--約聘僱人員與院外計畫等變數未納入、某些變數可能資料時間太短(如前瞻計畫)、某些變數可能應使用相對數值(如年輕著作獎)、人事變數比例是否過高等。筆者強調此資料之蒐集以方法介紹為主，非以資料分析為目的。我們將原始資料(人數、件

**Div. of Life Sci. (5+3)**



**Plant-Microb Bio**



**Cellul-Organ Bio**



**Bio Chemistry**



**Molecular Bio**



**Biomedical Sci**



**Agric Biot (Ctr)**



**Genomics (Ctr)**



**Biodiversity (Ctr)**

**Div. of Math-Physi Sci (8+3)**



**Mathematics**



**Physics**



**Chemistry**



**Earth**



**Information**



**Statistics**



**Atomic-Molecular**



**Astron-Astrophy**



**Applied Sci (Ctr)**



**Envir Change (Ctr)**



**Inf Tech Innov (Ctr)**

**Div. of Hum-Social Sci (11+1)**



**Hist-Philol**



**Ethnology**



**Mod Hist**



**Economic**



**Europ-Ame**



**Chi Liter-Phil**



**Taiwan Hist**



**Sociology**



**Linguistics**



**Political Sci**



**Iurisprudentiae**



**Hum-Soc Sci (Ctr)**



# Are there **major** and **minor** institutes in Academia Sinica?

## **A. Personnel Variables**

1. Research Fellow
2. Senior RF (%)
3. Female RF (%)
4. Research Scientist
5. Other Research
6. Administrative Staff
7. Total Personnel

## **B. Project Variables**

8. Thematic Project
9. Investigator Project
10. Career Project
11. Postdoc (AS)
12. Postdoc (Regular)

## **C. Other Variables**

13. Junior Award
14. Patent

## **D. Budget Variables**

15. Operating Expense
16. Equipment Expense

## **E. Institute Variables**

17. Year of Establish

## **Covariate**

18. Division
19. Research Center
20. Preparatory Office

Are there **major** and **minor** institutes in Academia Sinica? (original data)

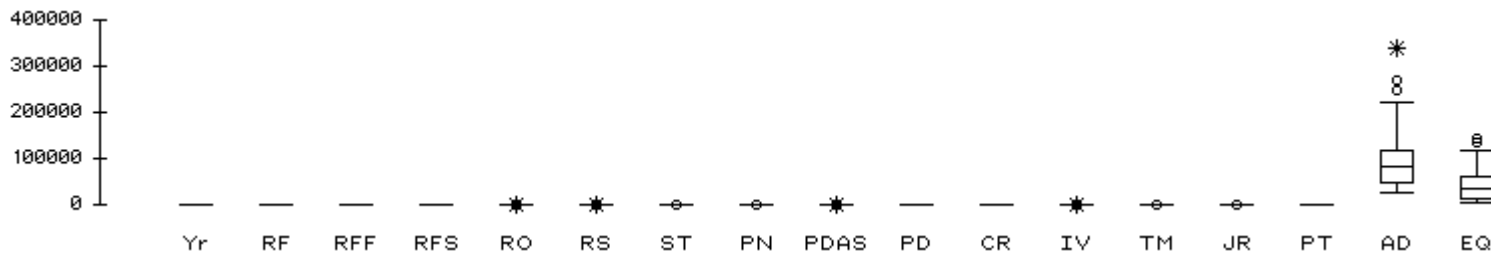
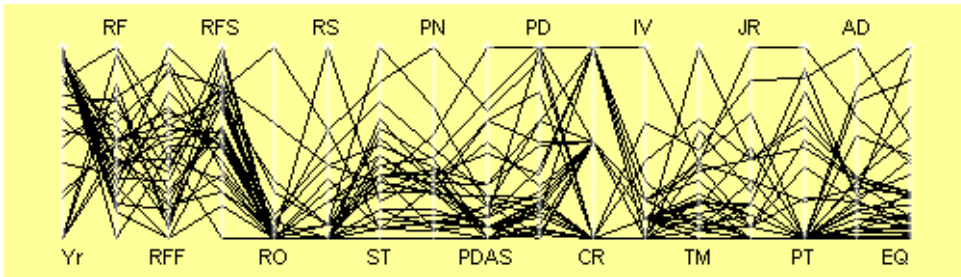
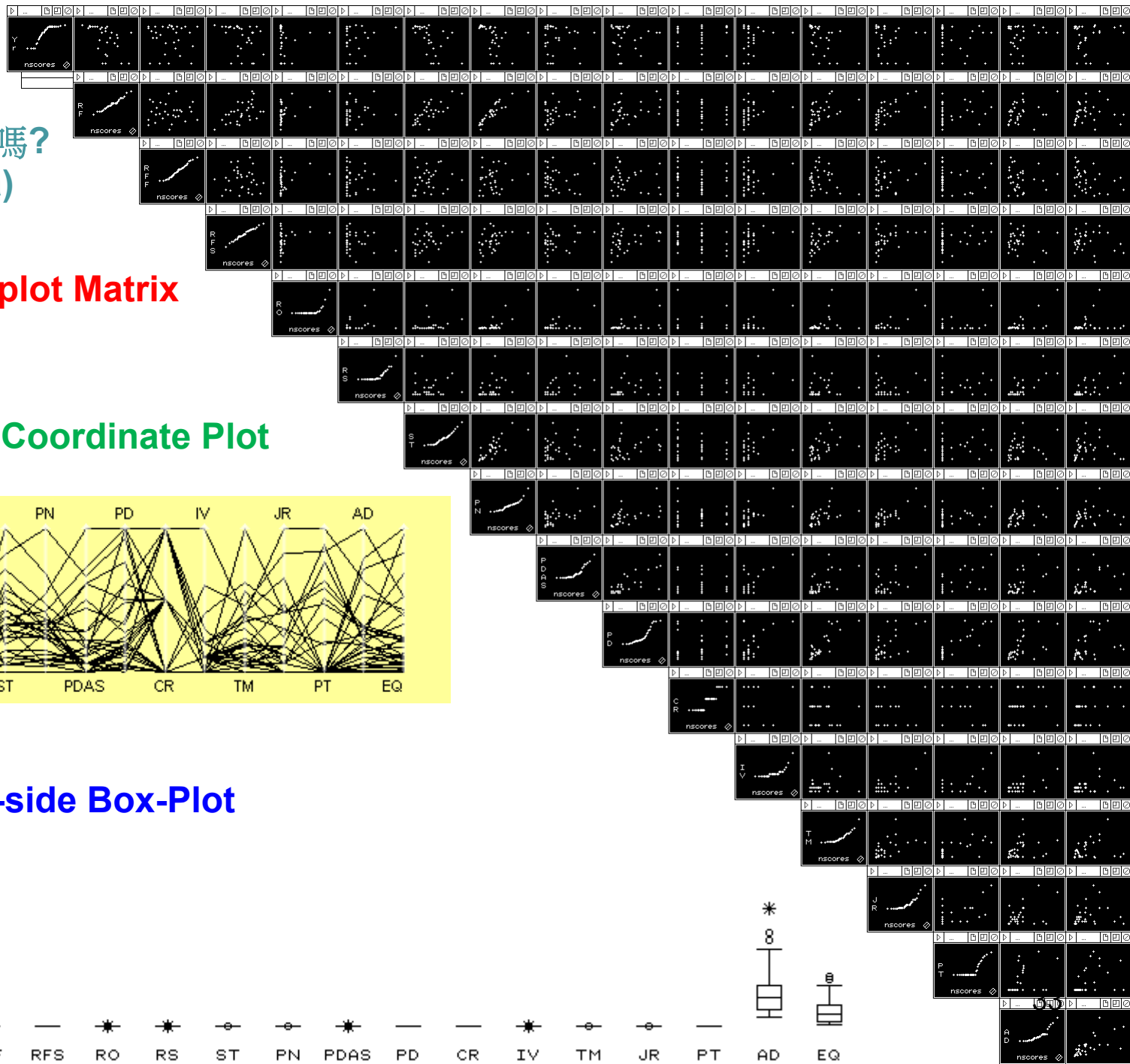
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Math	0	0	0	1947	26	0.769231	0.192308	1	0	7	34	2	1	1	1	22	1	0	45539	38375
Phys	0	0	0	1928	40	0.6	0.025	3	0	9	52	30.5	0	1	13	153	7	20	120657	99572
Chem	0	0	0	1928	23	0.434783	0.130435	0	3	10	36	17	0	0	2	150	4	38	95299	51506
Eart	0	0	0	1982	30	0.533333	0.1	7	4	16	57	7	1	1	4	31	2	0	69640	36377
Info	0	0	0	1982	37	0.540541	0.054054	3	0	6	46	10.7	2	1	5	94	7	25	127149	38979
Stat	0	0	0	1987	36	0.444444	0.138889	0	1	8	45	2	2	1	0	22	2	0	83410	19495
Atom	0	0	0	1995	26	0.730769	0.038462	0	0	14	40	19	4	0	11	117	7	27	124889	77647
Astr	0	0	1	1993	22	0.181818	0.090909	9	2	2	37	3	1	1	9	41	0	0	270658	144166
Appl	0	1	0	2004	16	0.125	0	1	0	2	19	1	0	2	1	19	1	9	85299	62790
Envi	0	1	0	2004	12	0.25	0.083333	0	0	2	14	1	0	1	3	2	1	0	73687	28500
Inno	0	1	0	2007	4	0	0.25	3	0	8	15	0	0	1	1	0	0	0	192877	64885
Plan	1	0	0	1944	27	0.37037	0.37037	6	3	11	47	9	2	2	3	92	1	22	121440	35152
Cell	1	0	0	1959	19	0.421053	0.315789	4	0	10	33	11.5	0	1	11	76	1	16	86000	17405
BiCm	1	0	0	1977	20	0.55	0.35	1	10	8	39	12	2	1	5	74	5	8	109008	37542
MoBi	1	0	0	1993	31	0.612903	0.483871	6	20	7	64	12	10	2	18	152	17	43	222961	60006
Biom	1	0	0	1993	49	0.591837	0.326531	9	33	18	109	14	6	0	12	142	14	36	339270	77403
Agri	1	1	0	2006	11	0.363636	0.272727	5	0	2	18	1	1	2	2	55	3	33	94698	137182
Geno	1	1	0	2003	22	0.227273	0.227273	16	0	5	43	1.5	3	2	8	50	1	0	250841	119556
Biod	1	1	0	2004	18	0.666667	0.166667	0	1	3	22	6	1	1	2	37	0	0	87864	11882
Hist	2	0	0	1928	47	0.617021	0.234043	1	8	22	78	12.5	1	0	3	31	11	0	121339	39163
Ethn	2	0	0	1928	26	0.269231	0.5	1	1	12	40	6	0	1	1	27	2	0	56133	13394
Mode	2	0	0	1965	37	0.378378	0.513514	0	1	13	51	4	1	0	0	28	3	0	70360	17630
Econ	2	0	0	1970	34	0.588235	0.205882	0	1	8	43	1.5	2	1	0	10	5	0	57589	20839
Euro	2	0	0	1972	27	0.518519	0.296296	0	1	9	37	1	1	0	1	7	3	0	42942	23016
Chin	2	0	0	2002	26	0.307692	0.384615	0	0	7	33	5	1	1	4	22	8	0	36622	15271
Taiw	2	0	0	2004	14	0.357143	0.428571	0	1	3	18	3.5	0	0	1	16	0	0	35569	9258
Soci	2	0	0	2000	21	0.428571	0.285714	0	0	4	25	5.5	1	0	1	28	3	0	42830	12468
Ling	2	0	0	2004	16	0.5625	0.5625	0	0	2	18	5.83	1	0	3	21	1	0	42733	8557
Poli	2	0	1	2002	10	0.4	0	0	0	1	11	1	0	0	0	6	0	0	27766	5823
Iuri	2	0	1	2004	11	0.181818	0.272727	0	0	4	15	0	1	0	0	2	0	0	30024	61782
Huma	2	1	0	2004	27	0.37037	0.333333	1	0	6	34	1	0	0	0	26	3	0	118930	20249

中研院有大小所嗎?  
(original data)

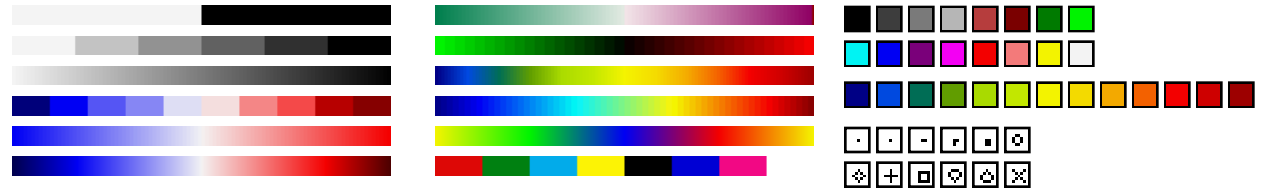
Scatter-plot Matrix

Parallel Coordinate Plot

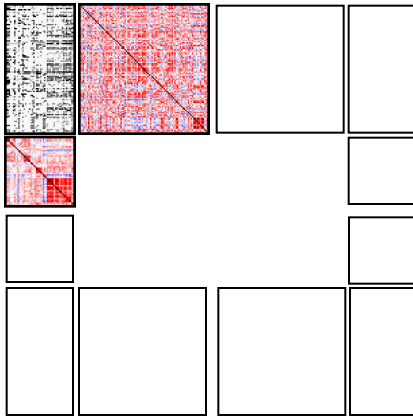
Side-by-side Box-Plot



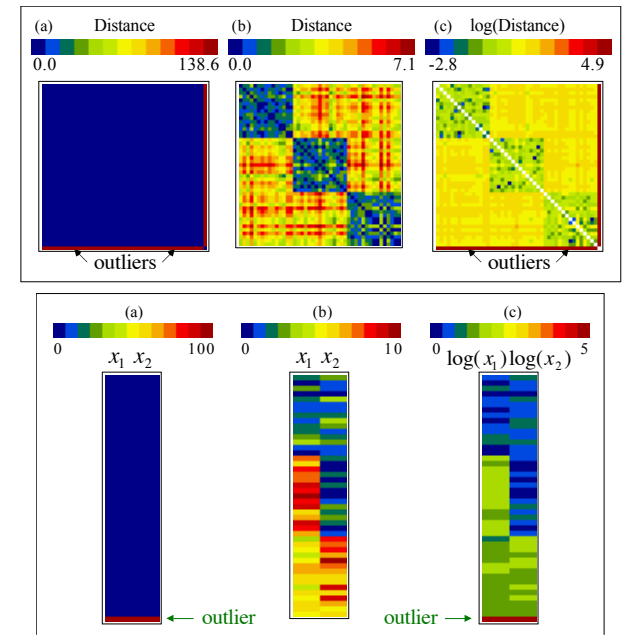
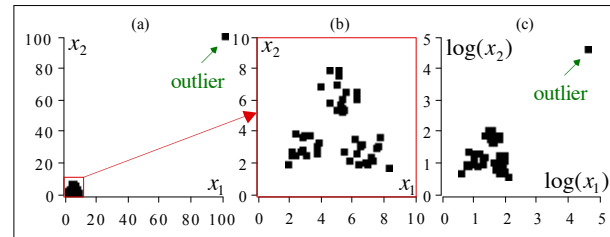
# 1. Selection of suitable color spectrum



## 1.presentation



## 2. Transformation/Standardization of data? ("Resolution" of a Statistical Graph)



## 3. Selection of proximities for variable/sample correlation/covariance/distance/...



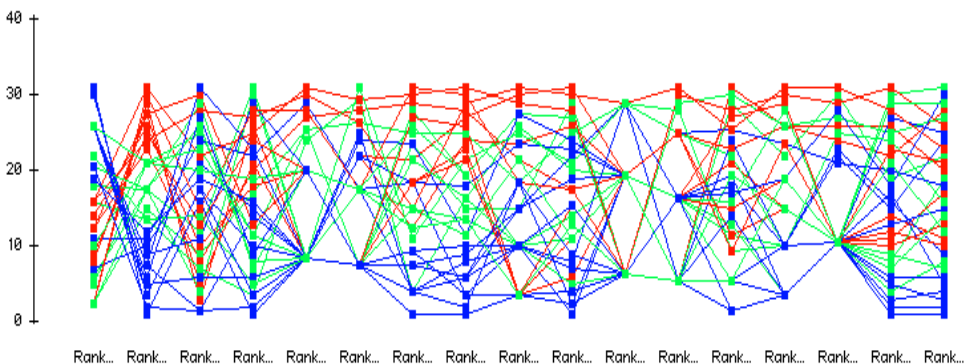
# Are there **major** and **minor** institutes in Academia Sinica? (**rank** data)

Name	Div	Cen	Pre	Yr	RF	RFS	RFF	RS	OR	AD	PN	TM	IV	CR	PDA	PD	JR	PT	OP	EQ
Math	0	0	0	6	17.5	31	12	17.5	8.5	15	13.5	11.5	16.5	19.5	10	11	10	10.5	8	19
Phys	0	0	0	2.5	29	26	3	22	8.5	21.5	27	31	5.5	19.5	30	31	26	24	22	28
Chem	0	0	0	2.5	15	17	9	7.5	25.5	23.5	15	29	5.5	6.5	15	29	22	30	19	22
Eart	0	0	0	12.5	23	20	8	28	27	29	28	21	16.5	19.5	21.5	17.5	15	10.5	11	17
Info	0	0	0	12.5	27.5	21	5	22	8.5	12.5	24	23	25	19.5	23.5	26	26	26	26	20
Stat	0	0	0	14	26	18	10	7.5	20	18.5	23	11.5	25	19.5	3.5	11	15	10.5	14	11
Atom	0	0	0	18	17.5	30	4	7.5	8.5	28	19.5	30	29	6.5	27.5	27	26	27	25	27
Astr	0	0	1	16	13.5	3.5	7	29.5	24	4	16.5	13	16.5	19.5	26	20	3.5	10.5	30	31
Appl	0	1	0	26	7.5	2	1.5	17.5	8.5	4	8	5.5	5.5	29	10	8	10	22	15	24
Envi	0	1	0	26	5	6	6	7.5	8.5	4	2	5.5	5.5	19.5	18.5	2.5	10	10.5	13	15
Inno	0	1	0	31	1	1	16	22	8.5	18.5	3.5	1.5	5.5	19.5	10	1	3.5	10.5	27	25
Plan	1	0	0	5	21	11.5	25	26.5	25.5	25	25	22	25	29	18.5	25	10	25	24	16
Cell	1	0	0	7	10	15	21	24	8.5	23.5	11.5	24	5.5	19.5	27.5	24	10	23	16	9
BiCm	1	0	0	11	11	22	24	17.5	29	18.5	18	25.5	25	19.5	23.5	23	23.5	21	20	18
MoBi	1	0	0	16	24	27	28	26.5	30	15	29	25.5	31	29	31	30	31	31	28	23
Biom	1	0	0	16	31	25	22	29.5	31	30	31	28	30	6.5	29	28	30	29	31	26
Agri	1	1	0	30	3.5	10	17.5	25	8.5	4	6	5.5	16.5	29	15	22	19	28	18	30
Geno	1	1	0	22	13.5	5	14	31	8.5	11	21.5	9.5	28	29	25	21	10	10.5	29	29
Biod	1	1	0	26	9	29	11	7.5	20	7.5	9	19.5	16.5	19.5	15	19	3.5	10.5	17	5
Hist	2	0	0	2.5	30	28	15	17.5	28	31	30	27	16.5	6.5	18.5	17.5	29	10.5	23	21
Ethn	2	0	0	2.5	17.5	7	29	17.5	20	26	19.5	19.5	5.5	19.5	10	14	15	10.5	9	7
Mode	2	0	0	8	27.5	13	30	7.5	20	27	26	15	16.5	6.5	3.5	15.5	19	10.5	12	10
Econ	2	0	0	9	25	24	13	7.5	20	18.5	21.5	9.5	25	19.5	3.5	6	23.5	10.5	10	13
Euro	2	0	0	10	21	19	20	7.5	20	21.5	16.5	5.5	16.5	6.5	10	5	19	10.5	7	14
Chin	2	0	0	20.5	17.5	8	26	7.5	8.5	15	11.5	16	16.5	19.5	21.5	11	28	10.5	4	8
Taiw	2	0	0	26	6	9	27	7.5	20	7.5	6	14	5.5	6.5	10	7	3.5	10.5	3	4
Soci	2	0	0	19	12	16	19	7.5	8.5	9.5	10	17	16.5	6.5	10	15.5	19	10.5	6	6
Ling	2	0	0	26	7.5	23	31	7.5	8.5	4	6	18	16.5	6.5	18.5	9	10	10.5	5	3
Poli	2	0	1	20.5	2	14	1.5	7.5	8.5	1	1	5.5	5.5	6.5	3.5	4	3.5	10.5	1	1
Iuri	2	0	1	26	3.5	3.5	17.5	7.5	8.5	9.5	3.5	1.5	16.5	6.5	3.5	2.5	3.5	10.5	2	35
Huma	2	1	0	26	21	11.5	23	17.5	8.5	12.5	13.5	5.5	5.5	6.5	3.5	13	19	10.5	21	12

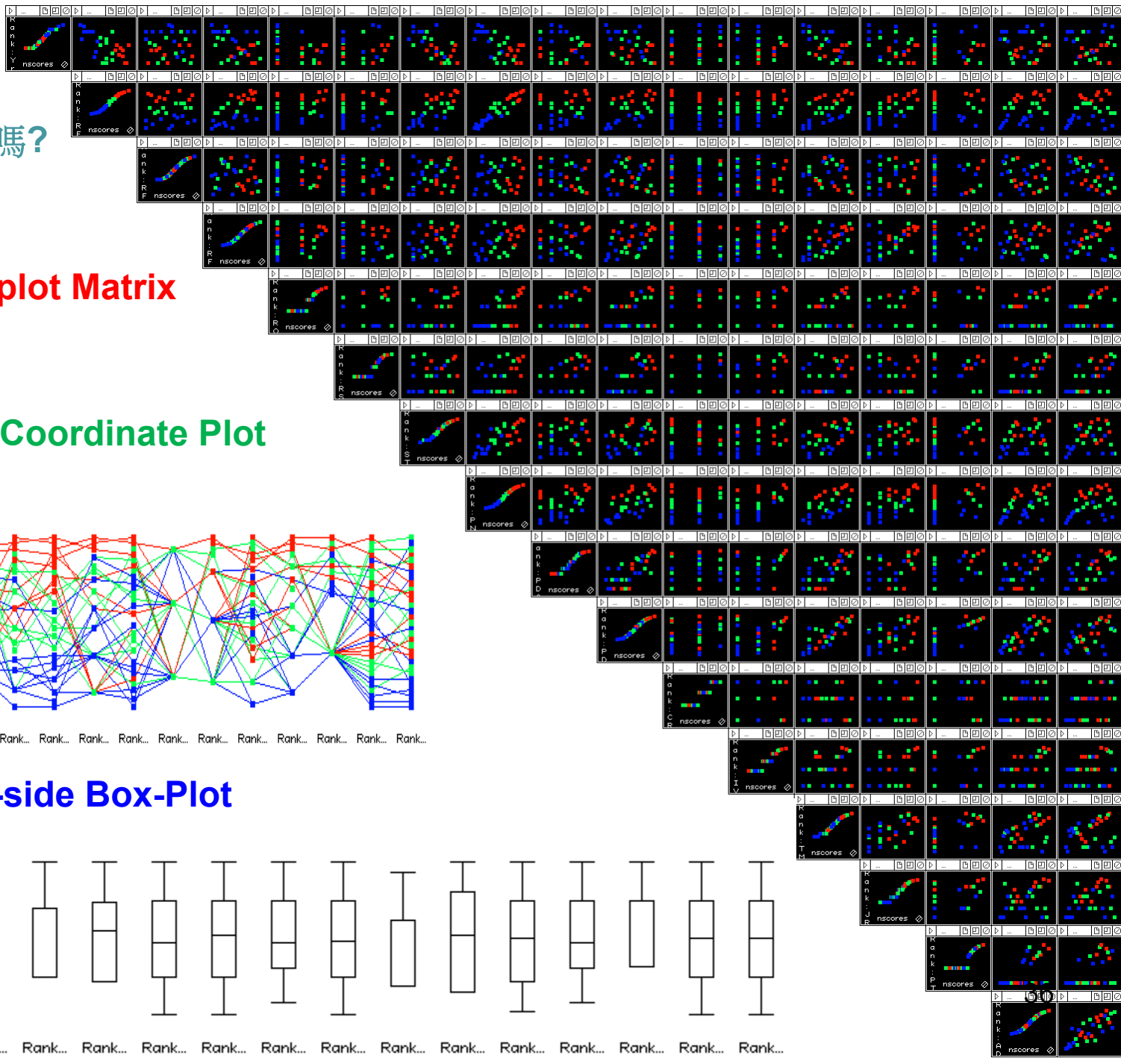
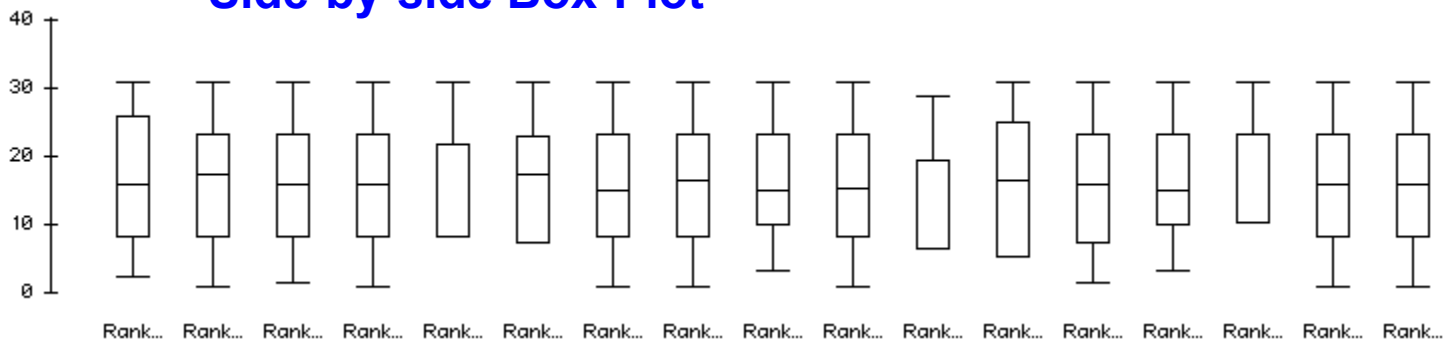
中研院有大小所嗎?  
(rank data)

Scatter-plot Matrix

Parallel Coordinate Plot



Side-by-side Box-Plot

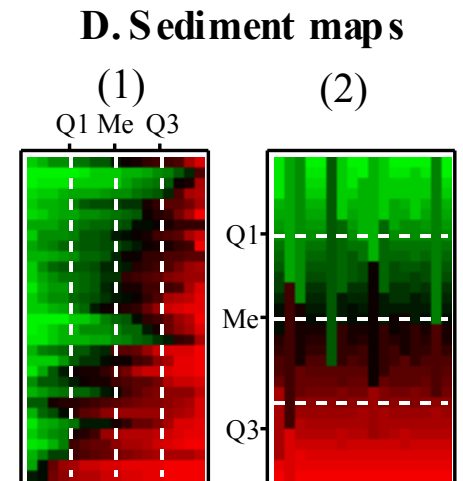
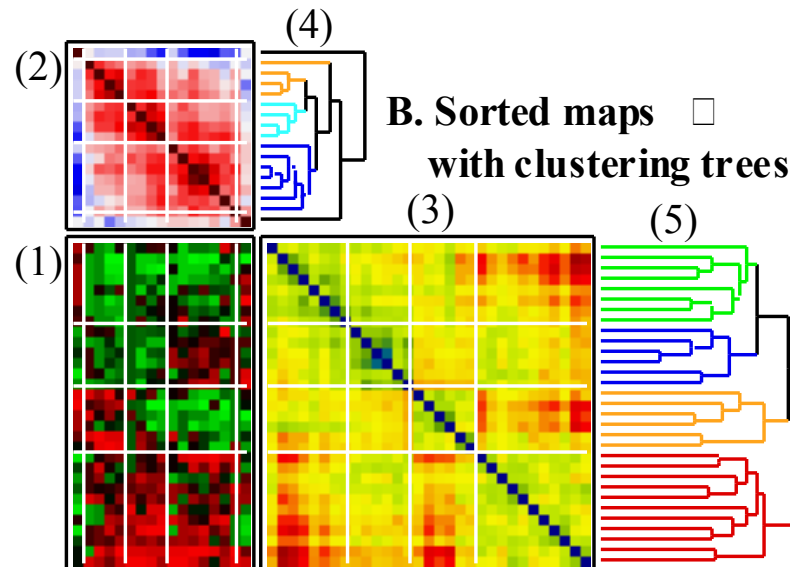
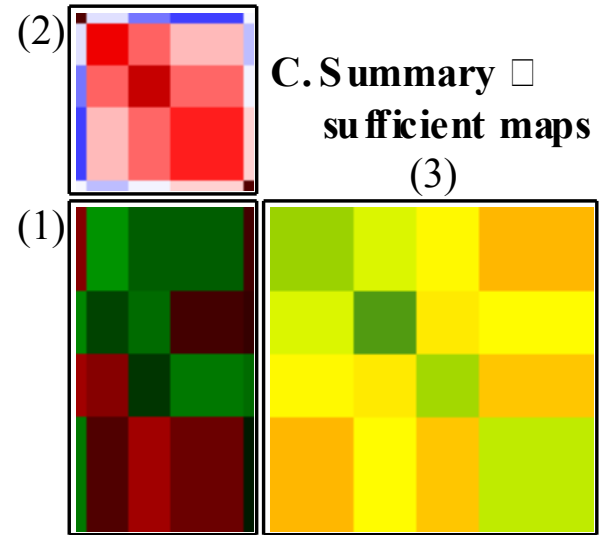
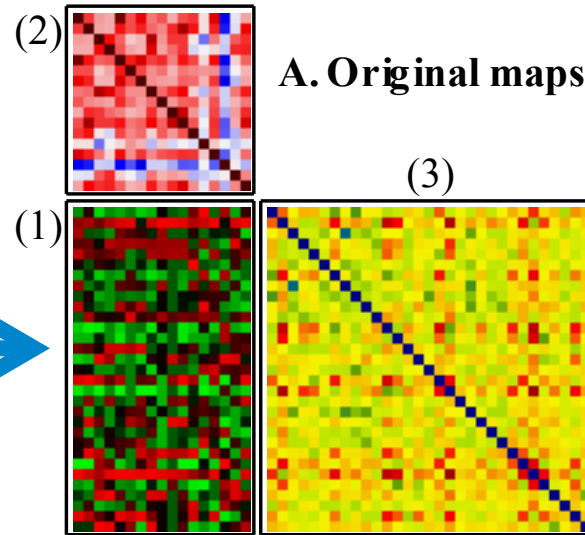
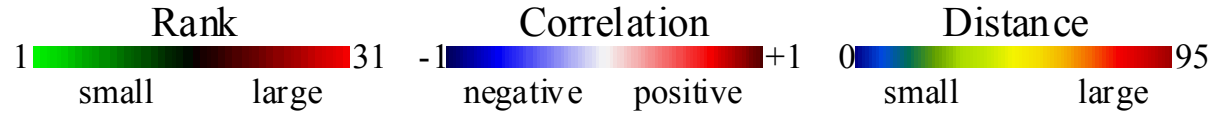


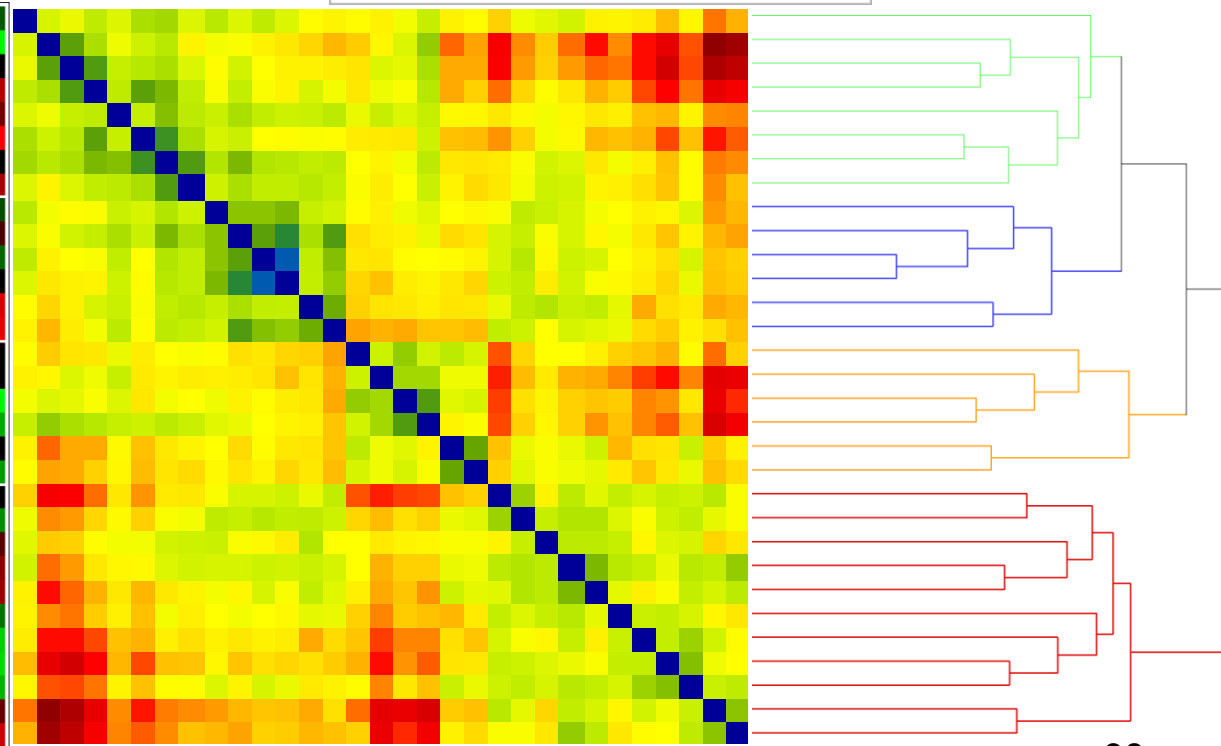
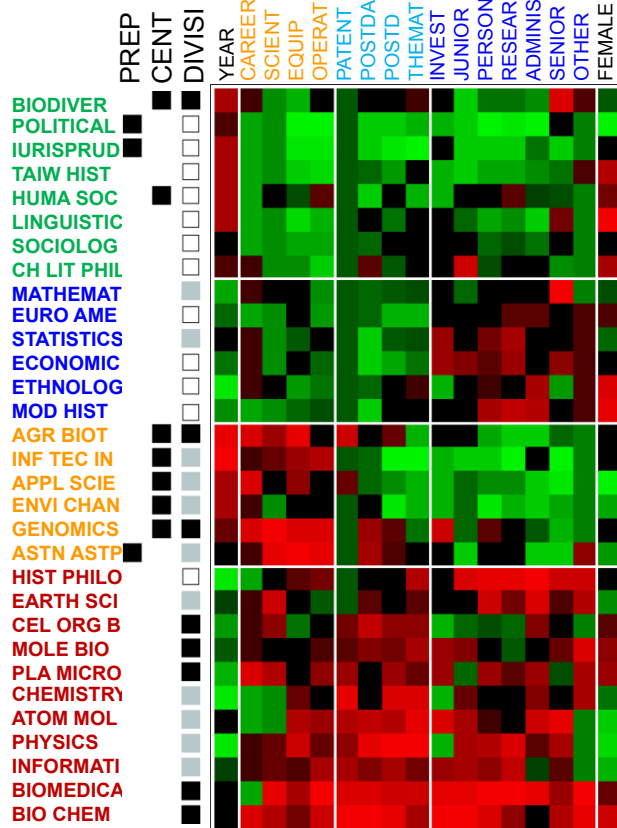
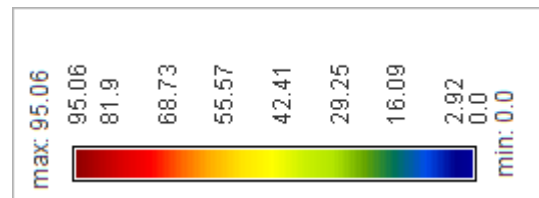
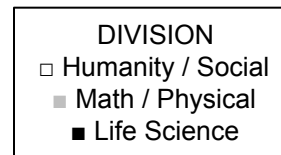
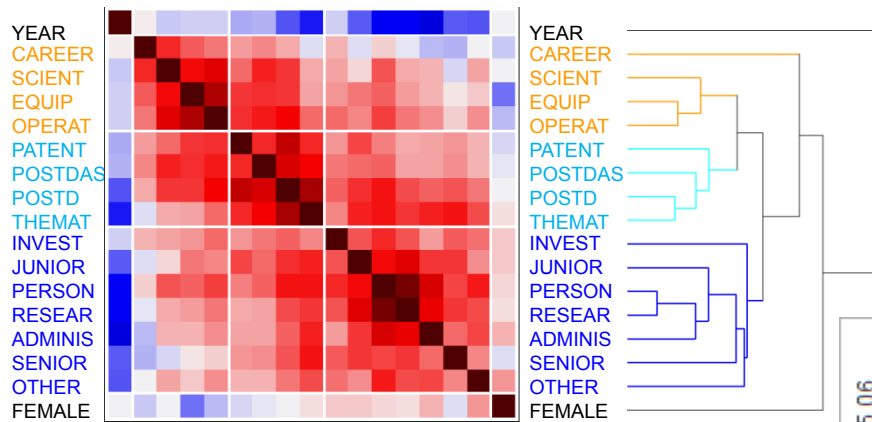
# Are there **major** and **minor** institutes in Academia Sinica?

## Rank Transformed Data

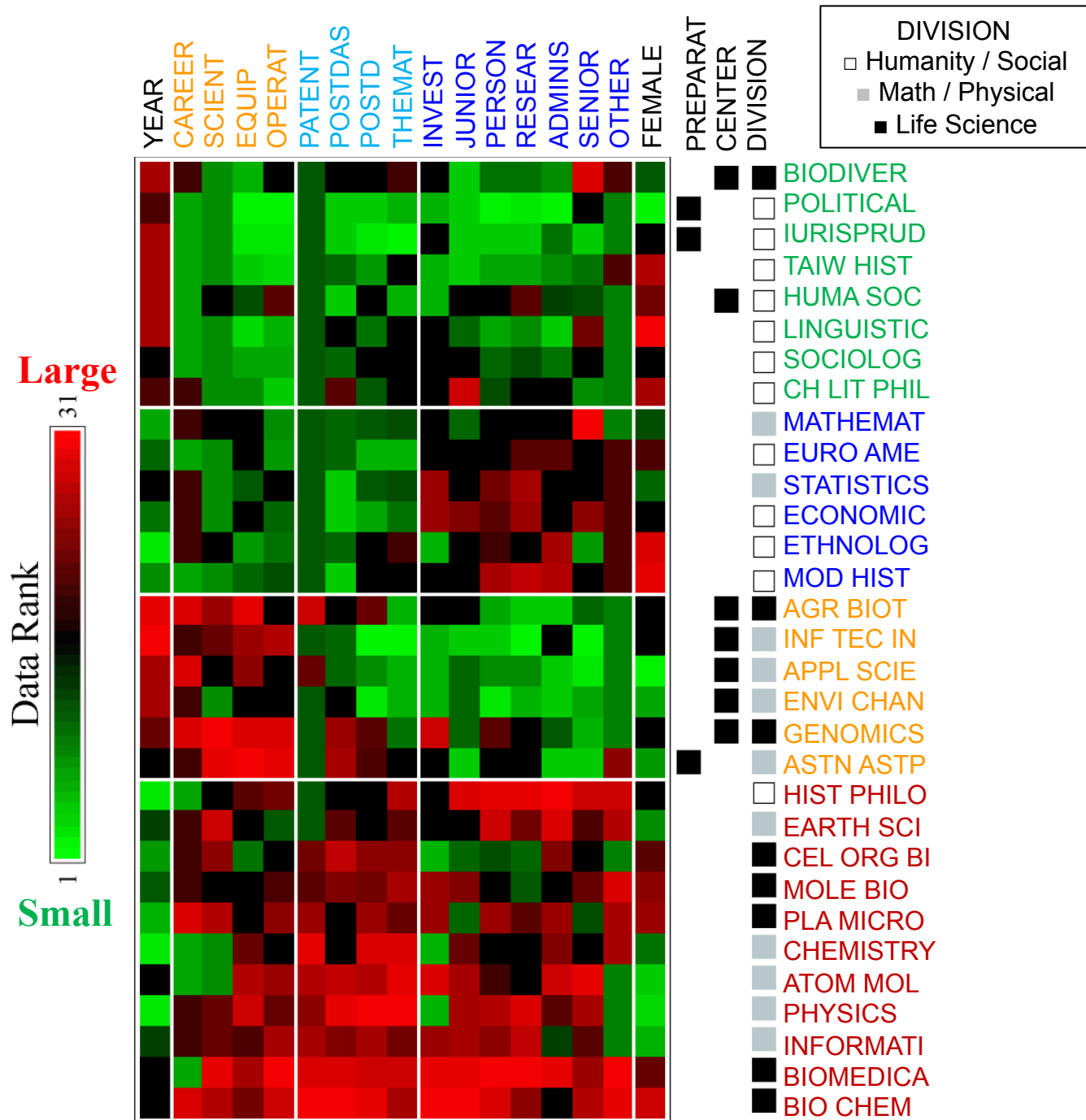
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3 15 17 9 8 26 24 15 29 6 7 15 29 22 30 19 22
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13 28 21 5 22 9 13 24 23 25 20 24 26 26 26 20
14 26 18 10 8 20 19 23 12 25 20 4 11 15 11 14 11
18 18 30 4 8 9 28 20 30 29 7 28 27 26 27 25 27
16 14 4 7 30 24 4 17 13 17 20 26 20 4 11 30 31
26 8 2 2 18 9 4 8 6 6 29 10 8 10 22 15 24
26 5 6 6 8 9 4 2 6 6 20 19 3 10 11 13 15
31 1 1 16 22 9 19 4 2 6 20 10 1 4 11 27 25
5 21 12 25 27 26 25 25 22 25 29 19 25 10 25 24 16
7 10 15 21 24 9 24 12 24 6 20 28 24 10 23 16 9
11 11 22 24 18 29 19 18 26 25 20 24 23 24 21 20 18
16 24 27 28 27 30 15 29 26 31 29 31 30 31 31 28 23
16 31 25 22 30 31 30 31 28 30 7 29 28 30 29 31 26
30 4 10 18 25 9 4 6 6 17 29 15 22 19 28 18 30
22 14 5 14 31 9 11 22 10 28 29 25 21 10 11 29 29
26 9 29 11 8 20 8 9 20 17 20 15 19 4 11 17 5
3 30 28 15 18 28 31 30 27 17 7 19 18 29 11 23 21
3 18 7 29 18 20 26 20 20 6 20 10 14 15 11 9 7
8 28 13 30 8 20 27 26 15 17 7 4 16 19 11 12 10
9 25 24 13 8 20 19 22 10 25 20 4 6 24 11 10 13
10 21 19 20 8 20 22 17 6 17 7 10 5 19 11 7 14
21 18 8 26 8 9 15 12 16 17 20 22 11 28 11 4 8
26 6 9 27 8 20 8 6 14 6 7 10 7 4 11 3 4
19 12 16 19 8 9 10 10 17 17 7 10 16 19 11 6 6
26 8 23 31 8 9 4 6 18 17 7 19 9 10 11 5 3
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26 21 12 23 18 9 13 14 6 6 7 4 13 19 11 21 12
    
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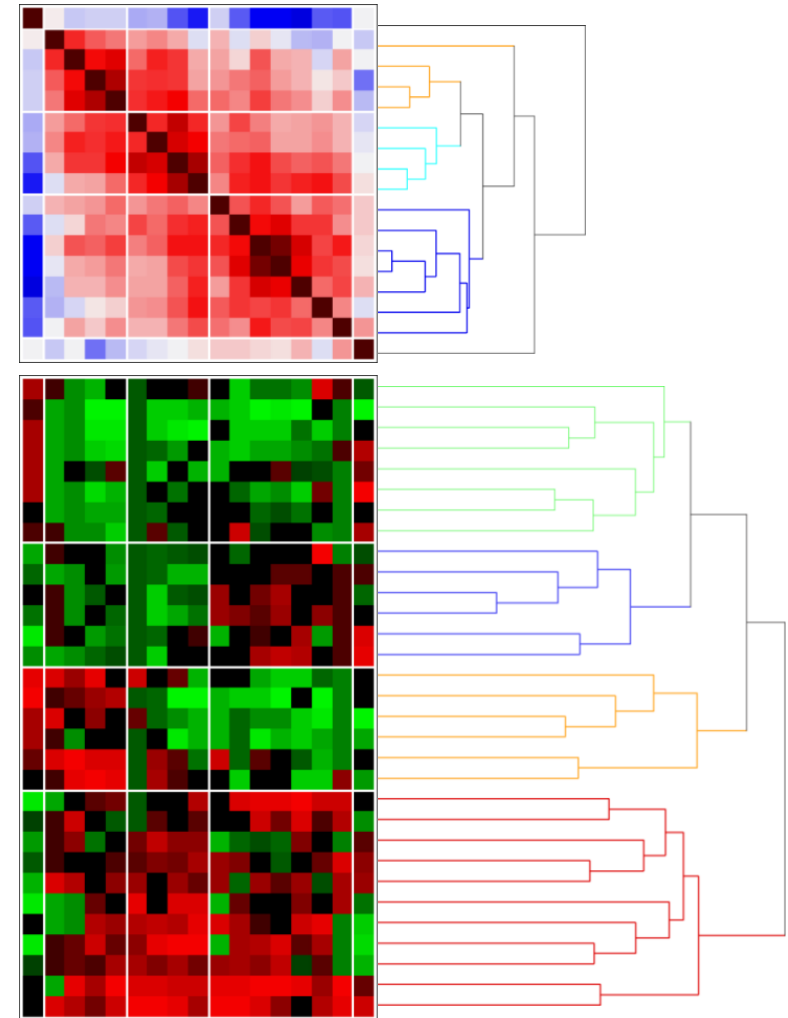






# Are there **major** and **minor** Instit. in Academia Sinica?

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3	15	17	9	8	26	24	15	29	6	7	15	29	22	30	19	22
13	23	20	8	28	27	29	28	21	17	20	22	18	15	11	11	17
13	28	21	5	22	9	13	24	23	25	20	24	26	26	26	26	20
14	26	18	10	8	20	19	23	12	25	20	4	11	15	11	14	11
18	18	30	4	8	9	28	20	30	29	7	28	27	26	27	25	27
16	14	4	7	30	24	4	17	13	17	20	26	20	4	11	30	31
26	8	2	2	18	9	4	8	6	6	29	10	8	10	22	15	24
26	5	6	6	8	9	4	2	6	6	20	19	3	10	11	13	15
31	1	1	16	22	9	19	4	2	6	20	10	1	4	11	27	25
5	21	12	25	27	26	25	25	22	25	29	19	25	10	25	24	16
7	10	15	21	24	9	24	12	24	6	20	28	24	10	23	16	9
11	11	22	24	18	29	19	18	26	25	20	24	23	24	21	20	18
16	24	27	28	27	30	15	29	26	31	29	31	30	31	31	28	23
16	31	25	22	30	31	30	31	28	30	7	29	28	30	29	31	26
30	4	10	18	25	9	4	6	6	17	29	15	22	19	28	18	30
22	14	5	14	31	9	11	22	10	28	29	25	21	10	11	29	29
26	9	29	11	8	20	8	9	20	17	20	15	19	4	11	17	5
3	30	28	15	18	28	31	30	27	17	7	19	18	29	11	23	21
3	18	7	29	18	20	26	20	20	6	20	10	14	15	11	9	7
8	28	13	30	8	20	27	26	15	17	7	4	16	19	11	12	10
9	25	24	13	8	20	19	22	10	25	20	4	6	24	11	10	13
10	21	19	20	8	20	22	17	6	17	7	10	5	19	11	7	14
21	18	8	26	8	9	15	12	16	17	20	22	11	28	11	4	8
26	6	9	27	8	20	8	6	14	6	7	10	7	4	11	3	4
19	12	16	19	8	9	10	10	17	17	7	10	16	19	11	6	6
26	8	23	31	8	9	4	6	18	17	7	19	9	10	11	5	3
21	2	14	2	8	9	1	1	6	6	7	4	4	4	11	1	1
26	4	4	18	8	9	10	4	2	17	7	4	3	4	11	2	2
26	21	12	23	18	9	13	14	6	6	7	4	13	19	11	21	12



Formulate more **appropriate Hypothesis** for answering that question.

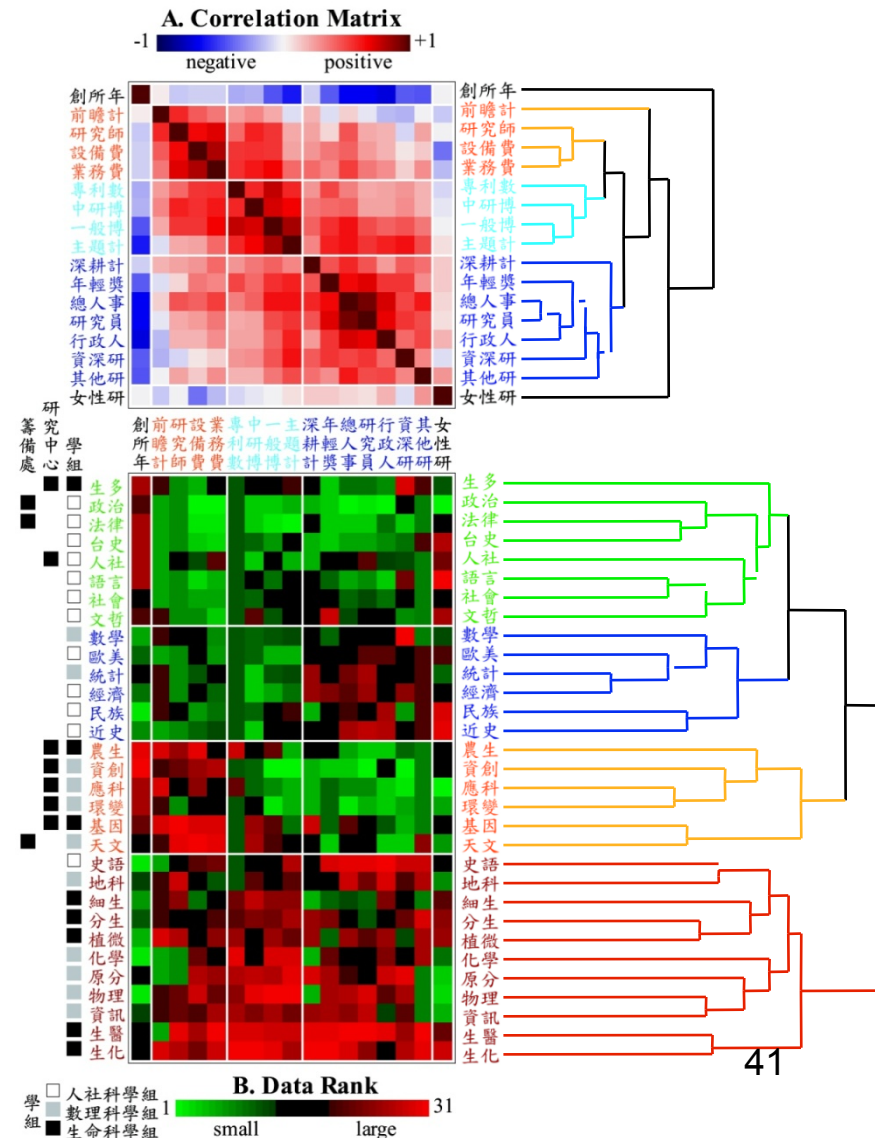
It is important to understand what you **CAN DO** before you learn to measure how **WELL** you seem to have **DONE** it.



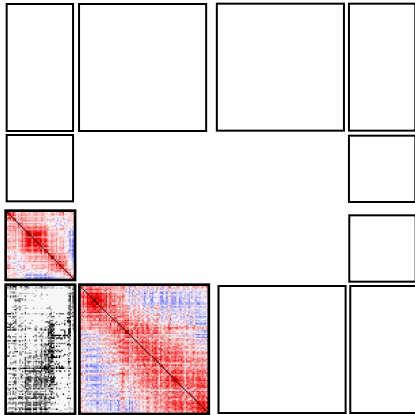
Name	Div	Cen	Pre	Yr	RF	RFF	RFS	RO	RS	ST	PN	PD	CR	IV	TM	JR	PT	AD	EQ
Math	0	0	0	6	17.5	12	31	8.5	17.5	15	13.5	10	11	19.5	16.5	11.5	10	10.5	8
Phys	0	0	0	2.5	29	3	26	8.5	22	21.5	27	30	31	19.5	5.5	31	26	24	22
Chem	0	0	0	2.5	15	9	17	25.5	7.5	23.5	15	15	29	6.5	5.5	29	22	30	19

The greatest value of a picture is when it forces us to notice what we never expected to see.

Poli	2	0	1	20.5	2	1.5	14	8.5	7.5	1	1	3.5	4	6.5	5.5	5.5	3.5	10.5	1
Juri	2	0	1	26	3.5	17.5	3.5	8.5	7.5	9.5	3.5	2.5	6.5	16.5	1.5	3.5	10.5	2	1
Huma	2	1	0	26	21	23	11.5	8.5	17.5	12.5	13.5	3.5	13	6.5	5.5	5.5	19	10.5	21



## 2.permutation



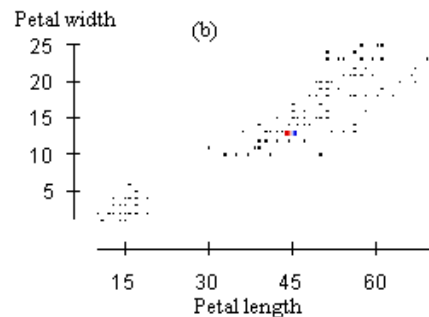
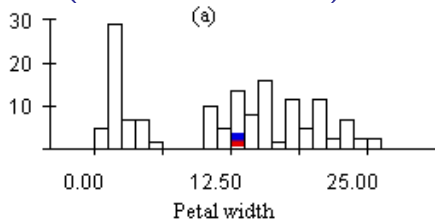
## Approaching Statistics

# Purpose of permutations

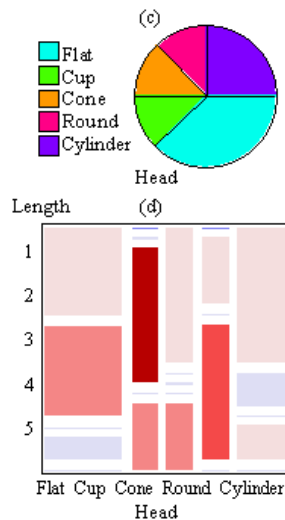
## Relativity of a statistical graph (Chen 2002)

Concept of placing **similar** (**different**) objects  
at **closer** (**distant**) positions

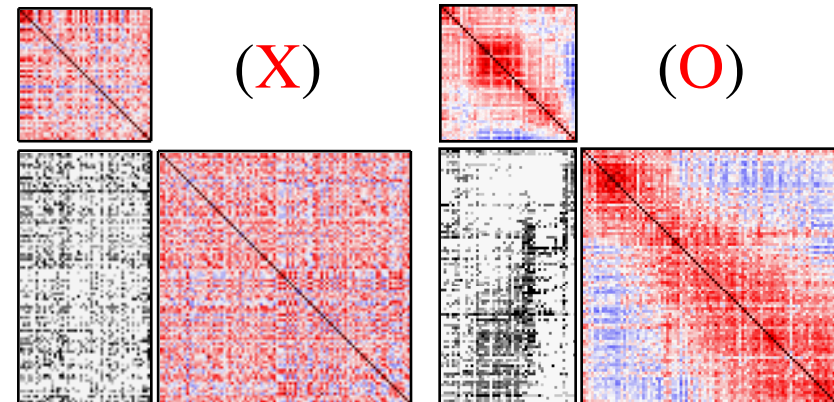
### 1. Continuous (V) (Fisher / Iris)



### 2. Categorical (X) (Hartigan / Tools)



### 3. Matrix Visualization (X) (MPGRP)



**Friendly & Kwan (2003): effect-ordered data display**  
**Hurley (2004): interesting displays in prominent positions**



# Statistical Approach

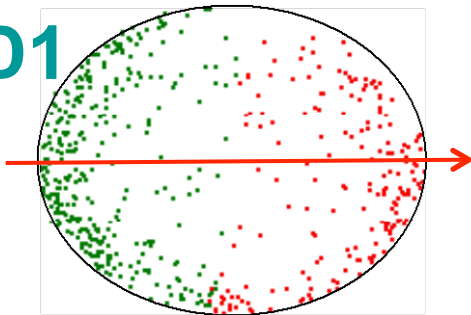
Identify **Global** Trend: **Singular Value** **D**ecomposition

**SVD**

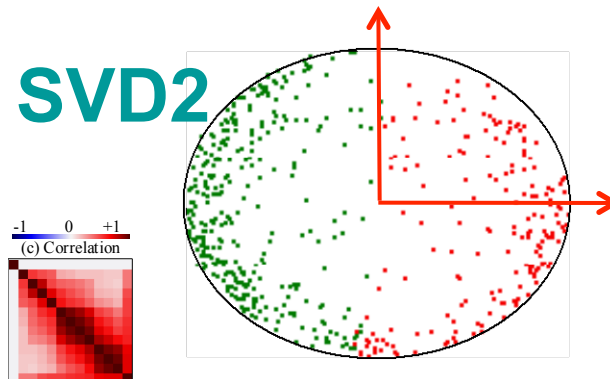
Alter O. et al  
2000, PNAS

Chen 2002,  
Statistica Sinica  
**Rank 2 Elliptical**

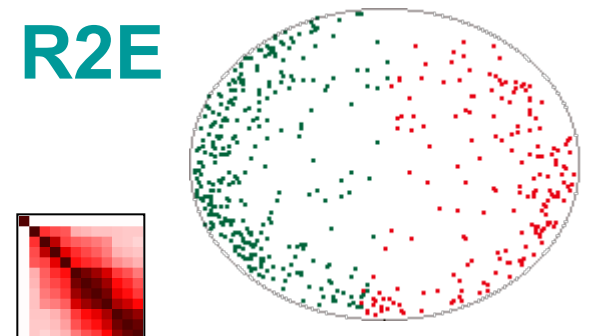
**SVD1**



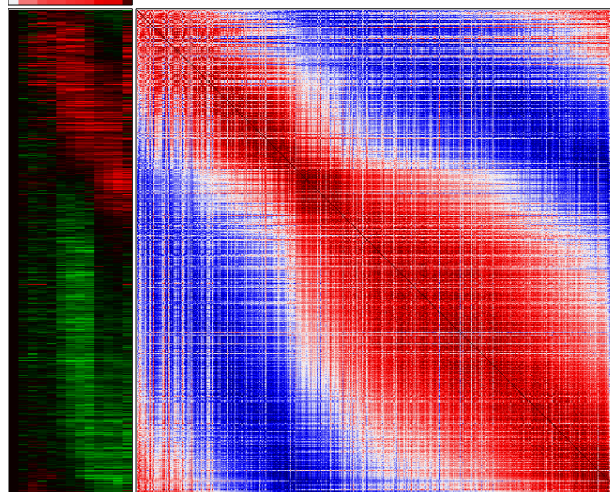
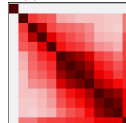
**SVD2**



**R2E**

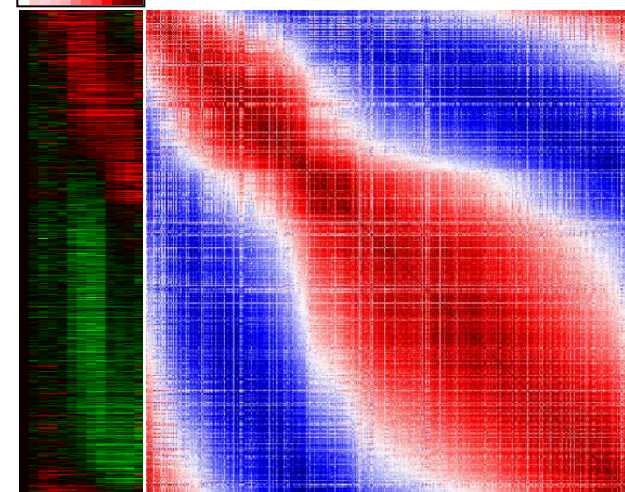


-1 0 +1  
(c) Correlation



-8 1:1 +8  
(a) Expression

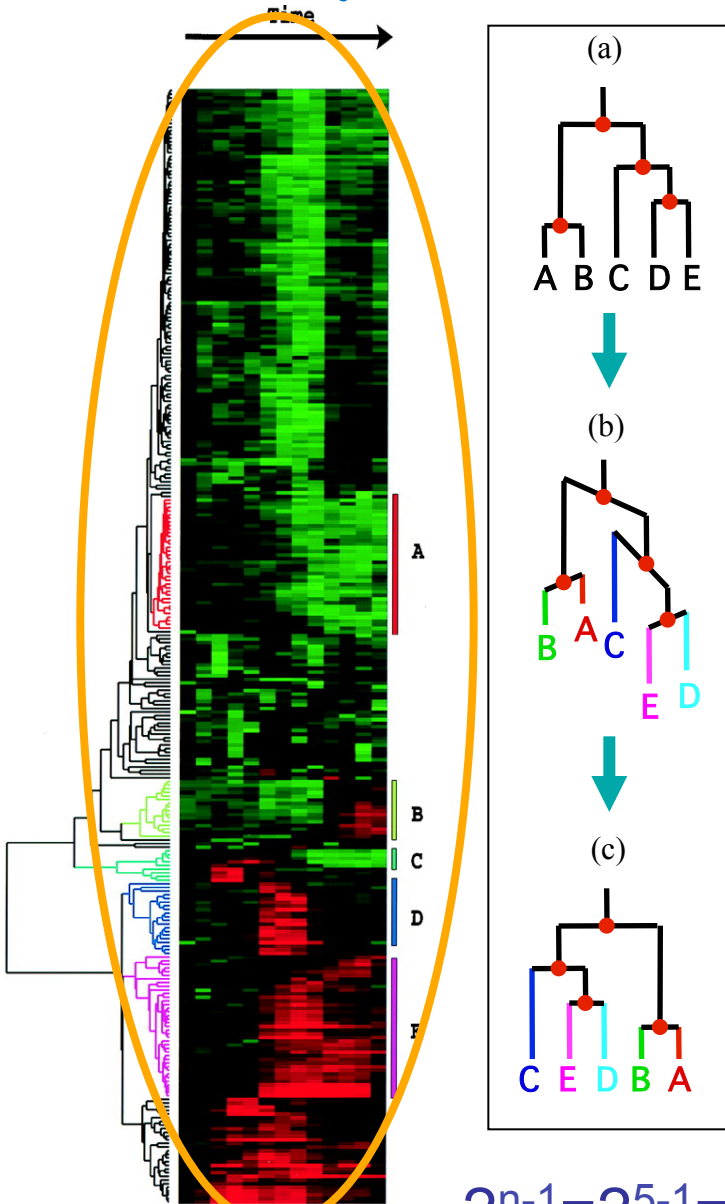
-1 0 +1  
(b) Correlation



# Statistical Approach: Identify **Local** Clusters

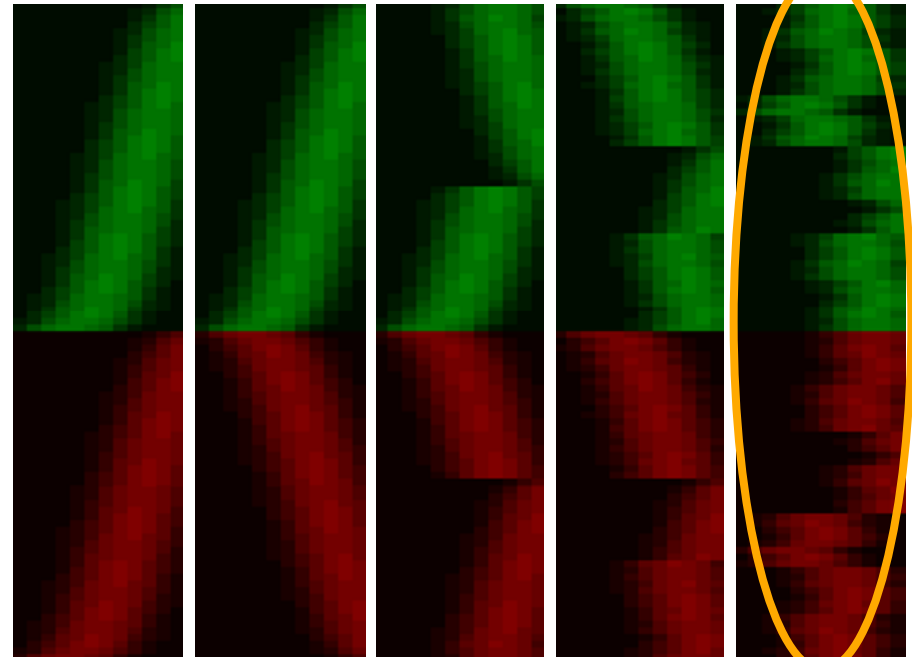
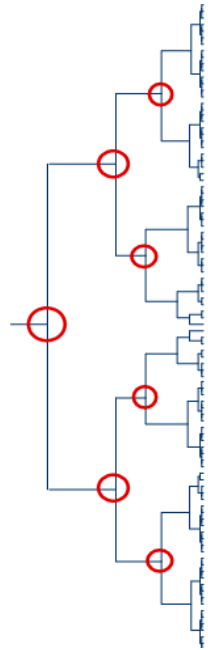
## Tree seriation & flipping of intermediate nodes

Different Seriations (Ordering of Terminal Nodes or  
Leaves) Generated from Identical Tree Structure



Eisen et al. (1998)

$$2^{n-1} = 2^{5-1} = 16$$



ideal  
model

1 flip

3 flips

5 flips

many  
flips

**external** and **internal** references  
for guiding flipping mechanism<sup>44</sup>



Methodology article

Open Access

### Methods for simultaneously identifying coherent local clusters with smooth global patterns in gene expression profiles

Yin-Jing Tien<sup>1</sup>, Yun-Shien Lee<sup>2,3</sup>, Han-Ming Wu<sup>4</sup> and Chun-Houh Chen<sup>\*5</sup>

Address: <sup>1</sup>Institute of Statistics, National Central University, Tao-Yuan, 32001, Taiwan, <sup>2</sup>Genomic Medicine Research Core Laboratory, Chang Gung Memorial Hospital (CGMH), Tao-Yuan, 33305, Taiwan, <sup>3</sup>Department of Biotechnology, Ming Chuan University, Tao-Yuan, 33348, Taiwan, <sup>4</sup>Department of Mathematics, Tamkang University, Tamsui 25137, Taiwan and <sup>5</sup>Institute of Statistical Science, Academia Sinica, Taipei, 11529, Taiwan

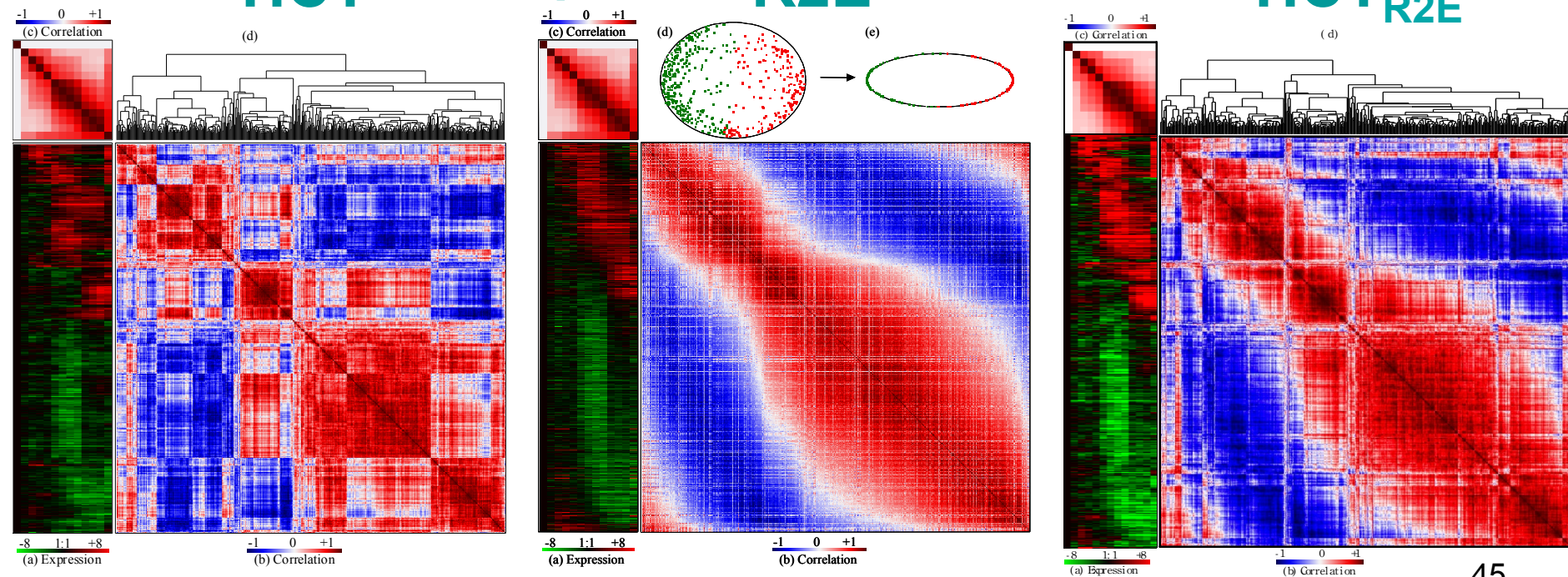
### HCT

+

### R2E

=

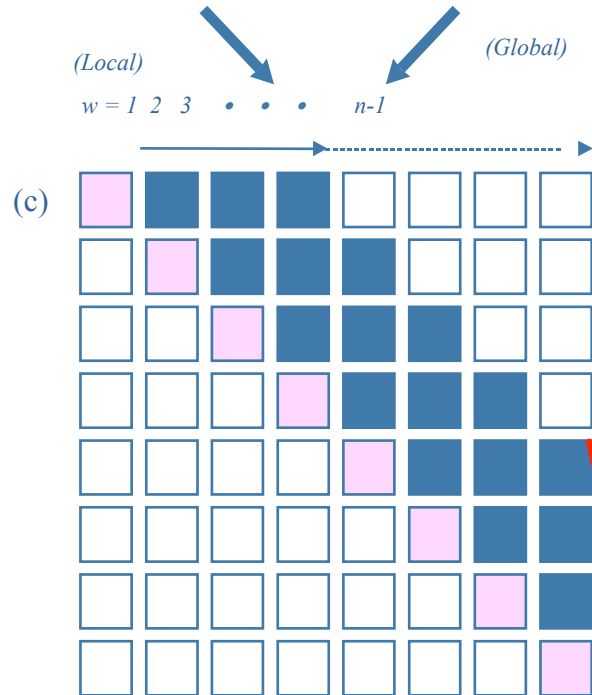
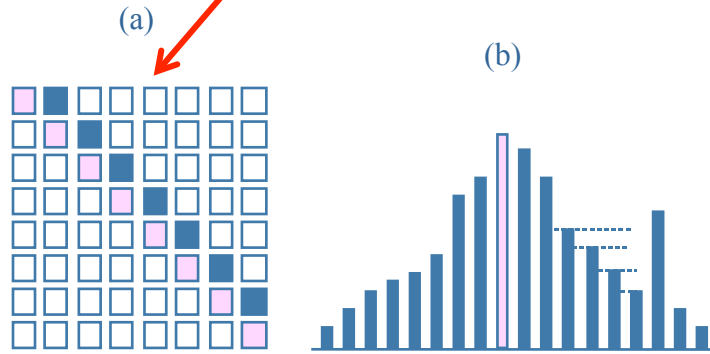
### HCT<sub>R2E</sub>



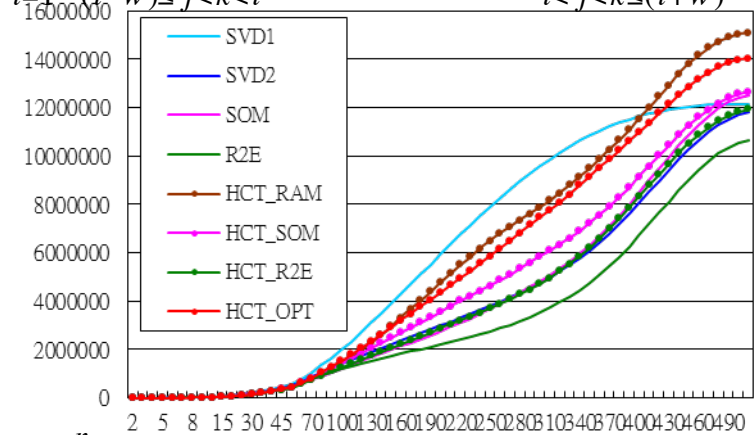
# Statistical Approach: Evaluation of permutation algorithms

## The Generalized anti-Robinson (GAR) criterion

$$AR = \sum_{i=1}^n \left[ \sum_{j < k < i} I(d_{ij} < d_{ik}) + \sum_{i < j < k} I(d_{ij} > d_{ik}) \right]$$

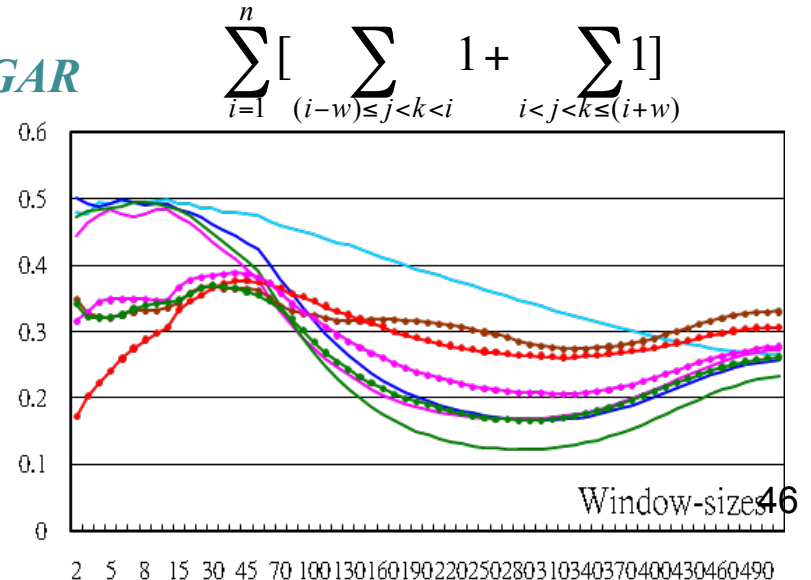


$$GAR = \sum_{i=1}^n \left[ \sum_{(i-w) \leq j < k < i} I(d_{ij} < d_{ik}) + \sum_{i < j < k \leq (i+w)} I(d_{ij} > d_{ik}) \right]$$



$$RGAR = \frac{\sum_{i=1}^n \left[ \sum_{(i-w) \leq j < k < i} I(d_{ij} < d_{ik}) + \sum_{i < j < k \leq (i+w)} I(d_{ij} > d_{ik}) \right]}{\sum_{i=1}^n \left[ \sum_{(i-w) \leq j < k < i} 1 + \sum_{i < j < k \leq (i+w)} 1 \right]}$$

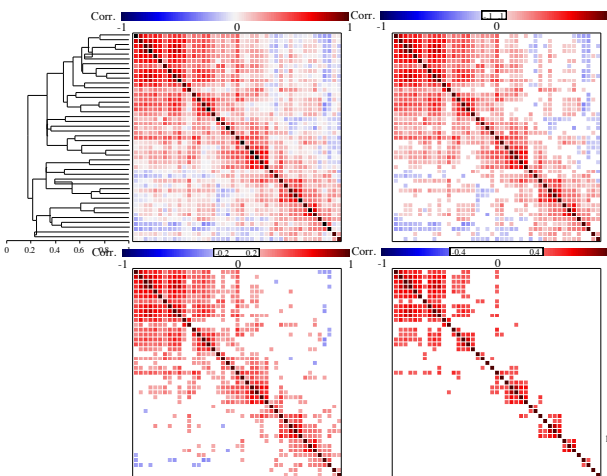
Relative **GAR**



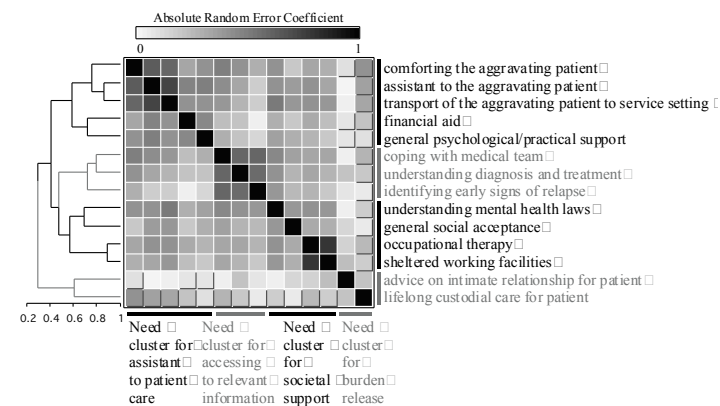


# GAP for Heritable (Genetic) Disease: **Schizophrenia** (National Taiwan University)

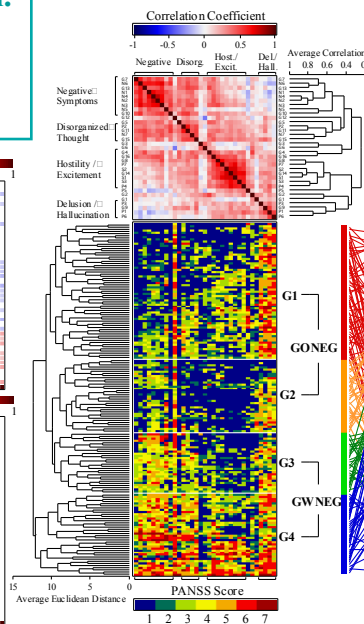
*Psychiatry Research* (1998) **Lin, Chen et al.**  
**Psychopathological Dimensions in Schizophrenia: A Correlational Approach**  
 to Items of the SANS and SAPS



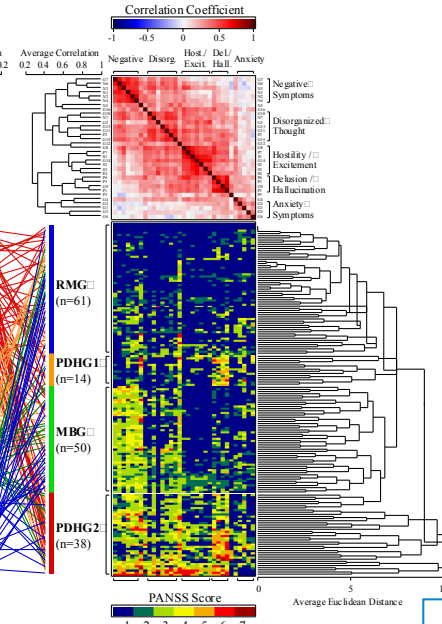
*J. of the Formosan Med. Ass.* (2008) **Yeh et al.**  
 Factors Related to **Perceived Needs** of Chief Caregivers of Patients with Schizophrenia



**Admission**



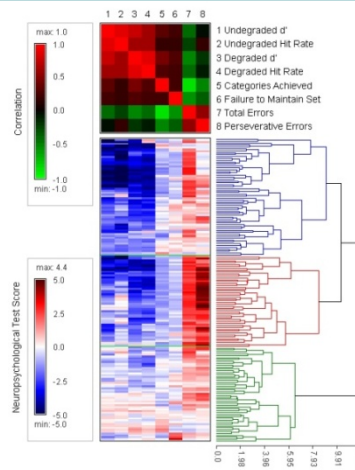
**6 month**



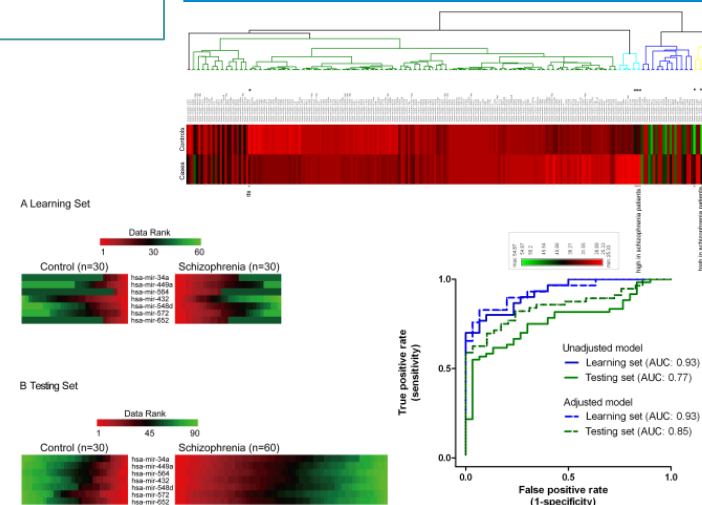
**Admission** **Hwu et al.**  
*Schizophrenia Research* (2002)  
**Symptom Patterns and Subgrouping of Schizophrenic Patients: Significance of Negative Symptoms Assessed on Admission**

**6 month** **Liu et al.**  
*J. of the Formosan Med. Ass.*  
**Validity of a 3-Subtype Model of Schizophrenia: Symptomatology, Social Function, and Neuropsychological Impairment**

*Genes, Brain and Behavior* (2009) **Lin et al.**  
 Clustering by neurocognition for **fine-mapping** of the schizophrenia susceptibility loci on chromosome 6p



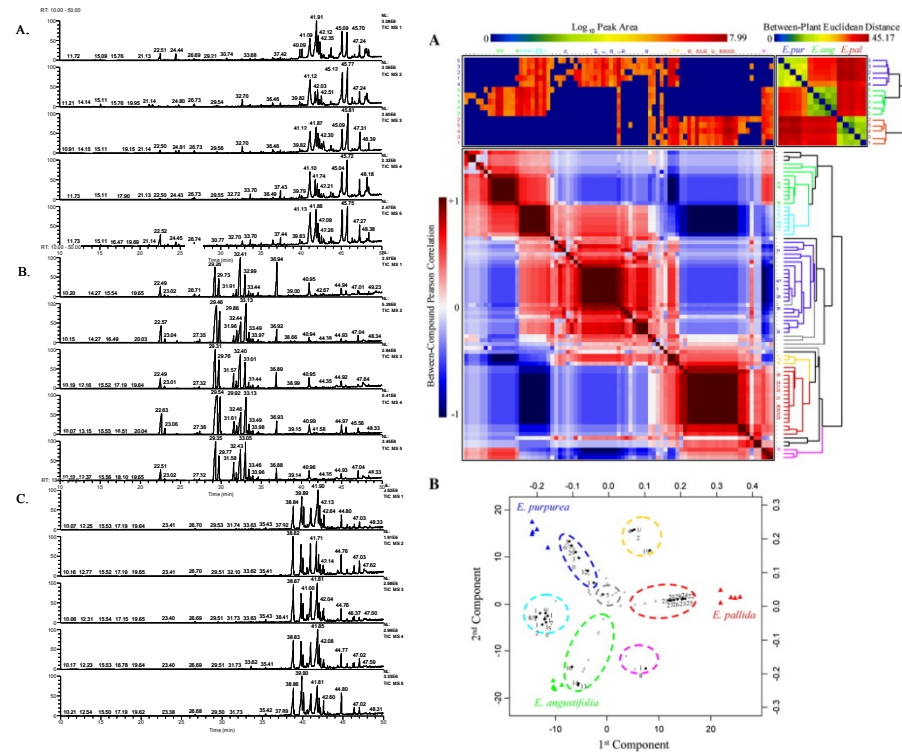
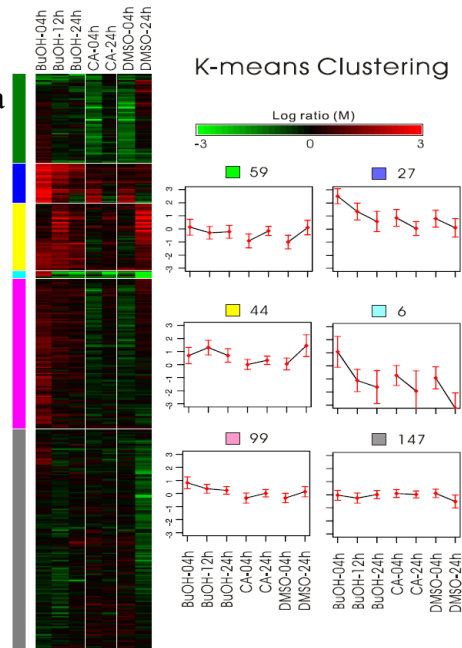
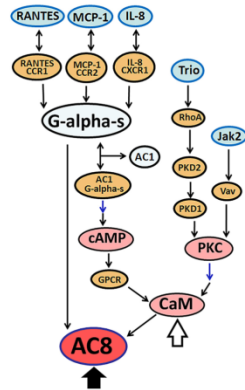
*PLoS ONE* (2011) **Lai et al.**  
**MicroRNA** expression aberration as potential peripheral blood biomarkers for schizophrenia



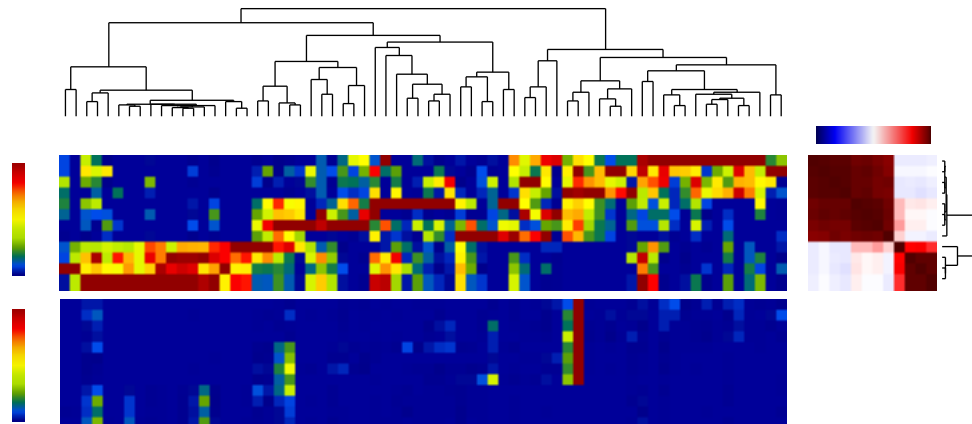
# GAP for Comparative Metabolome: Chinese Herbal Medicine

Drs. Ning-Sun Yang, Lie-Fen Shyur, Wen-Chin Yang  
Agricultural Biotechnology Research Center (ABRC) of Academia Sinica

**BMC Genomics** 9 (2008)  
Genomics and proteomics of immune modulatory effects of a butanol fraction of *Echinacea purpurea* in human dendritic cells **Wang et al.**



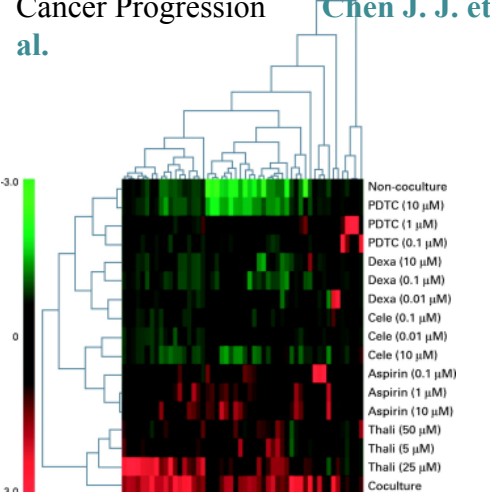
**Phytochemistry** 70 (2009)  
Anti-diabetic properties of three common *Bidens pilosa* variants in Taiwan  
**Chien et al.**



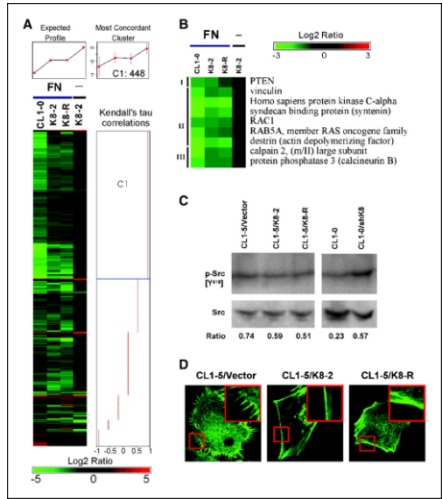
**Journal of Nutritional Biochemistry** (2010)  
Comparative metabolomics approach coupled with cell- and gene-based assays for species classification and anti-inflammatory bioactivity validation of *Echinacea* plants  
**Hou et al.**

# GAP for Cancer Study: Non-Small Cell Lung Cancer (National Taiwan University)

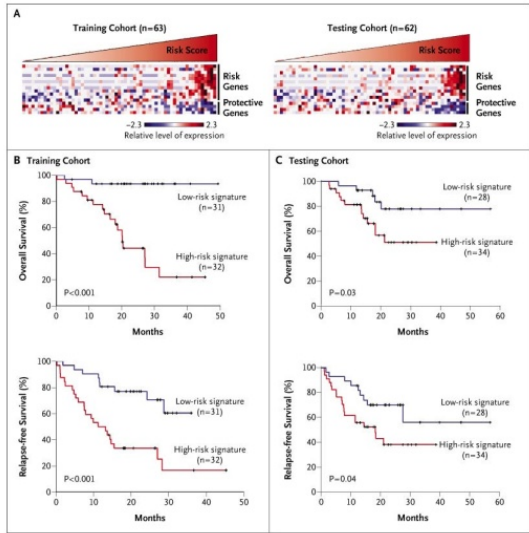
*Journal of Clinical Oncology* 23 (2005)  
Tumor-Associated Macrophages in Cancer Progression  
**Chen J. J. et al.**



*Cancer Research* 66 (2006)  
Non-Small Cell Lung Cancer with Tumor Cell Invasiveness  
**Sher Y. P. et al.**

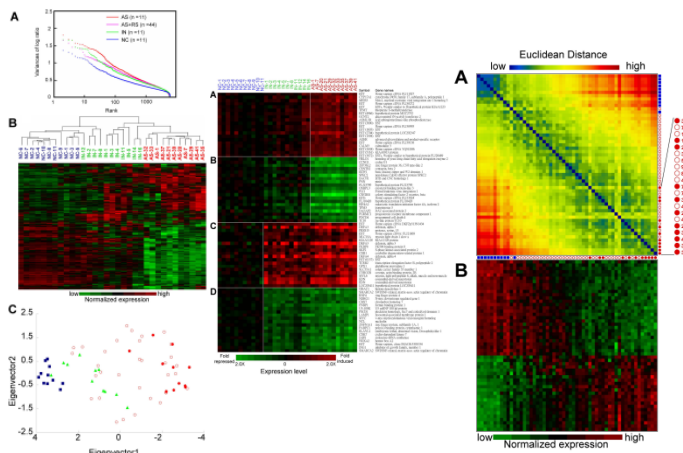


*The New England Journal of Medicine* 356 (2007) A Five-Gene Signature and Clinical Outcome in Non-Small-Cell Lung Cancer  
**Chen H. Y. et al.**



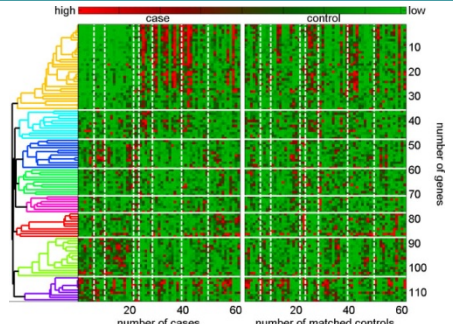
## GAP for Infectious Disease: SARS (Chang Gung Memorial Hospital)

*BMC Genomics* 6 (2005) Molecular signature of clinical severity in recovering patients with (SARS-CoV)  
**Lee Y. S. et al. (Chang Gung Hospital)**

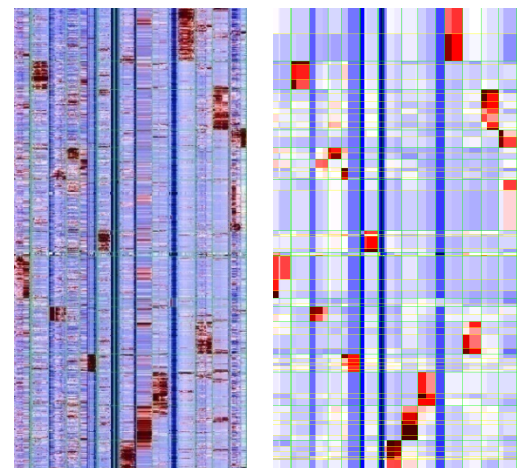


## GAP for Endophenotypes/Nutrition (Academia Sinica)

*Genetic Epidemiology* 30 (2006)  
Using endophenotypes for pathway clusters to map complex disease genes  
**Pan W. H. et al. (Academia Sinica)**

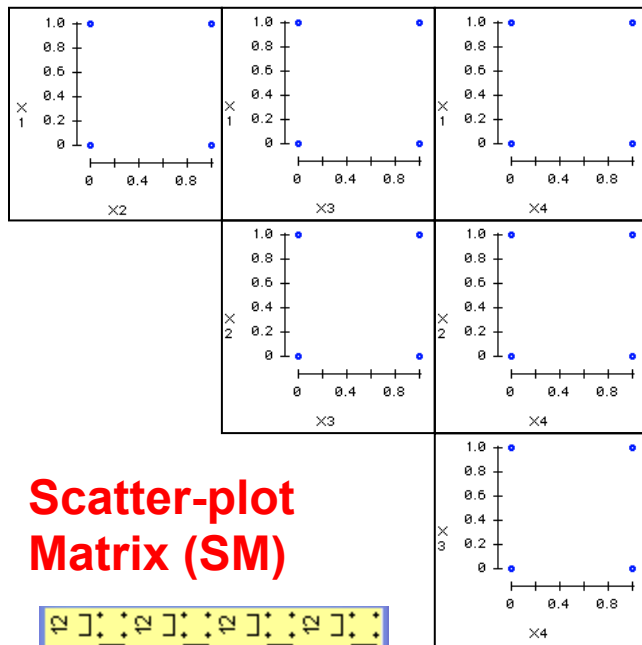


*Nutritional Sciences Journal* 30 (2006)  
Evaluating the DOH Food Guide Based on Taiwanese Food Choices  
**Pan W. H. et al.**



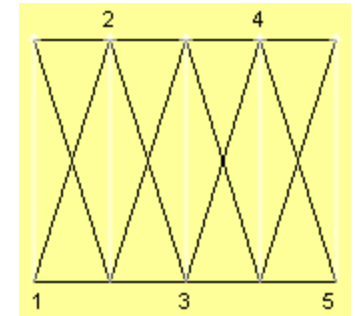
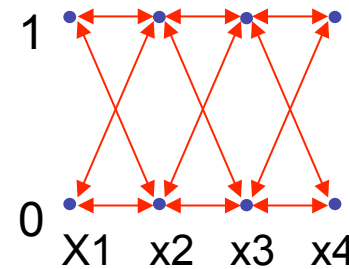
# Matrix visualization of **binary** data

Graphic tools for high-dimensional **non** continuous data visualization **w/o** dimension reduction

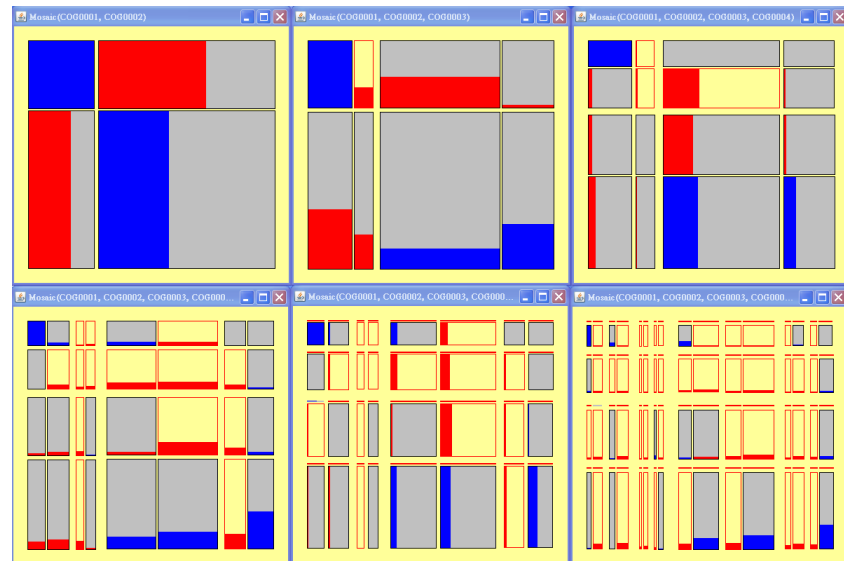


**Scatter-plot  
Matrix (SM)**

**Parallel  
Coordinates  
Plot (PCP)**



**Mosaic Plot (Display)**





# Approaching Statistics

## Essential elements in a GAP MV procedure?

### Continuous

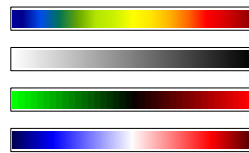
#### 3. Variable Proximity

Correlation  
Covariance  
polychoric  
Correlation . . .

#### 2. Subject Proximity

Euclidean Distance  
Manhattan Distance  
Correlation . . .

#### 1. Data Matrix



#### 3. Variable Proximity

#### 1. Data Matrix



### Binary

		Object B		
		1	0	
Object A	1	$a$	$b$	$(a + b)$
	0	$c$	$d$	$(c + d)$
		$(a + c)$	$(b + d)$	$(a + b + c + d)$

#### 2. Subject Proximity

## Selection of Proximity Measures for Matrix Visualization of Binary Data

Tzeng, S. L., Wu, H. M., and Chen, C. H. (2009)

*Proc. 2009 2nd Int'l Conf. on BioMed. Engin. & Info. (BMEI 2009), Tianjin, China*  
(available in *IEEE Xplore Digital Library*)

**Table 1. Commonly used similarity coefficients for binary data.**

		Object B		
		1	0	
Object A	1	$a$	$b$	$(a + b)$
	0	$c$	$d$	$(c + d)$
		$(a + c)$	$(b + d)$	$(a + b + c + d)$

Similarity	Formula
Braun	$\frac{a}{\max(a + b, a + c)}$
Dice	$\frac{2a}{2a + b + c}$
Hamman	$\frac{a + d - (b + c)}{a + b + c + d}$
Jaccard	$\frac{a}{a + b + c}$
Kappa	$\left(1 + \frac{(b + c)(a + b + c + d)}{2ad - 2bc}\right)^{-1}$
Kulczynski	$\frac{1}{2} \left( \frac{a}{a + b} + \frac{a}{a + c} \right)$

Similarity	Formula
Ochiai	$\frac{a}{\sqrt{((a + b)(a + c))}}$
Phi	$\frac{ad - bc}{\sqrt{(a + b)(a + c)(d + b)(d + c)}}$
Rao	$\frac{a}{a + b + c + d}$
Rogers	$\frac{a + d}{a + 2b + 2c + d}$
simple match	$\frac{a + d}{a + b + c + d}$
Simpson	$\frac{a}{\min(a + b, a + c)}$
Sneath	$\frac{a}{a + 2b + 2c}$
Yule	$\frac{ad - bc}{ad + bc}$

Object B

1 0

Object A

1	a	b	(a + b)
0	c	d	(c + d)
	(a + c)	(b + d)	(a + b + c + d)

Two issues for selecting similarity measures for binary:

I. **Symmetric** or **Asymmetric** :

**SYMMETRIC**: if both of its categories are **equally important**, i.e., there is no preference on which outcome should be coded as 0 or 1. Gender is an typical example of symmetric binary variable. (♀/ ♂, Bioif/Biost, ....)

**Symmetric binary variables should be treated as nominal ones.**

Similarity measures: often a function of both the co-occurrence and co-absence frequencies between two variables

e.g., simple matching  $\frac{a + d}{a + b + c + d}$  Rogers  $\frac{a + d}{a + 2b + 2c + d}$  Hamman  $\frac{a + d - (b + c)}{a + b + c + d}$

**ASYMMETRIC** if the outcomes of the two states are not equally important, such as the positive and negative **outcomes of a disease diagnosis**. Conventionally the most **important** outcome, which is usually the **uncommon** one is coded by **1** and the other by 0.

Therefore, **asymmetric** binary variables are often considered “**monary**” (as if there is only one state)

Similarity measures: a function of co-occurrence frequencies,

e.g.  $\frac{2a}{2a + b + c}$   $\frac{a}{a + b + c}$   $\frac{1}{2} \left( \frac{a}{a + b} + \frac{a}{a + c} \right)$   $\frac{a}{\sqrt{((a + b)(a + c))}}$   $\frac{a}{\max(a + b, a + c)}$   $\frac{a}{\min(a + b, a + c)}$   $\frac{a}{a + 2b + 2c}$

**Both symmetric and asymmetric**

$$\left( 1 + \frac{(b + c)(a + b + c + d)}{2ad - 2bc} \right)^{-1} \quad \frac{ad - bc}{\sqrt{(a + b)(a + c)(d + b)(d + c)}} \quad \frac{ad - bc}{ad + bc}$$



**CGMIM Online**

## Binary GAP Example

<http://www.bccrc.ca/ccr/CGMIM/>

**CGMIM performs automated text-mining of OMIM to identify genetically-related cancers**

**Online Mendelian In Man (OMIM) is a computerized database of information about **genes and heritable traits** in human populations**

**OMIM is maintained on the Internet by the National Center for Biotechnology Information at the US National Institutes of Health**

**CGMIM considers **21 anatomic sites** based on the major **cancers** identified by the National Cancer Institute of Canada**

**CGMIM compares each OMIM entry name and alternative name with a list of gene names assigned by HUGO (HUMAN Genome Organization).**

**CGMIM produces the number of genes for which an OMIM entry mentions each pair of cancers, as well as a ratio of the observed genes for the combination**



**BC Cancer Agency**

CARE & RESEARCH

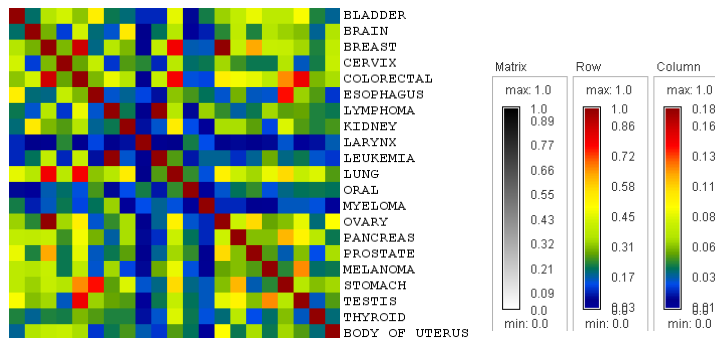
An agency of the Provincial Health Services Authority



# 21 Cancer Sites

BLADDER  
BRAIN  
BREAST  
CERVIX  
COLORECTAL  
ESOPHAGUS  
LYMPHOMA  
KIDNEY  
LARYNX  
LEUKEMIA  
LUNG  
ORAL  
MYELOMA  
OVARY  
PANCREAS  
PROSTATE  
MELANOMA  
STOMACH  
TESTIS  
THYROID  
BODY OF UTERUS

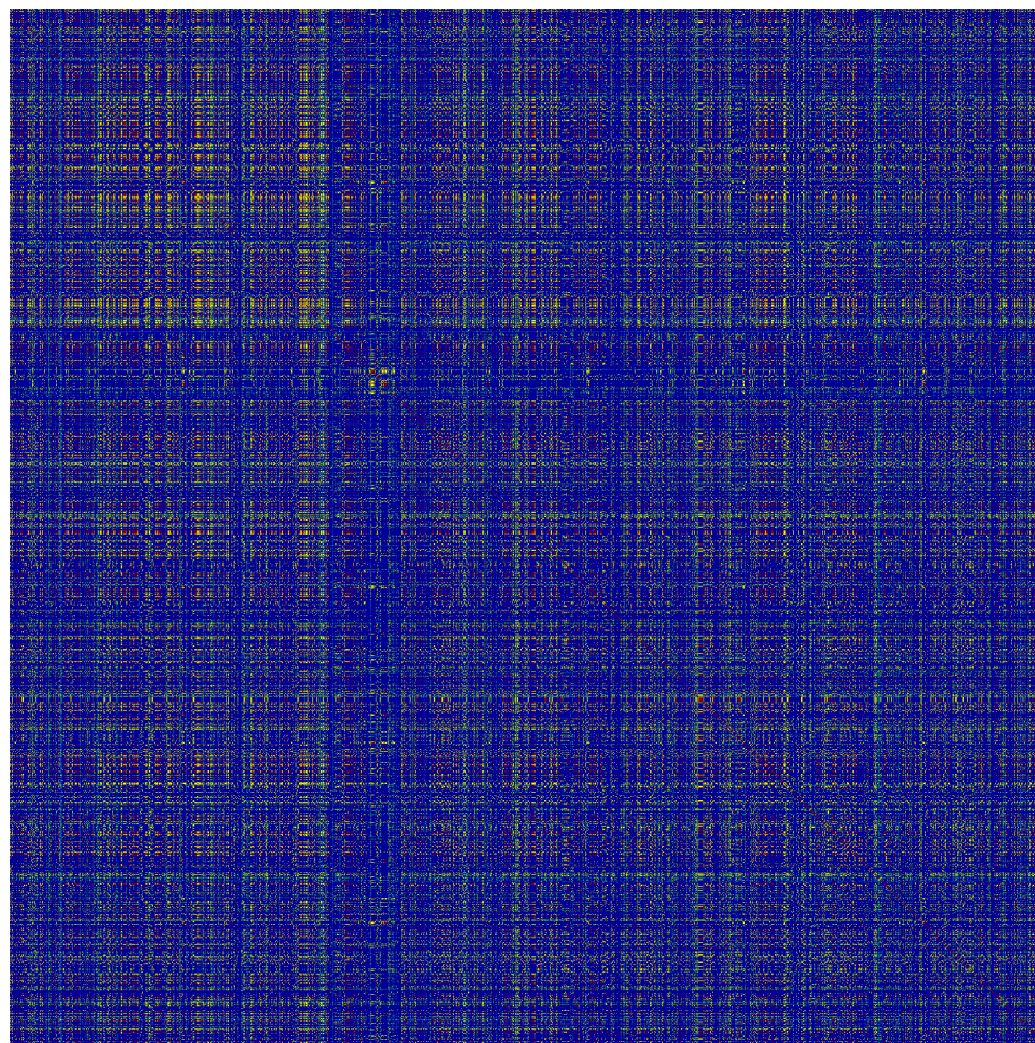
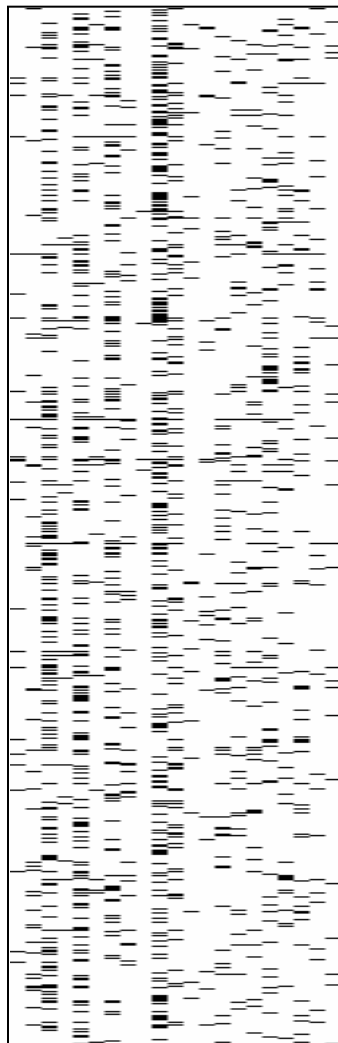
# 1948 Related Genes



## CGMIM

All Data (1948 genes \* 21 Sites)  
Original Order

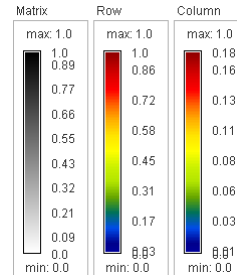
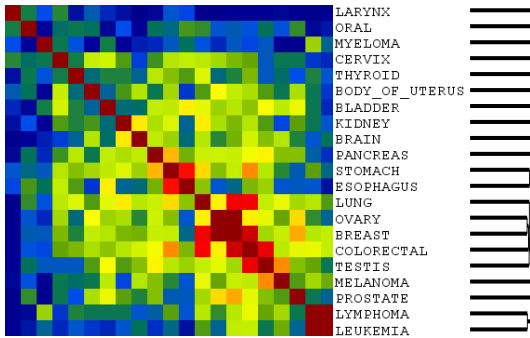
Jaccard:  $a/(a+b+c)$



# 21 Cancer Sites

LARYNX  
ORAL  
MYELOMA  
CERVIX  
THYROID  
BODY\_OF\_UTERUS  
BLADDER  
KIDNEY  
BRAIN  
PANCREAS  
STOMACH  
ESOPHAGUS  
LUNG  
OVARY  
BREAST  
COLORECTAL  
TESTIS  
MELANOMA  
PROSTATE  
LYMPHOMA  
LEUKEMIA

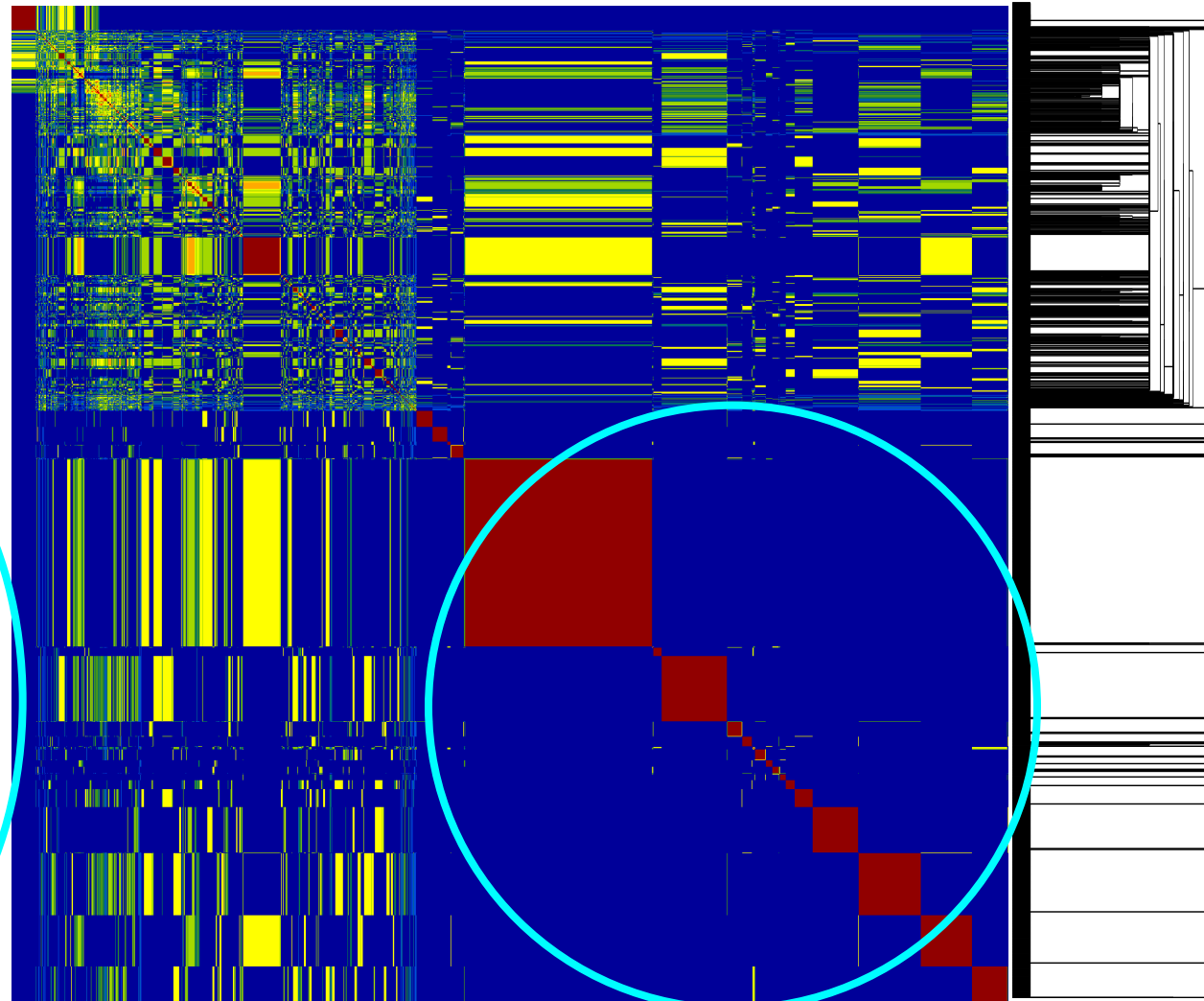
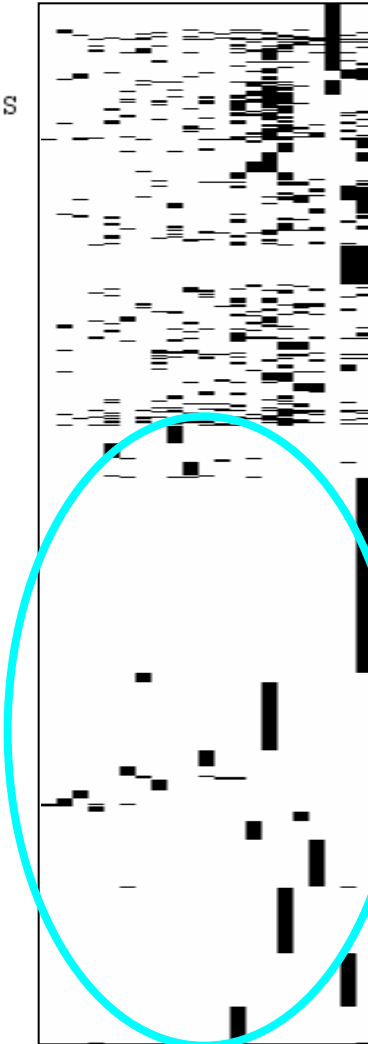
## 1948 Related Genes



## CGMIM

All Data (1948 genes \* 21 Sites)  
Single\_Tree\_GrandPa\_Guide

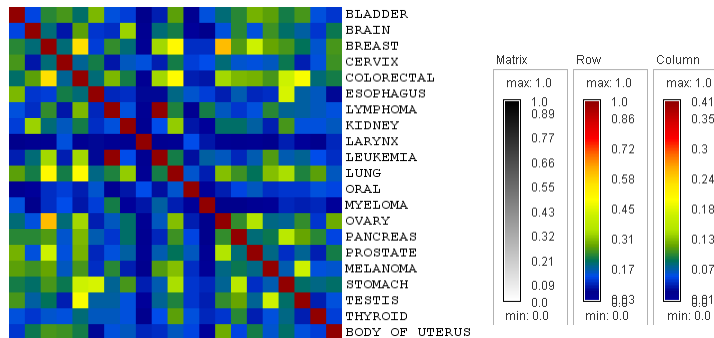
$$\text{Jaccard: } a/(a+b+c)$$





# 21 Cancer Sites

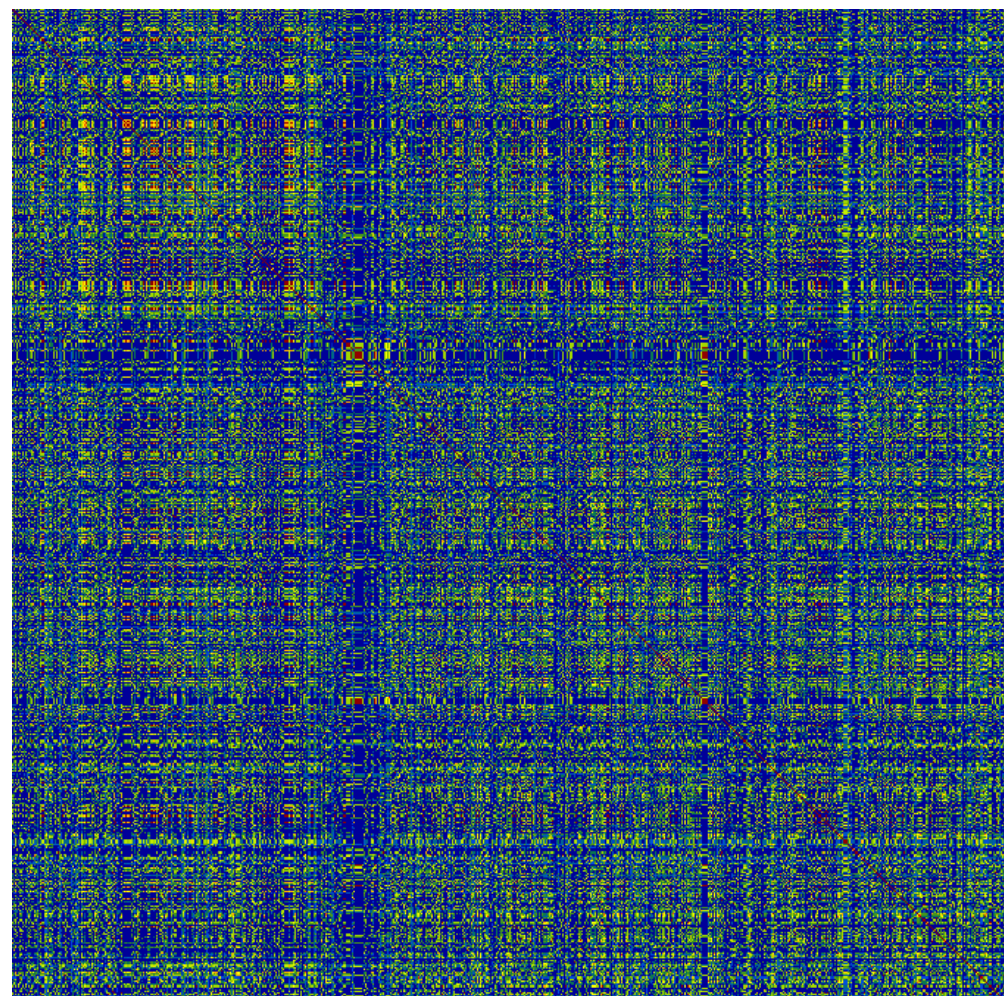
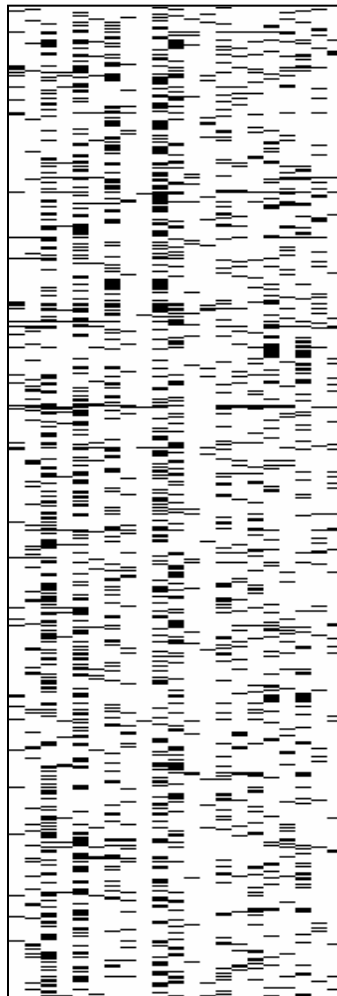
BLADDER  
BRAIN  
BREAST  
CERVIX  
COLORECTAL  
ESOPHAGUS  
LYMPHOMA  
KIDNEY  
LARYNX  
LEUKEMIA  
LUNG  
ORAL  
MYELOMA  
OVARY  
PANCREAS  
PROSTATE  
MELANOMA  
STOMACH  
TESTIS  
THYROID  
BODY OF UTERUS



CGMIM

768 genes at least at 2 Sites  
Original Order

$$\text{Jaccard: } a/(a+b+c)$$

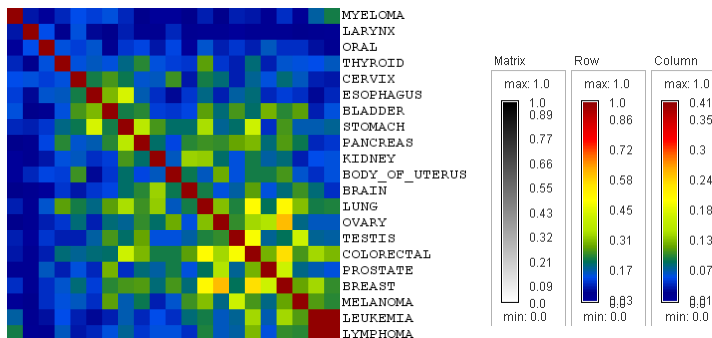


768  
Related  
Genes

# 21 Cancer Sites

MYELOMA  
 LARYNX  
 ORAL  
 THYROID  
 CERVIX  
 ESOPHAGUS  
 BLADDER  
 STOMACH  
 PANCREAS  
 KIDNEY  
 BODY\_OF\_UTERUS  
 BRAIN  
 LUNG  
 OVARY  
 TESTIS  
 COLORECTAL  
 PROSTATE  
 BREAST  
 MELANOMA  
 LEUKEMIA  
 LYMPHOMA

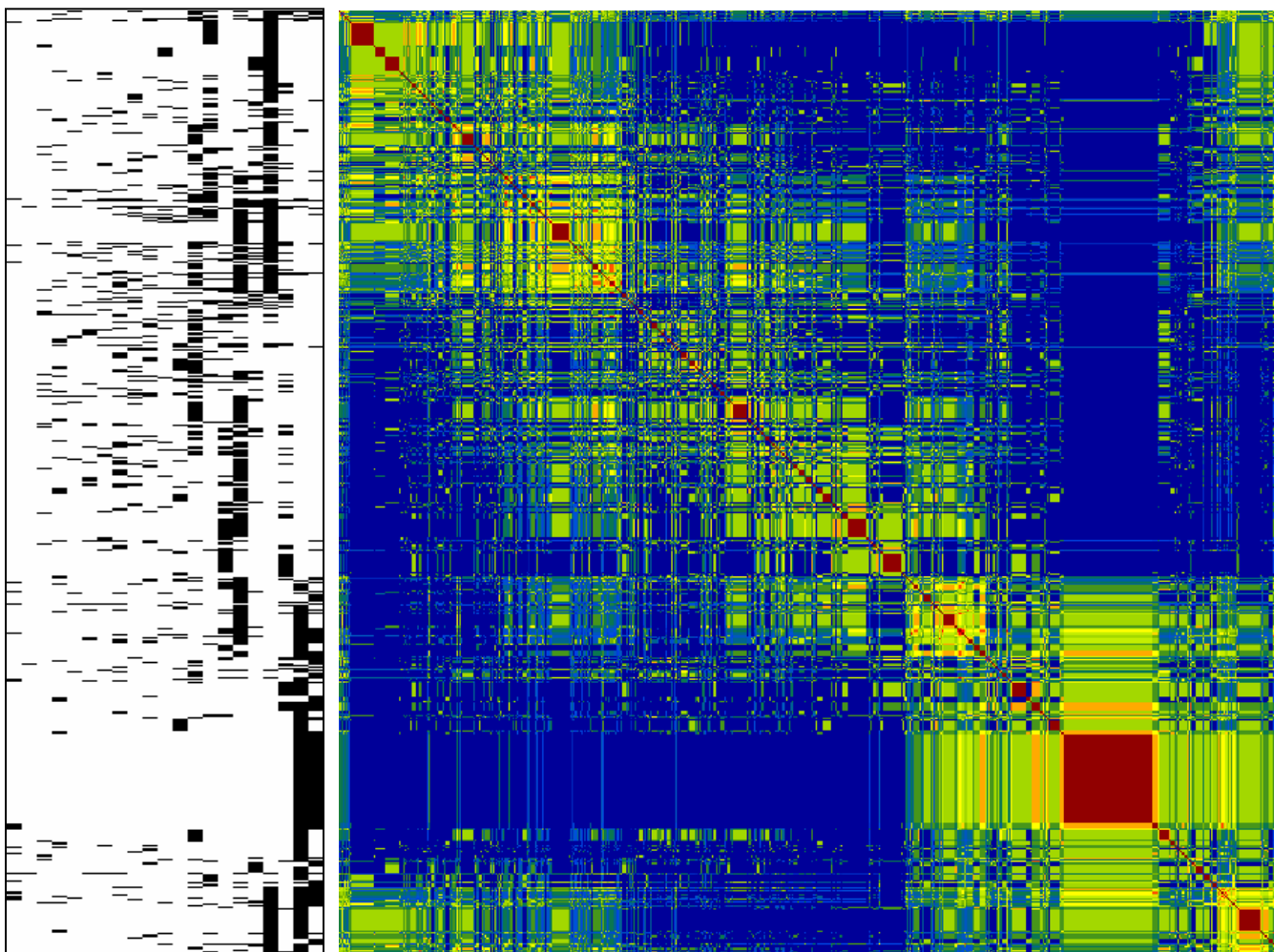
# 768 Related Genes



## CGMIM

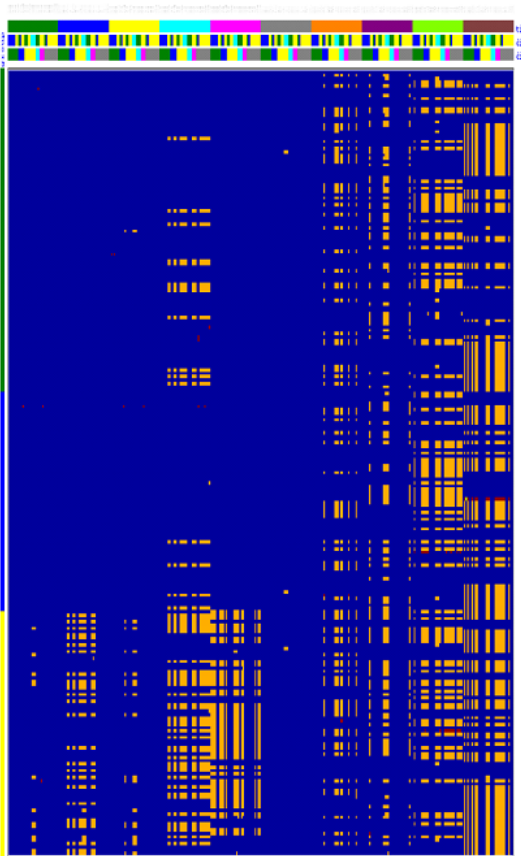
768 genes at least at 2 Sites  
 GAP\_Elliptical\_Order

$$\text{Jaccard: } a/(a+b+c)$$

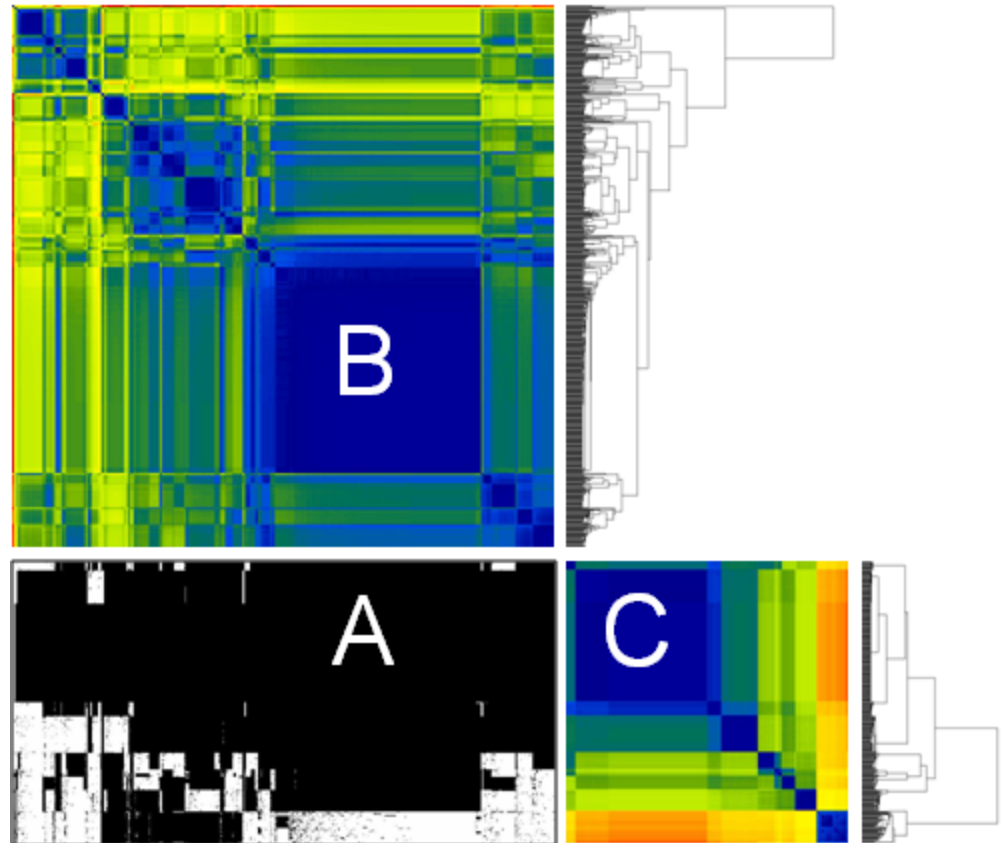




# Matrix Visualization for **data quality control** and **missing pattern exploration**



**Drop-out** structure of schizophrenia **patients** in a 5-year follow-up study



**Missing** and multi-level **stratification** structure of individual **SNPs profiles** for patients with a certain disease (simulation data generated from parameters estimated from a real data).

# Matrix visualization of **nominal** data (GAP approach)

Example:

**Classification of Animals Data**  
**Shizuhiko Nishisato 2006**

A typical nominal data

Shizuhiko Nishisato,  
2006

Classification of Animals

35 animals  
were sorted  
into piles of  
similar animals  
by 15 variables  
(Genotypes /  
Phenotypes ?)

What about  
3500 samples  
1500 variables



Animal/Subject	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
Alligator	8	1	6	9	6	1	3	4	1	4	3	4	3	6	4
Bear	6	3	2	6	6	1	3	4	4	5	5	1	4	2	2
Camel	4	3	9	3	1	4	3	5	4	2	5	1	7	7	8
Cat	6	3	7	4	0	1	1	2	3	3	1	1	6	3	5
Cheetah	3	3	7	4	0	1	3	5	4	3	6	1	6	2	2
Chicken	7	2	4	1	2	5	7	1	5	1	1	3	8	4	1
Chimpanzee	5	3	5	7	5	2	4	4	2	2	6	4	3	6	6
Cow	1	3	9	6	1	1	3	5	3	4	1	1	4	5	8
Crane	7	2	4	5	2	5	5	1	5	1	2	3	8	4	1
Crow	7	2	4	5	2	5	5	1	5	1	2	3	8	4	1
Dog	6	3	7	10	0	2	1	2	3	3	1	1	4	3	5
Duck	7	2	4	1	2	5	5	1	5	1	2	3	8	4	1
Elephant	4	3	6	3	1	4	3	5	4	5	3	1	7	7	2
Fox	6	3	7	4	0	1	6	2	3	3	3	1	4	3	5
Frog	8	1	3	2	3	3	2	3	1	4	4	2	1	1	3
Giraffe	1	3	8	3	1	4	3	5	4	2	5	1	7	7	8
Goat	3	3	9	6	1	4	6	5	3	3	1	1	5	3	5
Hawk	7	2	4	5	2	5	5	1	5	1	3	3	8	4	1
Hippopotamus	4	3	6	6	6	4	3	3	4	4	5	1	7	7	2
Horse	6	3	9	6	1	2	3	5	3	3	1	1	5	5	8
Leopard	1	3	7	4	0	1	3	5	4	3	3	1	6	2	2
Lion	5	3	7	4	6	1	3	5	4	3	3	1	7	2	2
Lizard	2	1	3	2	3	3	2	3	1	4	4	2	2	1	3
Monkey	6	3	5	7	5	2	4	4	2	2	6	4	3	6	6
Ostrich	3	2	4	1	2	5	3	1	5	1	5	3	8	7	8
Pig	1	3	9	6	1	1	6	5	3	3	1	1	5	5	5
Pigeon	7	2	4	5	2	5	5	1	5	1	2	1	8	4	1
Rabbit	6	3	1	6	0	4	6	2	3	3	1	1	5	3	5
Racoon	6	3	7	10	4	1	6	2	3	3	3	1	4	3	5
Rhinoceros	4	3	5	6	6	4	3	5	4	4	5	1	7	7	2
Snake	8	1	3	9	6	3	2	3	1	4	4	2	2	1	3
Sparrow	7	2	4	5	2	5	5	1	5	2	2	3	8	4	1
Tiger	5	3	7	4	0	1	3	5	4	3	3	1	6	2	2
Tortoise	8	1	3	9	3	3	2	3	1	5	4	2	1	1	3
Turkey	7	2	4	1	2	5	7	1	5	1	1	3	8	4	1

Alligator

Bear

Camel

Cat

Cheetah

Chicken

Chimpanzee

Cow

Crane

Crow

Dog

Duck

Elephant

Fox

Frog

Giraffe

Goat



Hawk

Hippopotamus

Horse

Leopard

Lion

Lizard

Monkey

Ostrich

Pig

Pigeon

Rabbit

Raccoon

Rhinoceros

Snake

Sparrow

Tiger

Tortoise

Turkey





**Alligator**



**Bear**



**Camel**



**Cat**



**Cheetah**



**Chicken**



**Cow**



**Crane**



**Chimpanzee**



**Crow**



**Dog**



**Duck**



**Elephant**



**Fox**



**Frog**



**Giraffe**



**Goat**



**Hawk**



**Hippopotamus**



**Horse**



**Leopard**



**Lion**



**Lizard**



**Ostrich**



**Pig**



**Pigeon**



**Rabbit**



**Raccoon**



**Rhinoceros**



**Snake**



**Sparrow**



**Tiger**



**Tortoise**



**Turkey**

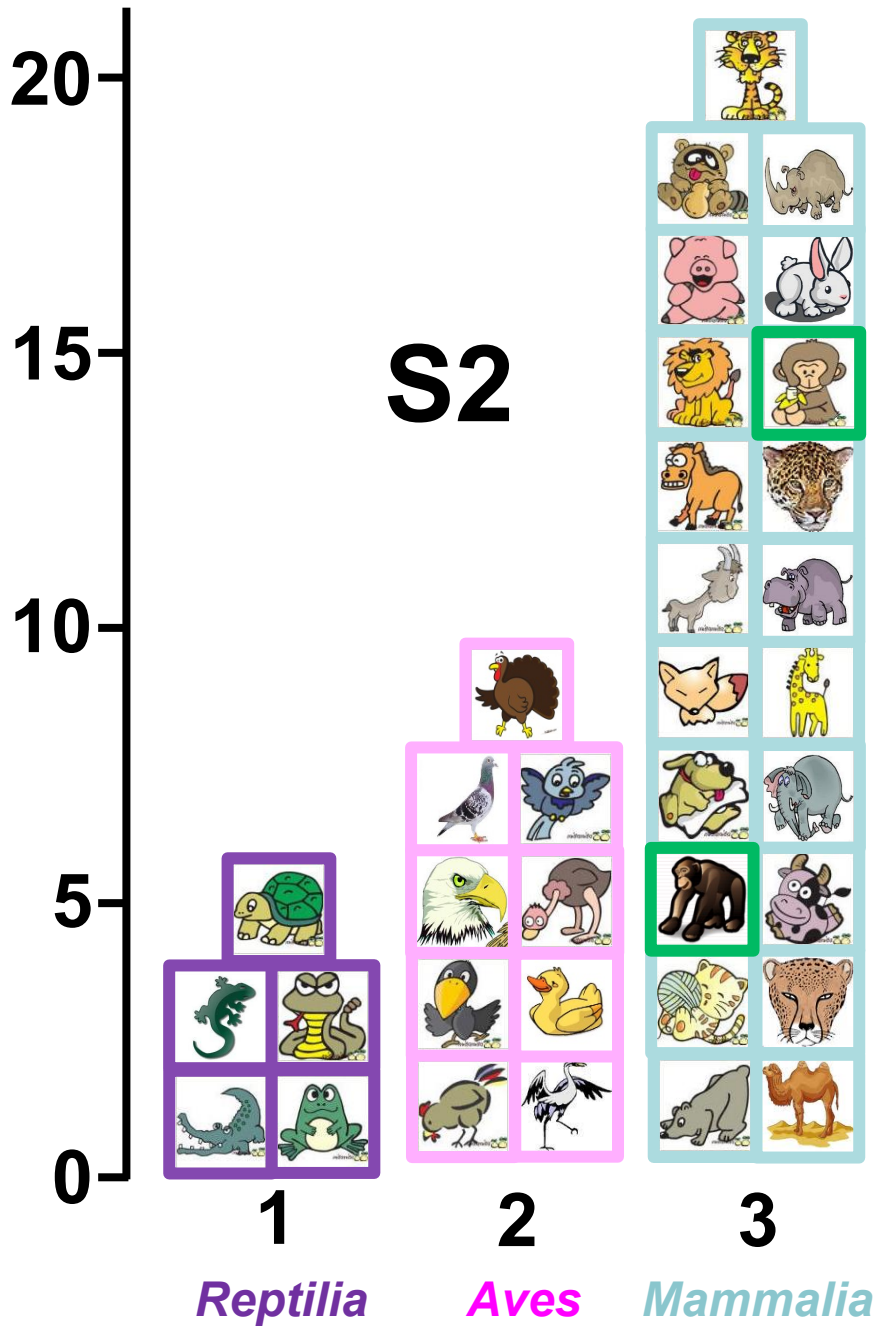


**Monkey**

# Uni-variate Display

Bar-Chart

Pie-Chart



# Bi-variate Display

# Mosaic Display

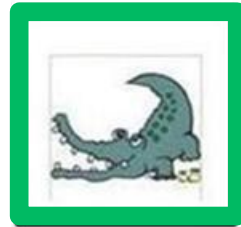
S12

4. Primate?

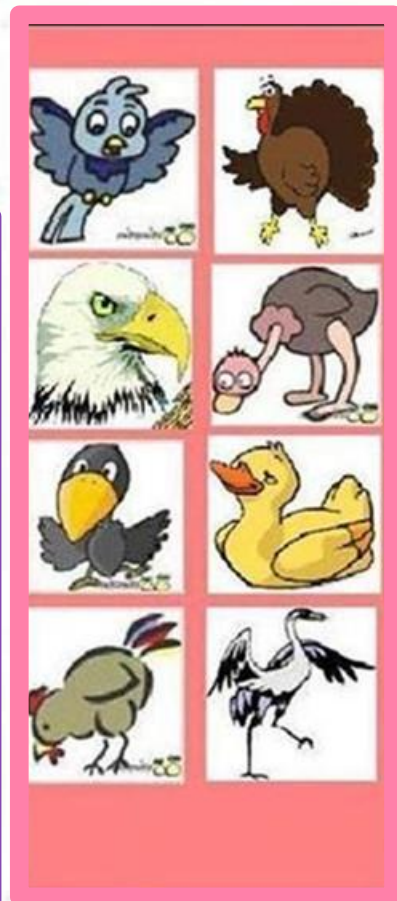
3. Bird

2. Reptile

1. Mammal



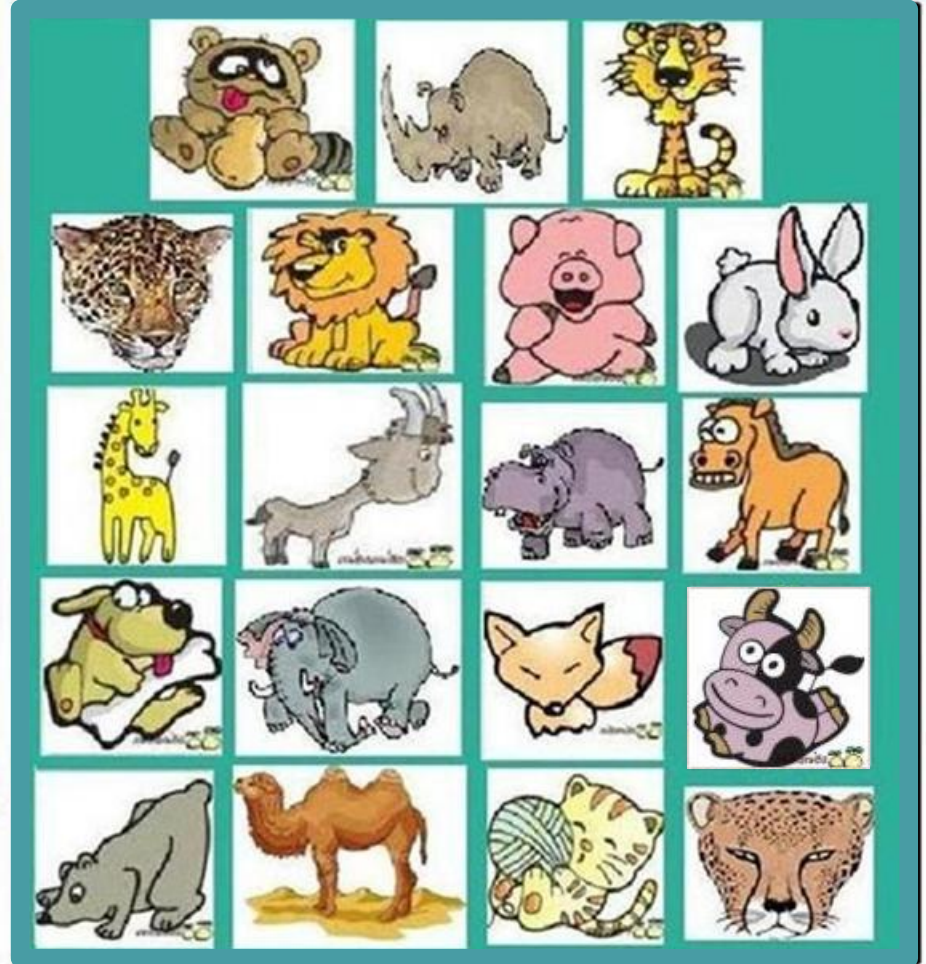
1. Reptile



2. Bird



S2

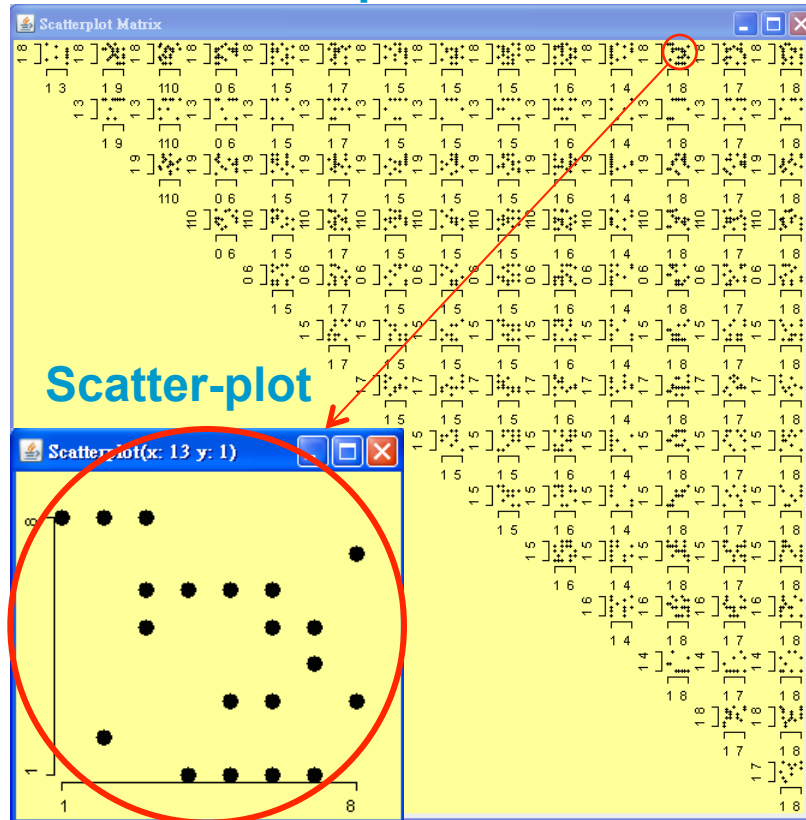


3. Mammal

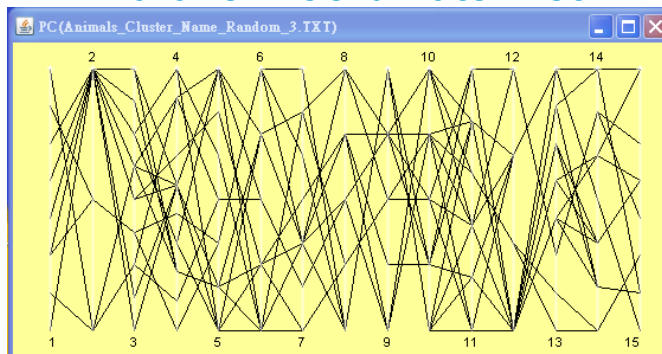


# Conventional statistical visualization for this data

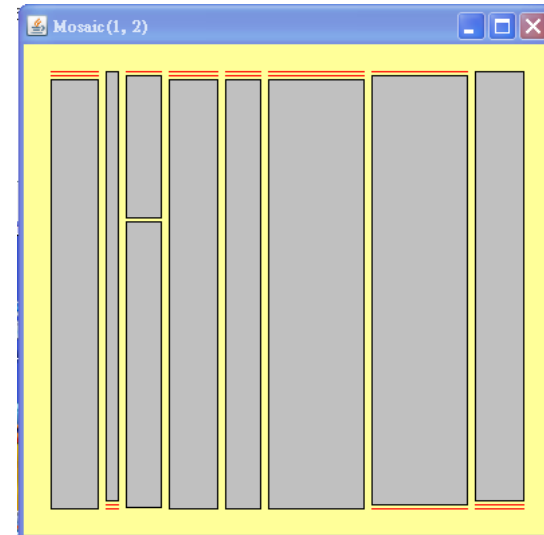
## Scatter-plot Matrix



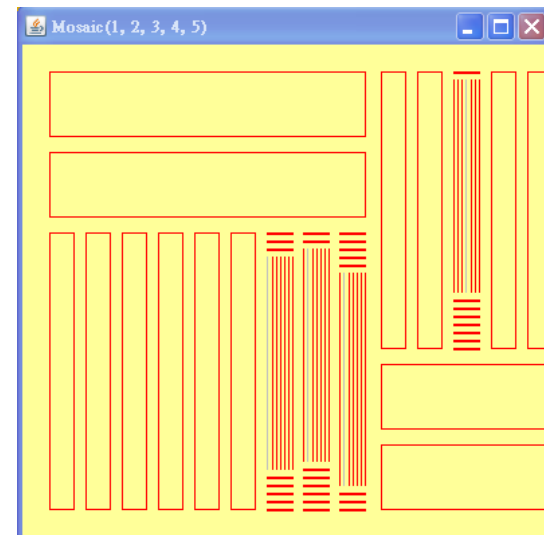
## Parallel Coordinate Plot



## 2D Mosaic Display



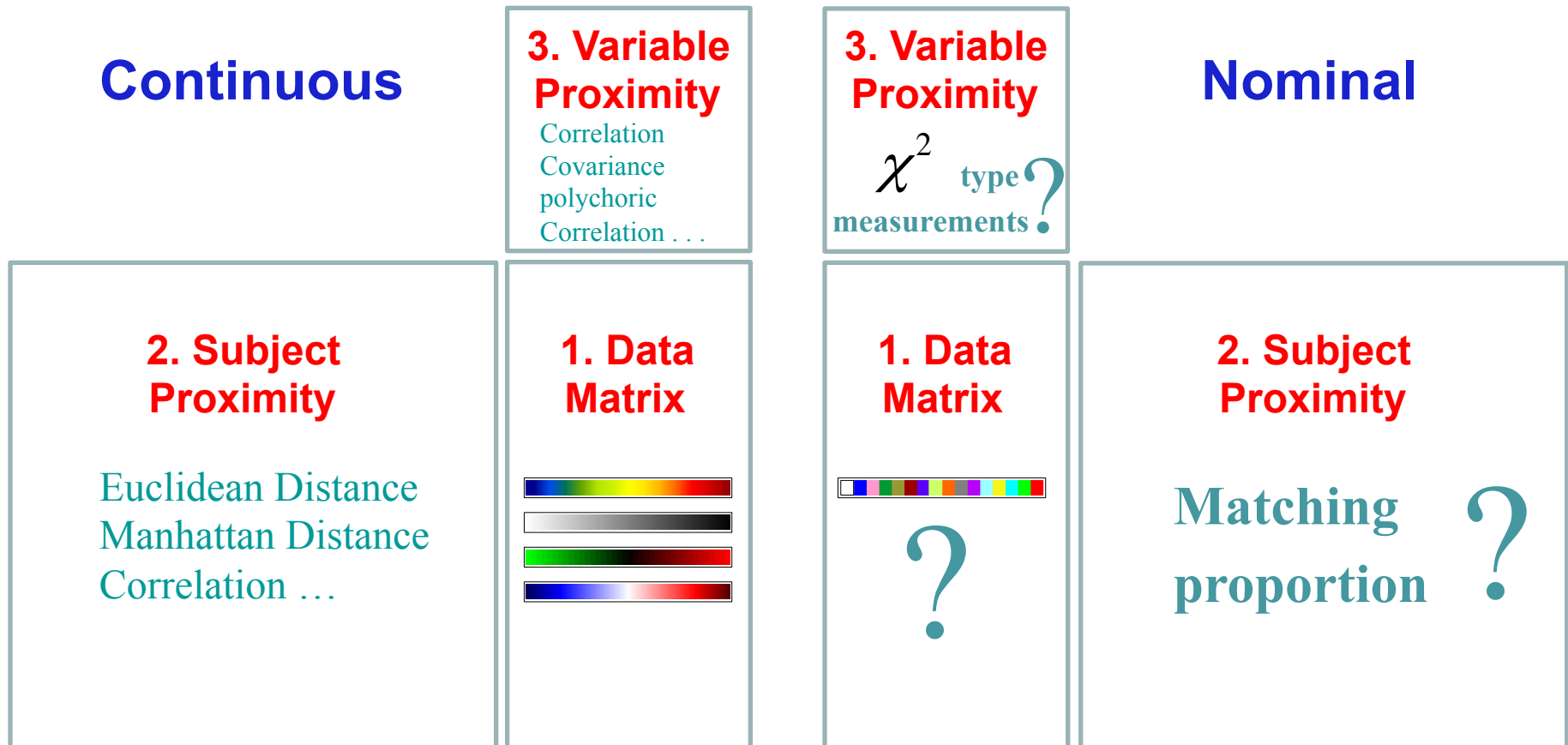
## 5D Mosaic Display





# Approaching Statistics & Statistical Approach

Essential elements in a GAP MV procedure?



Is there a natural way of taking care of all 3 problems?

## ***Statistical Approach: Dual Scaling/Homogeneity Analysis/MCA***

### Early Works:

Richardson & Kuder (1933)  
Hirschfeld (1935)  
Horst (1935)  
Edgerton & Kolbe (1936)  
Hotelling (1936)  
Wilks (1938)  
Fisher (1940)  
Maung (1941)  
Guttman (1941, 1946)  
Hayashi (1950, 1952)  
Bock (1956, 1960)

### Major Groups:

Hayashi school (1950-)  
Benzecri school (1960-)  
Gifi group (1967-)  
de Leeuw & others  
Toronto group (1969-)  
Nishisato & others

**PCA for  
categorical  
Data**

### Aliases:

Method of Reciprocal Average  
Simultaneous Linear Regression  
Appropriate Scoring, Additive Scoring  
Hayashi's Theory of Quantification  
Principal Component Analysis of  
Qualitative Data  
Optimal Scaling  
Analyse Factorielle des  
Correspondances  
Homogeneity Analysis  
Correspondence Analysis  
Correspondence Factor Analysis  
Basic Structure Content Scaling  
Dual Scaling  
Descriptive Multivariate Analysis  
Nonlinear Multivariate Analysis

Gifi, A (1990) Nonlinear Multivariate Analysis

Michailidis G, and de Leeuw, J. (1998), "The Gifi System of Descriptive Multivariate Analysis,"  
*Statistical Science*, **13**, 307-336.

Nishisato, S. (1996), "Gleaning in the field of dual scaling," *Psychometrika*, **61**, 559-599.68

Nishisato, S. (2006), Multidimensional Nonlinear Descriptive Analysis

# Mammals Dentition Example

The data for this example are taken from Hartigan (1975) (also discussed in Michailidis and De Leeuw, 1999). Dental characteristics are used in the classification of 66 different kinds of mammals. Mammals' teeth are divided into four groups: incisors, canines, premolars, and molars.

Description for Variables
<p>TI: Top incisors;  1: 0 incisors, 2: 1 incisors,  3: 2 incisors, 4: 3 or more incisors</p> <p>BI: Bottom incisors;  1: 0 incisors, 2: 1 incisors,  3: 2 incisors, 4: 3 incisors  5: 4 incisors</p> <p>TC: Top canine;  1: 0 canines, 2: 1 canines,</p> <p>BC: Bottom canine;  1: 0 canines, 2: 1 canines,</p> <p>TP: Top premolar;  1: 0 premolars, 2: 1 premolars,  3: 2 premolars, 4: 3 premolars  5: 4 premolars</p> <p>BP: Bottom premolar;  1: 0 premolars, 2: 1 premolars,  3: 2 premolars, 4: 3 premolars  5: 4 premolars</p> <p>TM: Top molar;  1: 0-2 molars, 2: 3 or more molars,</p> <p>BM: Bottom molar;  1: 0-2 molars, 2: 3 or more molars</p>

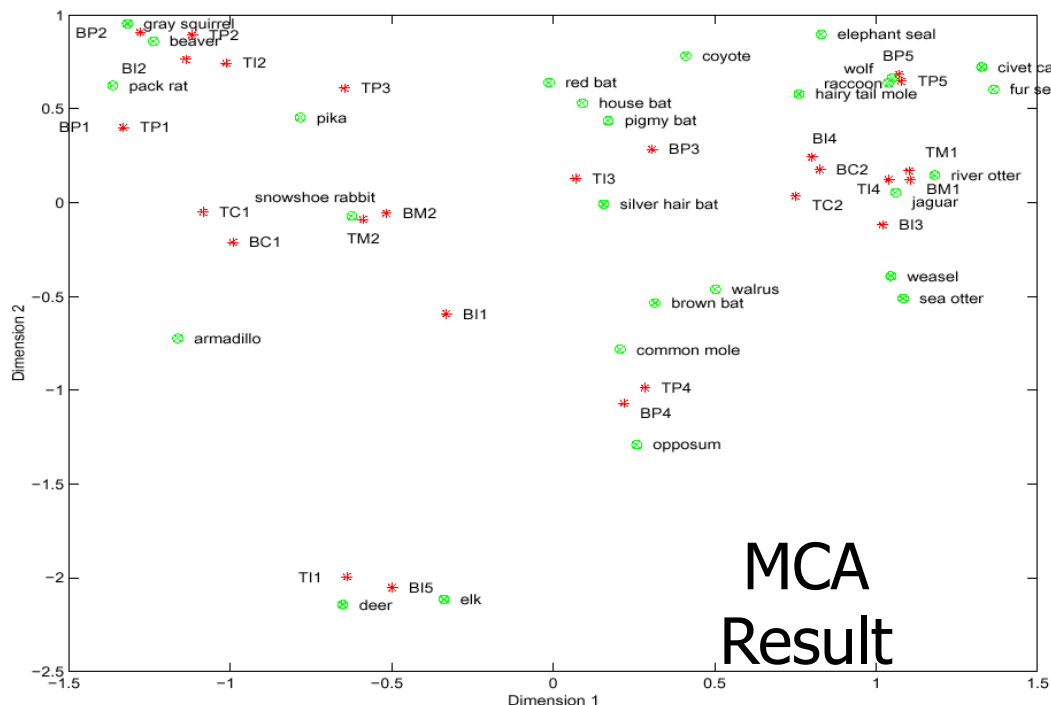
T	B	T	B	T	B	T	B	
I	I	C	C	P	P	M	M	
4	5	2	2	4	4	2	2	Opposum
4	4	2	2	5	5	2	2	Hairy-Tail-Mole
4	3	2	1	4	4	2	2	Common-Mole
4	4	2	2	5	5	2	2	Star-Nose-Mole
3	4	2	2	4	4	2	2	Brown-Bat
3	4	2	2	3	4	2	2	Silver-Hair-Bat
3	4	2	2	3	3	2	2	Pigmy-Bat
3	4	2	2	2	3	2	2	House-Bat
2	4	2	2	3	3	2	2	Red-Bat
2	4	2	2	3	3	2	2	Hoary-Bat
3	4	2	2	3	4	2	2	Lump-Nose-Bat
1	1	1	1	1	1	2	2	Armadillo
3	2	1	1	3	3	2	2	Pika
3	2	1	1	4	3	2	2	Snowshoe-Rabbit
2	2	1	1	3	2	2	2	Beaver
2	2	1	1	3	2	2	2	Marmot
2	2	1	1	3	2	2	2	Groundhog
2	2	1	1	3	2	2	2	Prairie-Dog
2	2	1	1	3	2	2	2	Ground-Squirrel
2	2	1	1	3	2	2	2	Chipmunk
2	2	1	1	2	2	2	2	Gray-Squirrel
2	2	1	1	2	2	2	2	Fox-Squirrel
2	2	1	1	2	2	2	2	Pocket-Gopher
2	2	1	1	2	2	2	2	Kangaroo-Rat
2	2	1	1	1	1	2	2	Pack-Rat
2	2	1	1	1	1	2	2	Field-Mouse
2	2	1	1	1	1	2	2	Muskrat
2	2	1	1	1	1	2	2	Black-Rat
2	2	1	1	1	1	2	2	House-Mouse
2	2	1	1	2	2	2	2	Porcupine
2	2	1	1	2	2	2	2	Guinea-Pig
2	4	2	2	5	5	2	2	Coyote
4	4	2	2	5	5	1	2	Wolf
4	4	2	2	5	5	1	2	Fox
4	4	2	2	5	5	1	2	Bear
4	4	2	2	5	5	1	1	Civet-Cat
4	4	2	2	5	5	2	1	Raccoon
4	4	2	2	5	5	1	1	Marten
4	4	2	2	5	5	1	1	Fisher
4	4	2	2	4	4	1	1	Weasel
4	4	2	2	4	4	1	1	Mink
4	4	2	2	4	4	1	1	Ferrer
4	4	2	2	5	5	1	1	Wolverine
4	4	2	2	4	4	1	1	Badger
4	4	2	2	4	4	1	1	Skunk
4	4	2	2	5	4	1	1	River-Otter
4	3	2	2	4	4	1	1	Sea-Otter
4	4	2	2	4	3	1	1	Jaguar
4	4	2	2	4	3	1	1	Ocelot
4	4	2	2	4	3	1	1	Cougar
4	4	2	2	4	3	1	1	Lynx
4	3	2	2	5	5	1	1	Fur-Seal
4	3	2	2	5	5	1	1	Sea-Lion
2	1	2	2	4	4	1	1	Walrus
4	3	2	2	4	4	1	1	Grey-Seal
3	2	2	2	5	5	1	1	Elephant-Seal
3	4	2	2	4	4	2	2	Peccary
1	5	2	1	4	4	2	2	Elk
1	5	1	1	4	4	2	2	Deer
1	5	1	1	4	4	2	2	Moose
1	5	2	1	4	4	2	2	Reindeer
1	5	1	1	4	4	2	2	Antelope
1	5	1	1	4	4	2	2	Bison
1	5	1	1	4	4	2	2	Mountain-Goat
1	5	1	1	4	4	2	2	Muskox
1	5	1	1	4	4	2	2	Mountain-Sheep

# Multiple Correspondence Analysis (MCA)

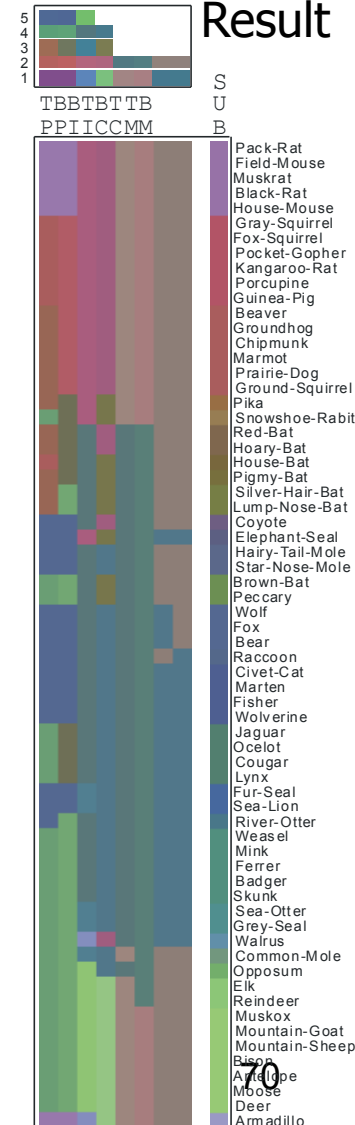
TB TBTB TBTB  
I I C C P P M M

4 5 2 2 4 4 2 2	Opposum
4 4 2 2 5 5 2 2	Hairy-Tail-Mole
4 3 2 1 4 4 2 2	Common-Mole
4 4 2 2 5 5 2 2	Star-Nose-Mole
3 4 2 2 4 4 2 2	Brown-Bat
3 4 2 2 3 4 2 2	Silver-Hair-Bat
3 4 2 2 3 3 2 2	Pigmy-Bat
3 4 2 2 3 3 2 2	House-Bat
2 4 2 2 3 3 2 2	Red-Bat
2 4 2 2 3 3 2 2	Hoary-Bat
3 4 2 2 3 4 2 2	Lump-Nose-Bat
1 1 1 1 1 1 2 2	Armadillo
3 2 1 1 3 3 2 2	Pika
3 2 1 1 4 3 2 2	Snowshoe-Rabbit
2 2 1 1 3 2 2 2	Beaver
2 2 1 1 3 2 2 2	Marmot
2 2 1 1 3 2 2 2	Groundhog
2 2 1 1 3 2 2 2	Prairie-Dog
2 2 1 1 3 2 2 2	Ground-Squirrel
2 2 1 1 3 2 2 2	Chipmunk
2 2 1 1 2 2 2 2	Gray-Squirrel
2 2 1 1 2 2 2 2	Fox-Squirrel
2 2 1 1 2 2 2 2	Pocket-Gopher
2 2 1 1 2 2 2 2	Kangaroo-Rat
2 2 1 1 1 1 2 2	Pack-Rat
2 2 1 1 1 1 2 2	Field-Mouse
2 2 1 1 1 1 2 2	Muskrat
2 2 1 1 1 1 2 2	Black-Rat
2 2 1 1 1 1 2 2	House-Mouse
2 2 1 1 2 2 2 2	Porcupine
2 2 1 1 2 2 2 2	Guinea-Pig
2 4 2 2 5 5 2 2	Coyote
4 4 2 2 5 5 1 2	Wolf
4 4 2 2 5 5 1 2	Fox
4 4 2 2 5 5 1 2	Bear
4 4 2 2 5 5 1 1	Civet-Cat
4 4 2 2 5 5 2 1	Raccoon
4 4 2 2 5 5 1 1	Marten
4 4 2 2 5 5 1 1	Fisher
4 4 2 2 4 4 1 1	Weasel
4 4 2 2 4 4 1 1	Mink
4 4 2 2 4 4 1 1	Ferrer
4 4 2 2 5 5 1 1	Wolverine
4 4 2 2 4 4 1 1	Badger
4 4 2 2 4 4 1 1	Skunk
4 4 2 2 5 4 1 1	River-Otter
4 3 2 2 4 4 1 1	Sea-Otter
4 4 2 2 4 3 1 1	Jaguar
4 4 2 2 4 3 1 1	Ocelot
4 4 2 2 4 3 1 1	Cougar
4 4 2 2 4 3 1 1	Lynx
4 3 2 2 5 5 1 1	Fur-Seal
4 3 2 2 5 5 1 1	Sea-Lion
2 1 2 2 4 4 1 1	Walrus
4 3 2 2 4 4 1 1	Grey-Seal
3 2 2 2 5 5 1 1	Elephant-Seal
3 4 2 2 4 4 2 2	Peccary
1 5 2 1 4 4 2 2	Elk
1 5 1 1 4 4 2 2	Deer
1 5 1 1 4 4 2 2	Moose
1 5 2 1 4 4 2 2	Reindeer
1 5 1 1 4 4 2 2	Antelope
1 5 1 1 4 4 2 2	Bison
1 5 1 1 4 4 2 2	Mountain-Goat
1 5 1 1 4 4 2 2	Muskox
1 5 1 1 4 4 2 2	Mountain-Sheep

Description for Variables
TI: Top incisors; 1: 0 incisors, 2: 1 incisors, 3: 2 incisors, 4: 3 or more incisors
BI: Bottom incisors; 1: 0 incisors, 2: 1 incisors, 3: 2 incisors, 4: 3 incisors 5: 4 incisors
TC: Top canine; 1: 0 canines, 2: 1 canines, 3: 2 canines, 4: 3 canines 5: 4 canines
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TM: Top molar; 1: 0-2 molars, 2: 3 or more molars, 3: 2 molars, 4: 3 molars 5: 4 molars
BM: Bottom molar; 1: 0-2 molars, 2: 3 or more molars, 3: 2 molars, 4: 3 molars 5: 4 molars



CateGAP  
Result





# UCI



## Machine Learning Repository



Each species is identified as definitely **edible**, definitely **poisonous**, or of **unknown** edibility and not recommended. This latter class was **combined with the poisonous** one.

### Mushroom Data Set

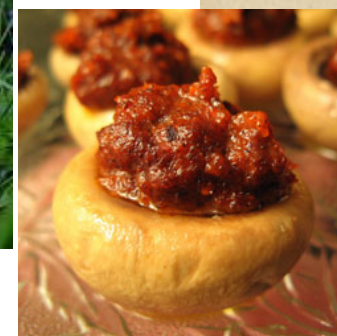
Download: [Data Folder](#), [Data Set Description](#)

**Abstract:** From Audobon Society Field Guide; mushrooms described in terms of physical characteristics; classification: poisonous or edible

<b>Data Set Characteristics:</b>	Multivariate	<b>Number of Instances:</b>	8124	<b>Area:</b>	Life
<b>Attribute Characteristics:</b>	Categorical	<b>Number of Attributes:</b>	22	<b>Date Donated</b>	1987 04/27
<b>Associated Tasks:</b>	Classification	<b>Missing Values?</b>	Yes	<b>Number of Web Hits:</b>	48017

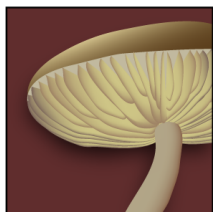
Origin:  
Mushroom records drawn from The Audubon Society Field Guide to North American Mushrooms (1981). G. H. Lincoff (Pres.), New York: Alfred A. Knopf  
Donor: Jeff Schlimmer ([Jeffrey.Schlimmer '@' a.gp.cs.cmu.edu](mailto:Jeffrey.Schlimmer@a.gp.cs.cmu.edu))





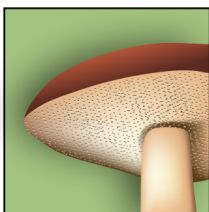


## Spore-bearing surface under cap



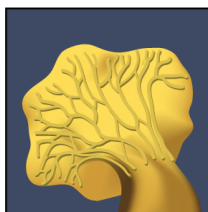
### Gills:

wide and thin sheet-like plates radiating from stem



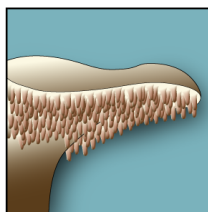
### Pores:

many small tubes ending in a spongy surface



### Ridges:

short, blunt elevated lines on stem and under cap



### Teeth:

many small finger-like projections

## Gill attachment



**Adnate** - gills widely attached widely to stem



**Adnexed** - gills attached narrowly to stem



**Decurrent** - gills running down stem for some length



**Emarginate** - gills notched immediately before attaching to stem



**Free** - gills not attached to stem



**Seceding** - gills attached, but breaking away from stem at margin (often older specimens)



**Sinuate** - gills smoothly notched and running briefly down stem



**Subdecurrent** - gills running briefly down stem

## Cap morphology



**Campanulate** - bell-shaped



**Conical** - triangular



**Convex** - outwardly rounded



**Depressed** - with a low central region



**Flat** - with top of uniform height



**Infundibuliform** - deeply depressed, funnel-shaped



**Ovate** - shaped like half an egg



**Umbilicate** - with a small, deep depression



**Umbonate** - with a central bump or knob

## LAMELLAE TUBE ATTACHMENT



Free



Adnexed



Sinuate



Adnate



Narrowly adnate



Sub-decurrent



Adnate with a decurrent tooth



Strongly decurrent



Arcuate



Ovoid



Globose



Ellipsoidal



Cylindrical



Hemispherical



Convex



Broadly convex

## PILEUS MARGINS IN SECTION



Recurved



Incurved/decurved



Inrolled/involute



Plane/applanate



Depressed



Plane



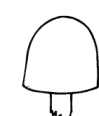
Straight



Umbonate



Conic



Bell-shaped/  
campanulate



Funnel shaped/  
sunken

## SHAPES OF THE STIPE



Equal



Clavate/  
club shaped



Ventricose/  
swollen



Smooth/entire



Crenate/scalloped



Striate



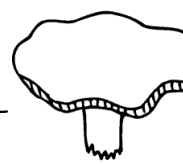
Bulbous



Fusoid



Radicate



Wavy



Appendiculate



Rimose/cracked

## MARGINS OF THE PILEUS

## Attribute Information:

1. **cap-shape**: bell=b,conical=c,convex=x,flat=f, knobbed=k,sunken=s
2. **cap-surface**: fibrous=f,grooves=g,scaly=y,smooth=s
3. **cap-color**: brown=n,buff=b,cinnamon=c,gray=g,green=r,  
pink=p,purple=u,red=e,white=w,yellow=y
4. **bruises?**: bruises=t,no=f
5. **odor**: almond=a,anise=l,creosote=c,fishy=y,foul=f, musty=m,none=n,pungent=p,spicy=s
6. **gill-attachment**: attached=a,descending=d,free=f,notched=n
7. **gill-spacing**: close=c,crowded=w,distant=d
8. **gill-size**: broad=b,narrow=n
9. **gill-color**: black=k,brown=n,buff=b,chocolate=h,gray=g,  
green=r,orange=o,pink=p,purple=u,red=e, white=w,yellow=y
10. **stalk-shape**: enlarging=e,tapering=t
11. **stalk-root**: bulbous=b,club=c,cup=u,equal=e, rhizomorphs=z,rooted=r,missing=?
12. **stalk-surface-above-ring**: fibrous=f,scaly=y,silky=k,smooth=s
13. **stalk-surface-below-ring**: fibrous=f,scaly=y,silky=k,smooth=s
14. **stalk-color-above-ring**: brown=n,buff=b,cinnamon=c,gray=g,orange=o,  
pink=p,red=e,white=w,yellow=y
15. **stalk-color-below-ring**: brown=n,buff=b,cinnamon=c,gray=g,orange=o,  
pink=p,red=e,white=w,yellow=y
16. **veil-type**: partial=p,universal=u
17. **veil-color**: brown=n,orange=o,white=w,yellow=y
18. **ring-number**: none=n,one=o,two=t
19. **ring-type**: cobwebby=c,evanescent=e,flaring=f,large=l,  
none=n,pendant=p,sheathing=s,zone=z
20. **spore-print-color**: black=k,brown=n,buff=b,chocolate=h,green=r,  
orange=o,purple=u,white=w,yellow=y
21. **population**: abundant=a,clustered=c,numerous=n, scattered=s,several=v,solitary=y
22. **habitat**: grasses=g,leaves=l,meadows=m,paths=p, urban=u,waste=w,woods=d



# Approaching Statistics & Statistical Approach

## Matrix Visualization with cartography links

THE WORLD FACTBOOK 2002

CIA



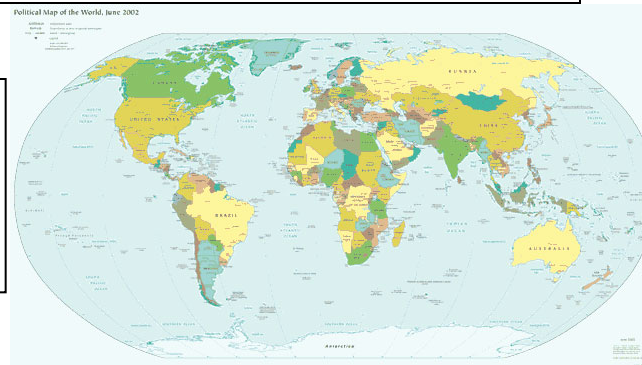
160 international organization

Data:

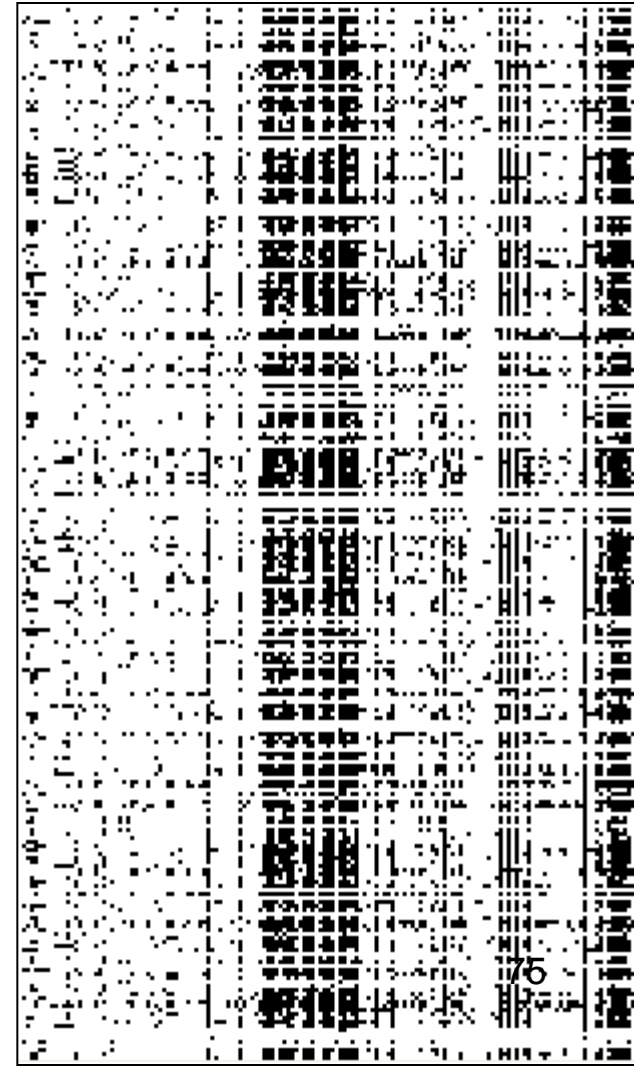
**160** international organization membership pattern (**variables**) for **230** countries/regions (**subjects**)

0. non-member ☐ 1. member ☒  
2. observer 3. associate member  
4. guest 5. dialogue partner

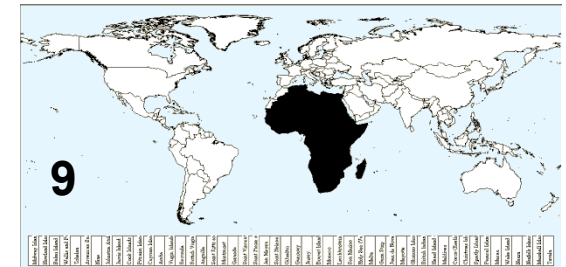
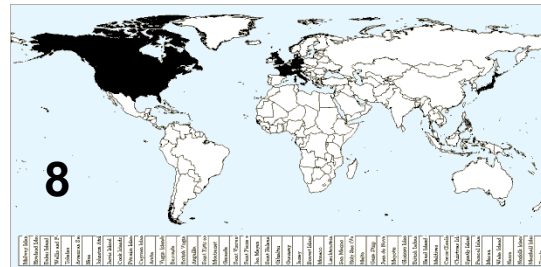
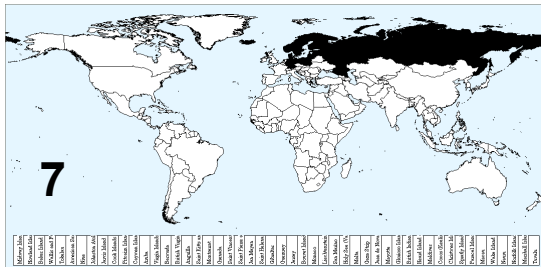
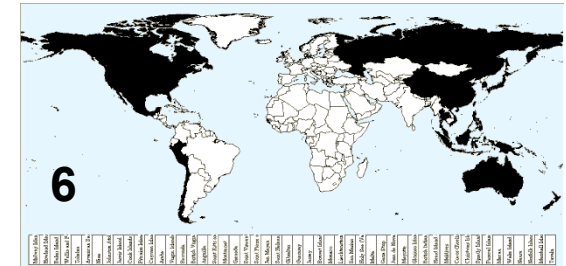
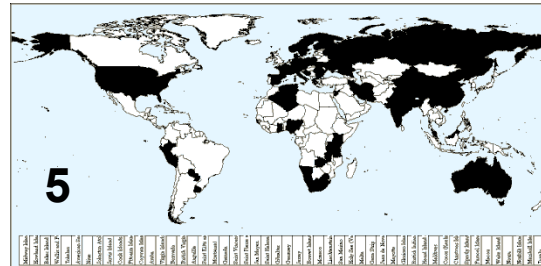
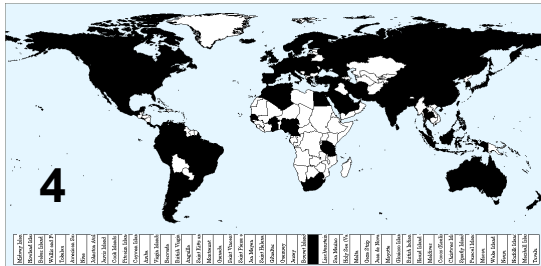
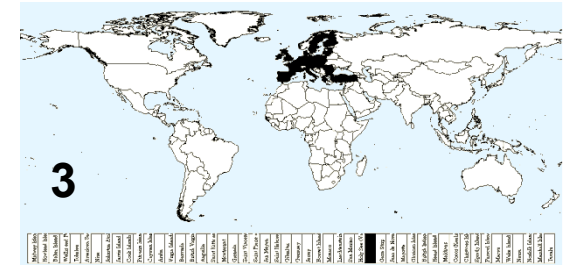
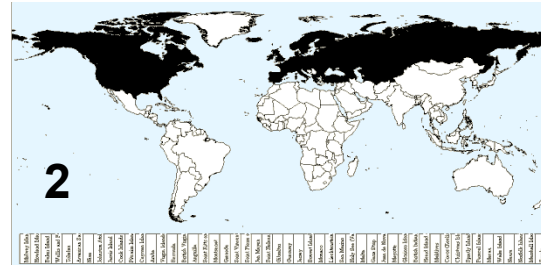
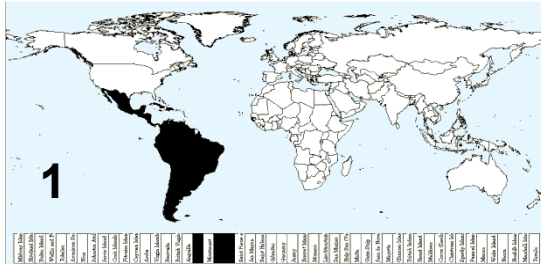
230  
countries  
(regions)



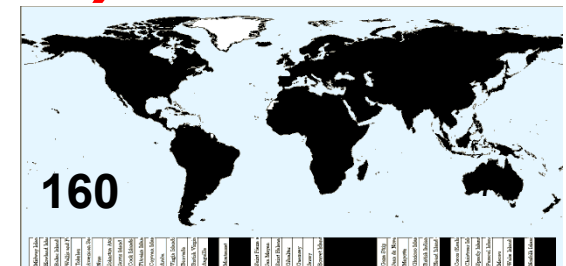
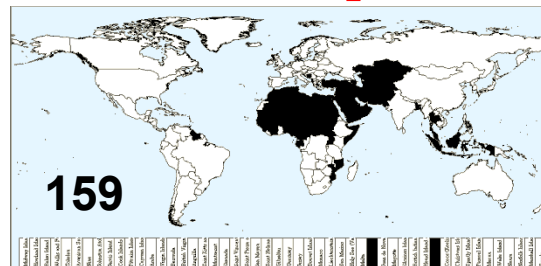
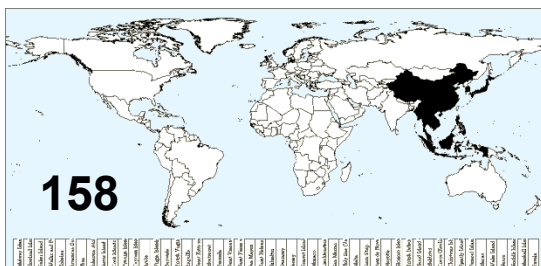
CIA Political Map of the World



# Draw one membership map for each organization (variable)?

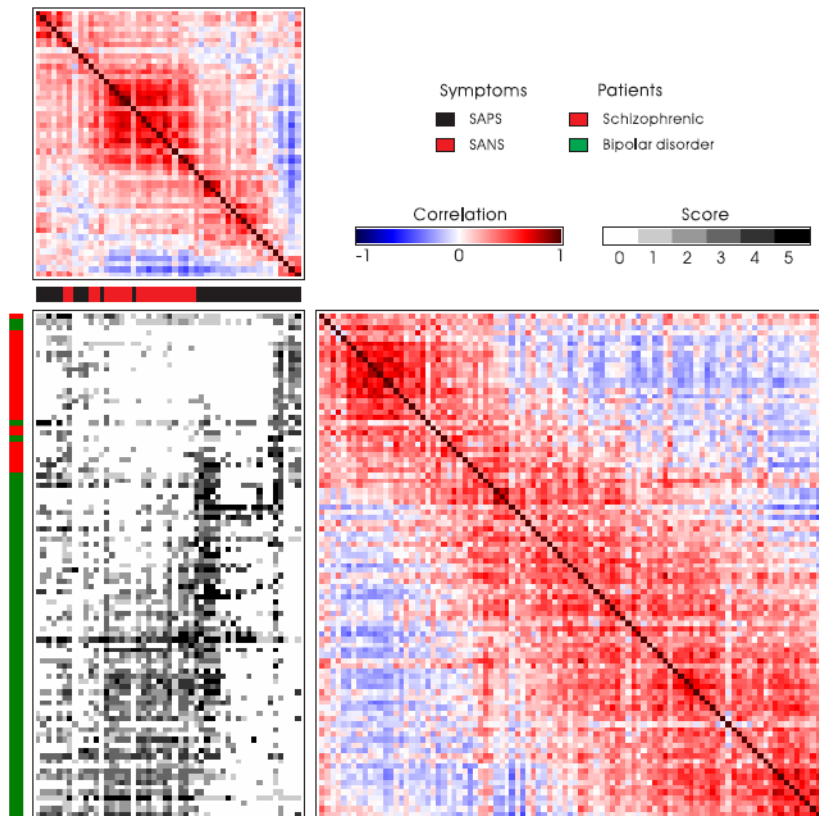


• • • 160 maps (?) • • •

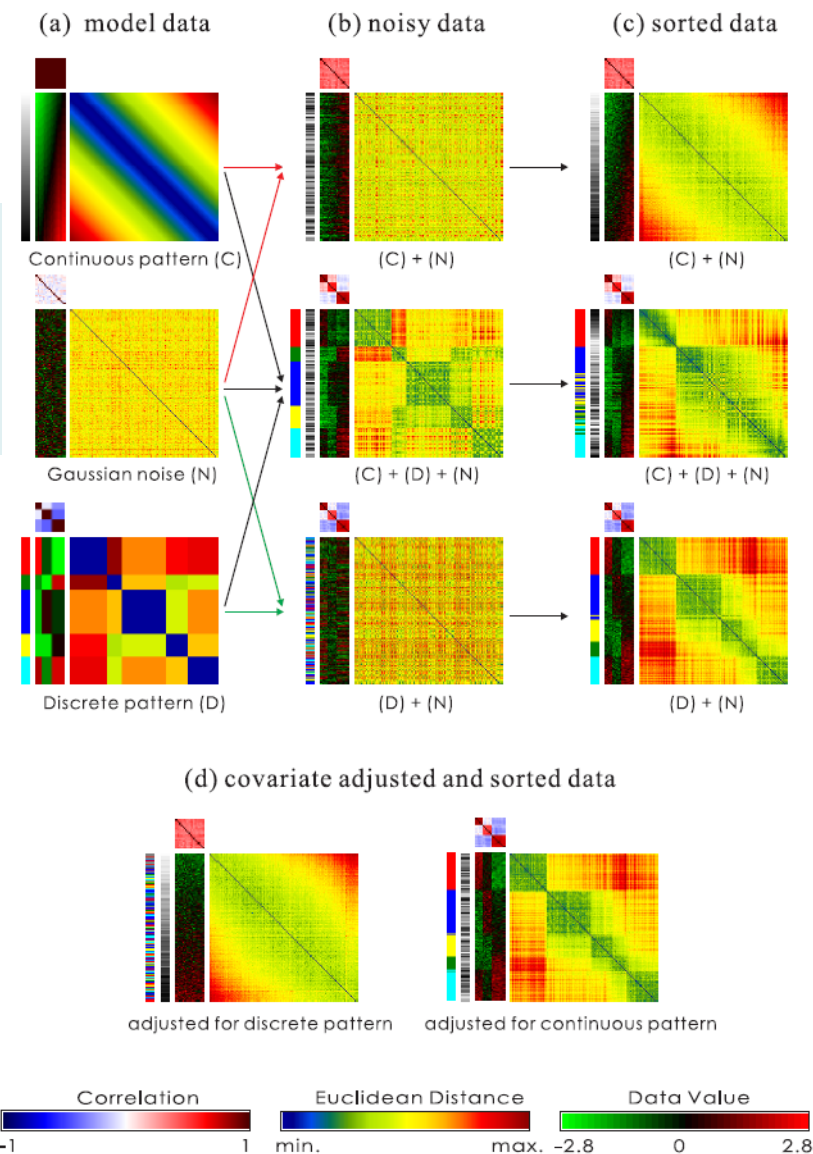


# Approaching Statistics & Statistical Approach

## Covariate-adjusted Matrix Visualization

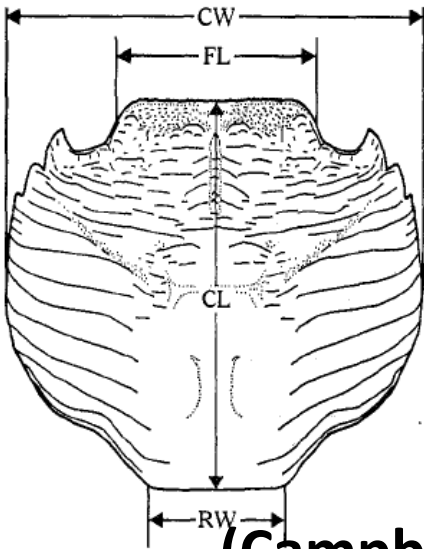


Psychosis disorder data  
with covariates



A simulation data;  
(a) the model data sets, (b) noisy data sets,  
(c) sorted data sets and  
(d) covariate adjusted and sorted data  
sets

# Morphological Measurements on Leptograpsus Crabs



The crabs data (Campbell and Mahon 1974) in MASS package in R has **200 crabs** with **5 morphological measurements** ( frontal lobe size (FL), rear width (RW), carapace length (CL), carapace width (CW), and body depth (BD)) on **50 crabs each of two species** ( **blue (B)** and **orange (O)** ) and both **sexes**.

sp	sex	FL	RW	CL	CW	BD
B	M	8.1	6.7	16.1	19	7
B	M	8.8	7.7	18.1	20.8	7.4
B	M	9.2	7.8	19	22.4	7.7
B	M	9.6	7.9	20.1	23.1	8.2
B	M	9.8	8	20.3	23	8.2
⋮						
B	F	7.2	6.5	14.7	17.1	6.1
B	F	9	8.5	19.3	22.7	7.7
B	F	9.1	8.1	18.5	21.6	7.7
B	F	9.1	8.2	19.2	22.2	7.7
B	F	9.5	8.2	19.6	22.4	7.8
⋮						
O	M	9.1	6.9	16.7	18.6	7.4
O	M	10.2	8.2	20.2	22.2	9
O	M	10.7	8.6	20.7	22.7	9.2
O	M	11.4	9	22.7	24.8	10.1
O	M	12.5	9.4	23.2	26	10.8
⋮						
O	F	11.4	9.2	21.7	24.1	9.7
O	F	12.5	10	24.1	27	10.9
O	F	12.6	11.5	25	28.1	11.5
O	F	12.9	11.2	25.8	29.1	11.9
O	F	14	11.9	27	31.4	12.6



# *Approaching Statistics & Statistical Approach*

## Matrix Visualization for **MANCOVA** modeling

Y. J. Tien, H. M. Wu,  
Y. S. Lee, and C. H. Chen (2010)

## MANOVA model examples

### Model 1: one-factor fixed effect model

$$Y_{(i)jk} = \mu_{(i)} + \rho_{(i)j} + \varepsilon_{(i)jk}$$

$$i = 1, 2, \dots, p \quad j = 1, 2, \dots, n \quad k = 1, 2, \dots, n_j \quad E(\varepsilon_{(i)jk}) = 0 \quad \text{Var}(\varepsilon_{(i)jk}) = \sigma^2$$

$$H_0 : \rho_{(i)j} = 0$$

**Overall**

**Individual variable**

**Post-hoc analysis (multiple comparison)**

# Extensions of MANOVA MV

*Approaching Statistics Statistical Approach*

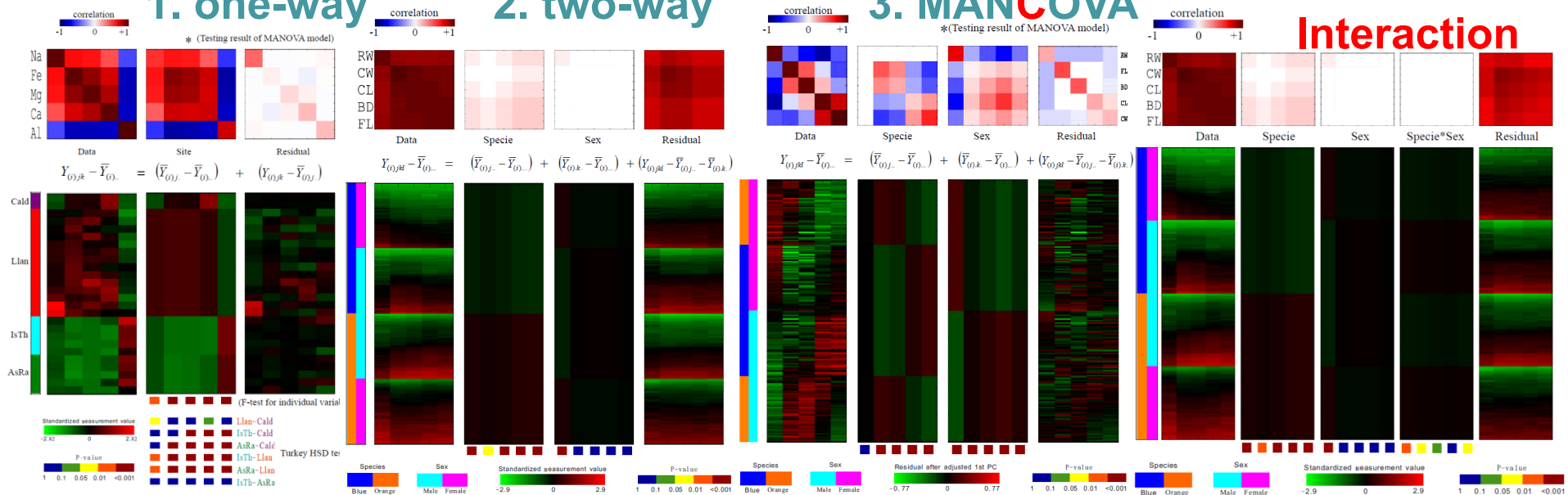
4. with

**Interaction**

1. one-way

2. two-way

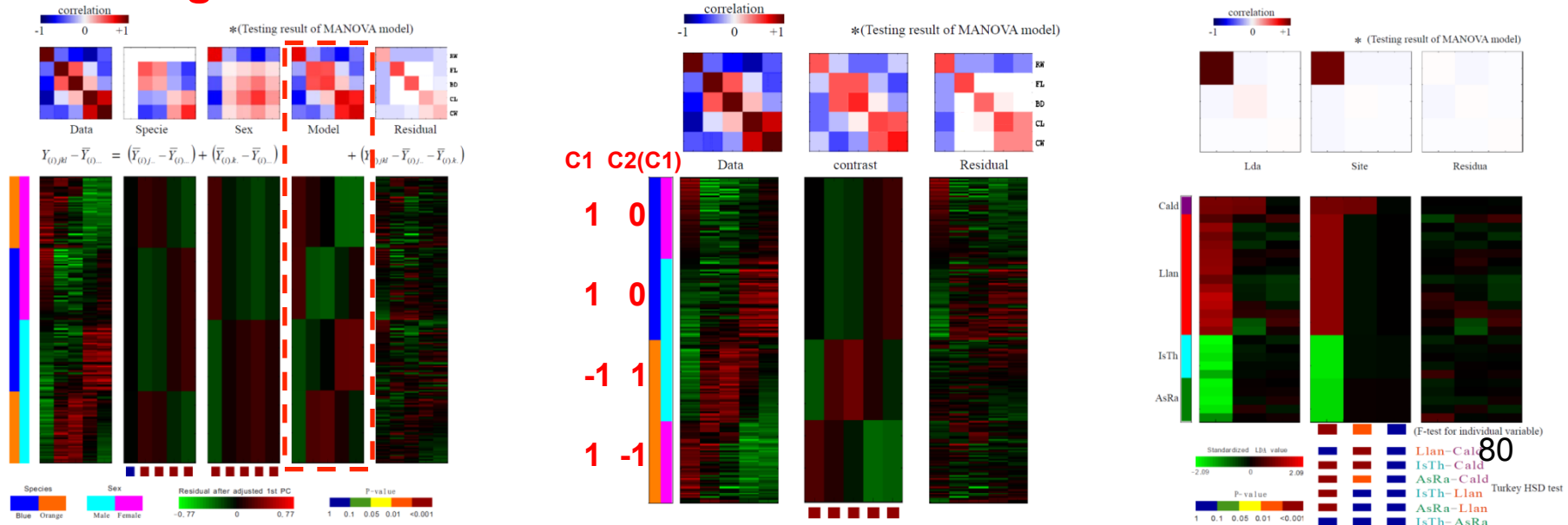
3. MANCOVA



5. with Regression Model

6. with Contrast

7. with Reduced rank



# Extension of Matrix Visualization for Symbolic Data (Analysis):

The GAP Approach (with Junji Nakano)

## Clustered (non-independent) Data

- 1. Hierarchical (multi-level) Data*
- 2. Genetic Familial Data*

## Huge Data Sets

- 1. Large  $n$*
- 2. Large  $p$*
- 3. Large  $n$  &  $p$*

## Other Types of Symbolic Data

# 1.1 Symbolic Data Analysis (SDA) and 1.2 Matrix Visualization (MV)

(a)  
Conventional  
Data  
Matrix

$$P^S_{n \times n}$$

Euclidean Distance  
Manhattan Distance  
⋮

Proximity for  
Conventional Variables

$$P^V_{p \times p}$$

Correlation  
Covariance  
⋮

$$\begin{matrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{matrix}$$

$$X_{n \times p}$$

Proximity for  
Symbolic (Interval) Variables

$$P^I_{p \times p}$$

Empirical Correlation  
Empirical Covariance  
⋮

$$\begin{matrix} [a_{11} \ b_{11}] & \cdots & [a_{1p} \ b_{1p}] \\ [a_{21} \ b_{21}] & \cdots & [a_{2p} \ b_{2p}] \\ \vdots & & \vdots \\ [a_{k1} \ b_{k1}] & \cdots & [a_{kp} \ b_{kp}] \end{matrix}$$

$$I_{k \times p}$$

$$\begin{matrix} [a_{k1} \ b_{k1}] & \cdots & [a_{kp} \ b_{kp}] \end{matrix}$$

(b)  
Symbolic  
Interval  
Data Table

$$P^C_{k \times k}$$

Gowda-Diday Distance  
Ichino-Yaguchi Distance  
Hausdorff Distance  
⋮

Proximity for  
Symbolic Concepts

Proximity for Individual Samples

**Fig. 1. Diagram for related conventional data matrix and symbolic (interval type) data table with their corresponding proximity matrices for samples/concepts and variables.**



## 2.1 Proximity matrix for interval (range) variables

The empirical covariance function between  $I_i$  and  $I_j$  is given by

$$\begin{aligned} Cov(I_i, I_j) &= \frac{1}{4k} \sum_{c=1}^k [(a_{ci} + b_{ci})(a_{cj} + b_{cj})] \\ &\quad - \frac{1}{4k^2} \left[ \sum_{c=1}^k (a_{ci} + b_{ci}) \right] \left[ \sum_{c=1}^k (a_{cj} + b_{cj}) \right]. \end{aligned}$$

The empirical correlation coefficient between  $I_i$  and  $I_j$  is given by

$$r(I_i, I_j) = \frac{Cov(I_i, I_j)}{S_{Z_i} S_{Z_j}},$$

$$S_{Z_i}^2 = \frac{1}{3k} \sum_{c=1}^k (b_{ci}^2 + b_{ci}a_{ci} + a_{ci}^2) - \frac{1}{4k^2} \left[ \sum_{c=1}^k (b_{ci} + a_{ci}) \right]^2.$$

Table 1. Distance measures for interval type symbolic data proposed in Billard and Diday (2006).

## 2.2 Proximity matrix for concepts with interval variables

Measure Name	Formula	Component detail
The Gowda-Diday distance (Gowda and Diday, 1991)	$\sum_{r=1}^p D(I_{ir}, I_{jr})$	$D(I_{ir}, I_{jr}) = \frac{ a_{ir} - a_{jr} }{ \max_c b_{cr} - \min_c a_{cr} }$ $+ \frac{  b_{ir} - a_{ir}  -  b_{jr} - a_{jr}  }{\max(b_{ir}, b_{jr}) - \min(a_{ir}, a_{jr})}$ $+ \frac{ b_{ir} - a_{ir}  +  b_{jr} - a_{jr}  - 2I_r}{\max(b_{ir}, b_{jr}) - \min(a_{ir}, a_{jr})}$ <p>where <math>I_r =  \max(a_{ir}, a_{jr}) - \min(b_{ir}, b_{jr}) </math></p>
The Ichino-Yaguchi distance (Ichino and Yaguchi, 1994)	$\sqrt[q]{\sum_{r=1}^p D(I_{ir}, I_{jr})^q}$	$D(I_{ir}, I_{jr}) =  [a_{ir}, b_{ir}] \cup [a_{jr}, b_{jr}]  -  [a_{ir}, b_{ir}] \cap [a_{jr}, b_{jr}] $ $+ \gamma(2 [a_{ir}, b_{ir}] \cap [a_{jr}, b_{jr}]  -  [a_{ir}, b_{ir}]  -  [a_{jr}, b_{jr}] )$ <p>where <math>0 \leq \gamma \leq 0.5</math></p>
The $L_1$ distance	$\sum_{r=1}^p D(I_{ir}, I_{jr})$	$D(I_{ir}, I_{jr}) = \left  \frac{a_{ir} + b_{ir}}{2} - \frac{a_{jr} + b_{jr}}{2} \right $
The $L_2$ distance (de Carvalho et al., 2006)	$\sum_{r=1}^p D(I_{ir}, I_{jr})$	$D(I_{ir}, I_{jr}) = \left( \frac{a_{ir} + b_{ir}}{2} - \frac{a_{jr} + b_{jr}}{2} \right)^2$
The City-Block distance (de Souza and de Carvalho, 2004)	$\sum_{r=1}^p D(I_{ir}, I_{jr})$	$D(I_{ir}, I_{jr}) =  a_{ir} - a_{jr}  +  b_{ir} - b_{jr} $
The Hausdorff distance (Chavent and Lechevallier, 2002)	$\sum_{r=1}^p D(I_{ir}, I_{jr})$	$D(I_{ir}, I_{jr}) = \max( a_{ir} - a_{jr} ,  b_{ir} - b_{jr} )$
The Euclidean Hausdorff distance	$\sqrt[2]{\sum_{r=1}^p D(I_{ir}, I_{jr})^2}$	$D(I_{ir}, I_{jr}) = \max( a_{ir} - a_{jr} ,  b_{ir} - b_{jr} )$
The normalized Euclidean Hausdorff distance	$\sqrt[2]{\sum_{r=1}^p \left[ \frac{D(I_{ir}, I_{jr})}{ R_r } \right]^2}$	$D(I_{ir}, I_{jr}) = \max( a_{ir} - a_{jr} ,  b_{ir} - b_{jr} )$ $H_r^2 = \frac{1}{2k^2} \sum_{i=1}^k \sum_{j=1}^k D(I_{ir}, I_{jr})^2$
The span normalized Euclidean Hausdorff distance	$\sqrt[2]{\sum_{r=1}^p \left[ \frac{D(I_{ir}, I_{jr})}{ R_r } \right]^2}$	$D(I_{ir}, I_{jr}) = \max( a_{ir} - a_{jr} ,  b_{ir} - b_{jr} )$ $ R_r  = \max_c b_{cr} - \min_c a_{cr}$

## 2.3 Color coding for interval (range) data table

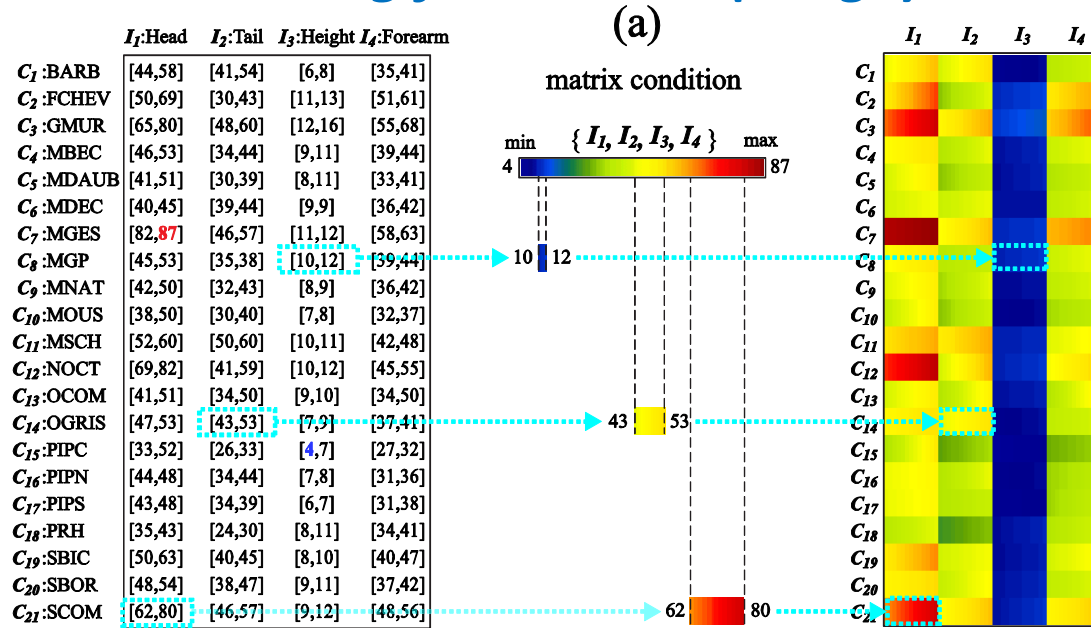
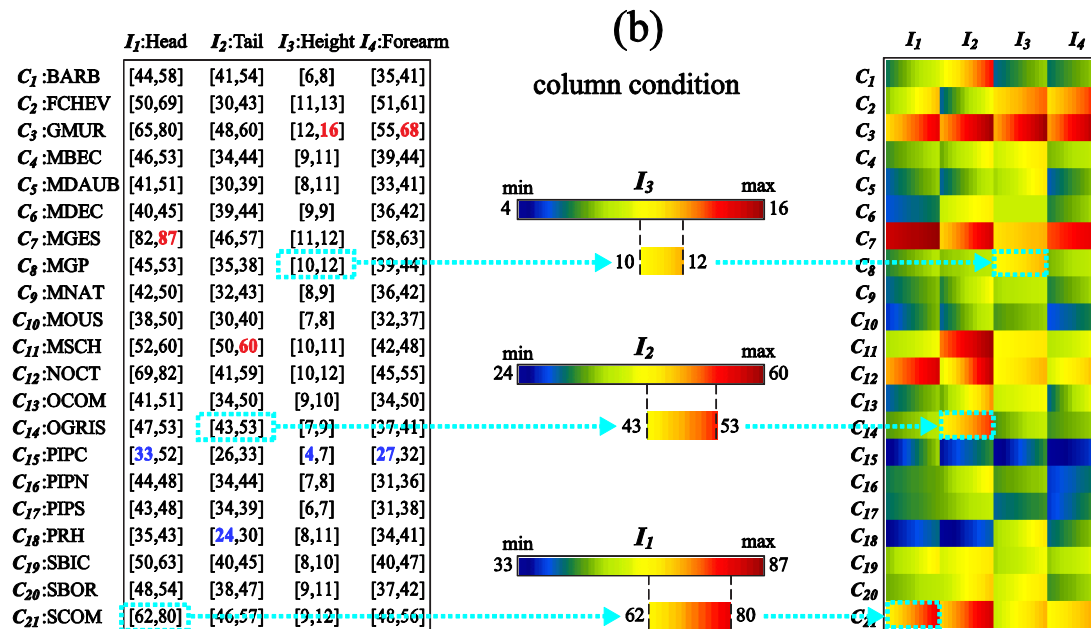


Fig. 2. Color-coding scheme for interval-valued SDA data using the Bats example.



(a) matrix condition;  
(b) column condition.

## Ongoing / Future Directions for GAP\_MV:

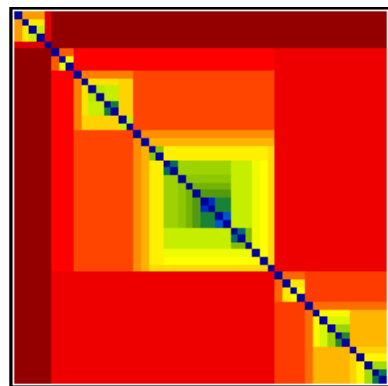
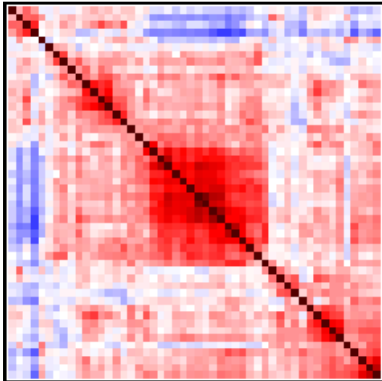
1. MV for diagnosing **proximity** matrix modeling
2. MV with **covariate** adjustment
3. MV with dependent (**clustered**) structure
4. MV for data with **missing** values
5. MV with **nonlinear** proximity measurement
6. MV for **longitudinal** multivariate data
7. MV for **multi-conditioned** multivariate data
8. MV with **dependent** variable
9. MV for **mixed** data
10. MV for **huge** data set
11. MV for **time** series data
12. MV for color-**blind** people



# Matrix Visualization

## Statistical Modeling of Proximity Matrix

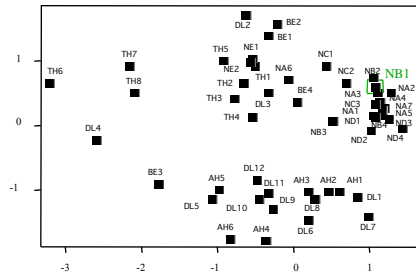
**Input**  
Proximity Matrix



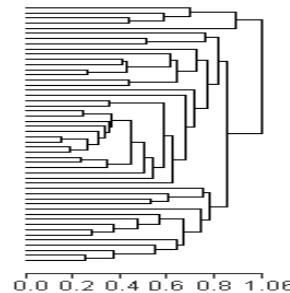
**Output**  
Distance Matrix

### Statistical Models:

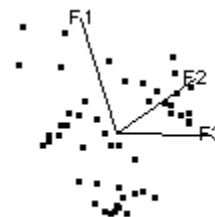
#### 1. Multidimensional Scaling (MDS)



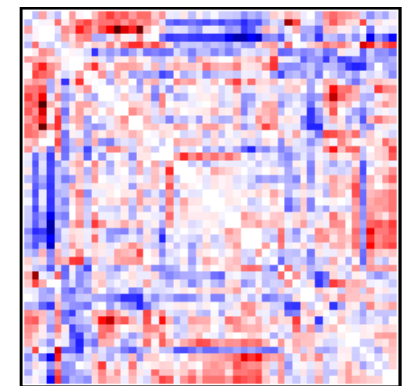
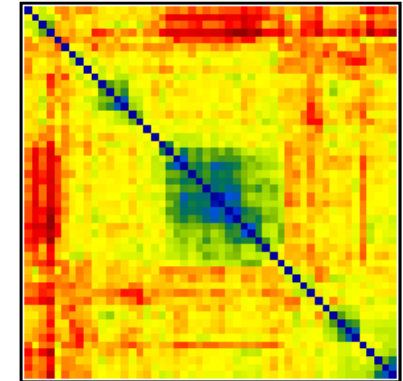
#### 2. Hierarchical Clustering Tree (HCT)



#### 3. Factor Analysis (FA)



**Transformed**  
Disparity Matrix



**Residual**  
Stress Matrix

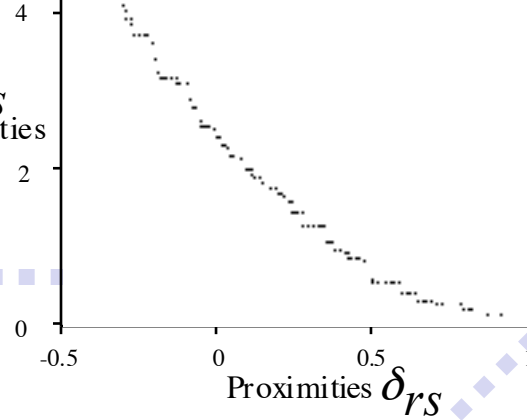
# MDS Modeling

## Classical MDS Diagnostic Plots

Input  
Proximity

(a). Plot of Transformation

$\hat{d}_{rs}$   
Disparities

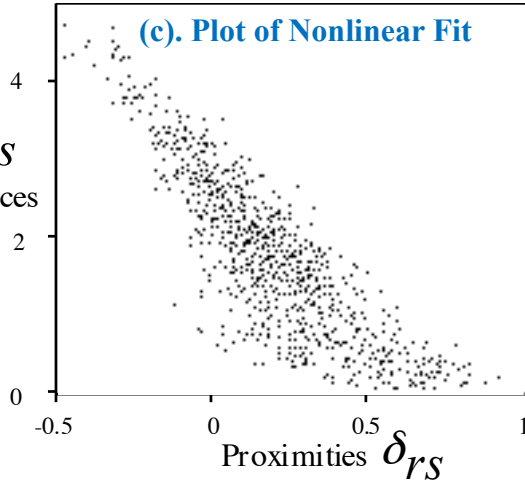


Transformed  
Disparity

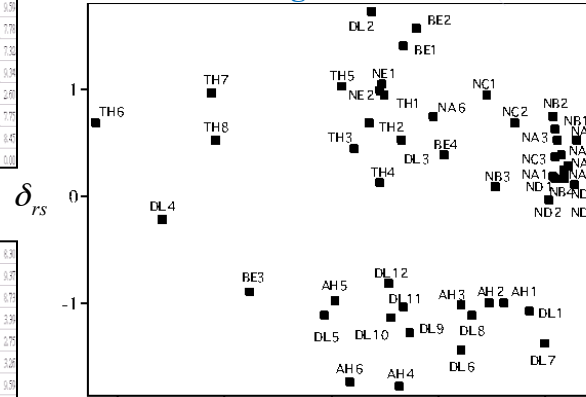
(c). Plot of Nonlinear Fit

$d_{rs}$

Distances

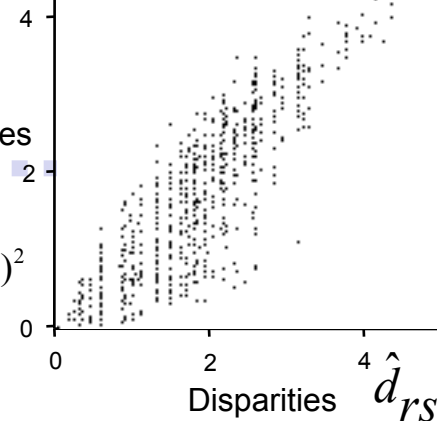


Configuration Plot



(b). Plot of Linear Fit

(b). Plot of Linear Fit



Residual  
Stress

## Classical MDS Diagnostic Indices

Output  
Distance

$d_{rs}$   
Distances

0.00	3.69	0.74	10.27	9.43	9.69	4.26	10.20	9.39	4.95	8.96	7.47	1.79	8.38
3.69	0.00	7.97	11.27	10.10	11.52	2.67	9.08	9.14	3.85	10.34	6.27	4.37	9.37
0.74	7.97	0.00	9.69	10.73	10.32	6.96	3.07	3.66	9.85	9.22	3.17	9.11	8.73
10.27	11.27	9.69	0.00	3.03	2.59	11.43	8.23	7.47	11.21	2.91	9.08	10.51	3.34
9.43	11.09	10.73	3.03	0.00	1.83	11.38	9.50	8.07	10.94	2.51	9.01	9.47	2.75
9.69	11.52	10.32	2.59	1.83	0.00	10.78	9.01	8.43	11.56	2.60	9.04	10.11	3.26
4.26	2.67	6.96	11.43	11.38	10.78	0.00	9.02	8.23	5.22	10.48	5.27	4.69	9.59
10.20	9.08	3.07	8.23	9.50	9.01	9.02	0.00	11.35	7.99	4.42	10.26	7.76	10.20
9.39	9.14	3.66	7.47	8.07	8.43	8.23	11.35	0.00	10.65	7.19	4.12	9.05	7.32
4.95	3.85	9.85	11.21	10.94	11.56	5.22	11.35	10.65	0.00	10.42	6.22	5.76	9.34
8.96	10.34	9.22	2.91	2.51	2.60	10.48	7.99	7.19	10.42	0.00	8.52	9.10	2.60
7.47	6.27	3.17	9.08	9.01	9.04	5.27	4.42	4.12	6.22	8.52	0.00	7.56	7.75
1.79	4.37	9.11	10.51	9.47	10.11	4.69	10.26	9.05	5.76	9.10	7.56	0.00	8.45
8.38	9.37	8.73	3.34	2.75	3.26	9.59	7.76	7.32	9.34	2.60	7.75	8.45	0.00

0.00	3.69	0.74	10.27	9.43	9.69	4.26	10.20	9.39	4.95	8.96	7.47	1.79	8.38
3.69	0.00	7.97	11.27	10.10	11.52	2.67	9.08	9.14	3.85	10.34	6.27	4.37	9.37
0.74	7.97	0.00	9.69	10.73	10.32	6.96	3.07	3.66	9.85	9.22	3.17	9.11	8.73
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4.95	3.85	9.85	11.21	10.94	11.56	5.22	11.35	10.65	0.00	10.42	6.22	5.76	9.34
8.96	10.34	9.22	2.91	2.51	2.60	10.48	7.99	7.19	10.42	0.00	8.52	9.10	2.60
7.47	6.27	3.17	9.08	9.01	9.04	5.27	4.42	4.12	6.22	8.52	0.00	7.56	7.75
1.79	4.37	9.11	10.51	9.47	10.11	4.69	10.26	9.05	5.76	9.10	7.56	0.00	8.45
8.38	9.37	8.73	3.34	2.75	3.26	9.59	7.76	7.32	9.34	2.60	7.75	8.45	0.00

0.00	3.69	0.74	10.27	9.43	9.69	4.26	10.20	9.39	4.95	8.96	7.47	1.79	8.38
3.69	0.00	7.97	11.27	10.10	11.52	2.67	9.08	9.14	3.85	10.34	6.27	4.37	9.37
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7.47	6.27	3.17	9.08	9.01	9.04	5.27	4.42	4.12	6.22	8.52	0.00	7.56	7.75
1.79	4.37	9.11	10.51	9.47	10.11	4.69	10.26	9.05	5.76	9.10	7.56	0.00	8.45
8.38	9.37	8.73	3.34	2.75	3.26	9.59	7.76	7.32	9.34	2.60	7.75	8.45	0.00

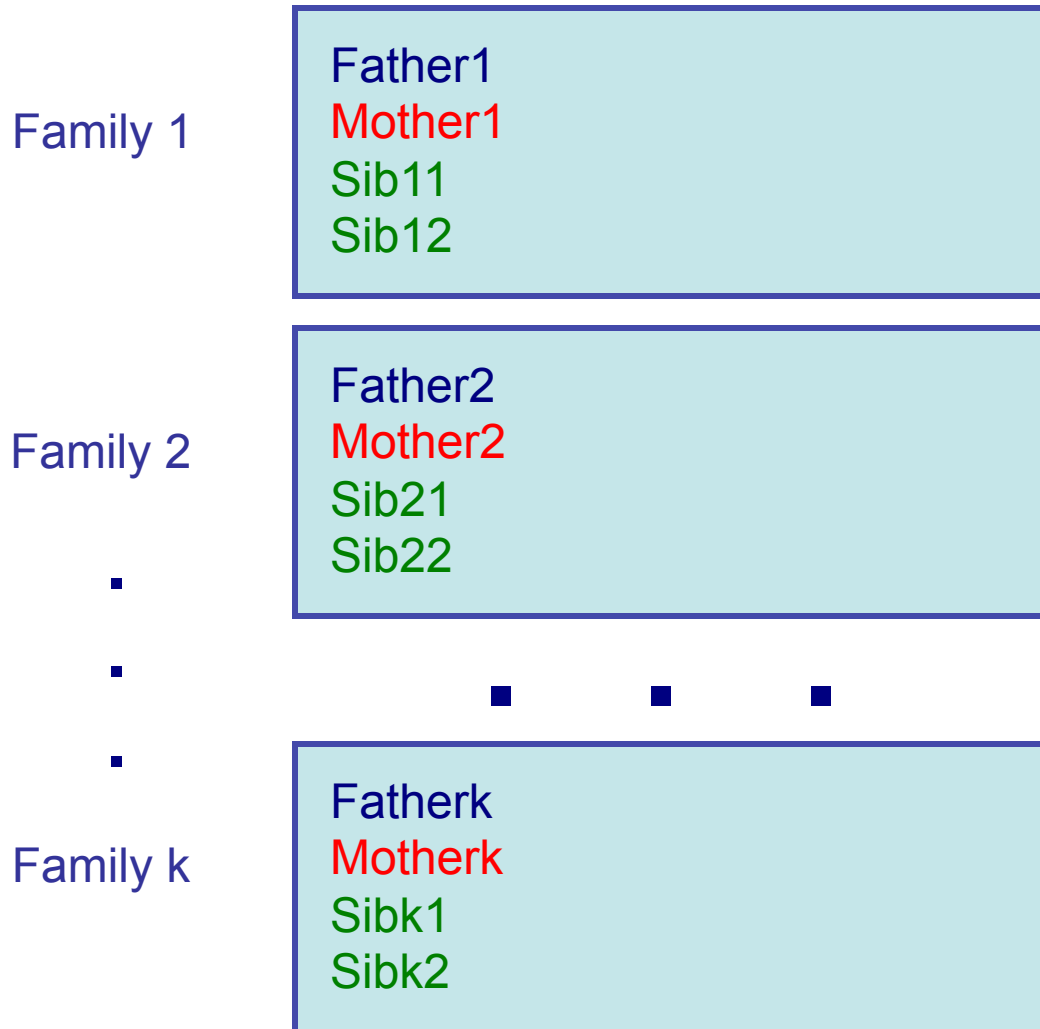
0.00	3.69	0.74	10.27	9.43	9.69	4.26	10.20	9.39	4.95	8.96	7.47	1.79	8.38
3.69	0.00	7.97	11.27	10.10	11.52	2.67	9.08	9.14	3.85	10.34	6.27	4.37	9.37
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8.96	10.34	9.22	2.91	2.51	2.60	10.48	7.99	7.19	10.42	0.00	8.52	9.10	2.60
7.47	6.27	3.17	9.08	9.01	9.04	5.27	4.42	4.12	6.22	8.52	0.00	7.56	7.75
1.79	4.37	9.11	10.51	9.47	10.11	4.69	10.26	9.05	5.76	9.10	7.56	0.00	8.45
8.38	9.37	8.73	3.34	2.75	3.26	9.59	7.76	7.32	9.34	2.60	7.75	8.45	0.00

$$STRESS(q) = \left\{ \frac{\sum_r \sum_s (d_{rs}^q - \hat{d}_{rs}^q)^2}{\sum_r \sum_s (d_{rs}^q)^2} \right\}^{1/2}$$

$$SSTRESS(q) = \sum_r \sum_s ((d_{rs}^q)^2 - (\hat{d}_{rs}^q)^2)^2$$

cophenetic correlation (cc) = correlation of  $d_{rs}$  and  $\hat{d}_{rs} = 0.883$

### 3. MV with dependent (clustered) structure



How to:

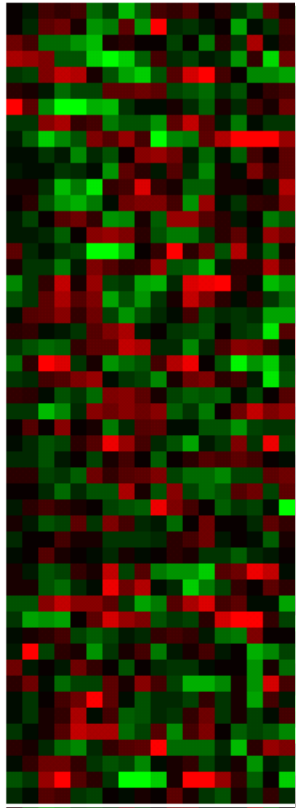
Compute proximity  
for individuals?  
for clusters?

Display matrices  
for individuals?  
for clusters?

# Approaching Statistics & Statistical Approach

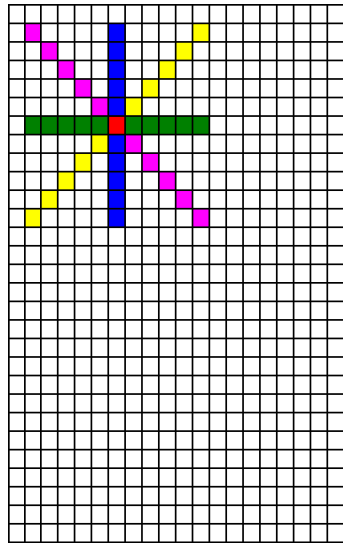
## 4. MV for data with missing values

### Step 4 Evaluation



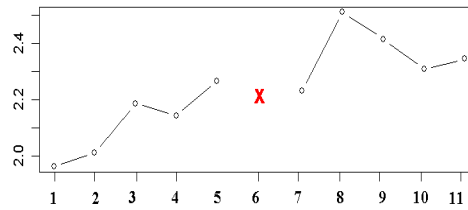
no. directions = 4

no. elements = 10

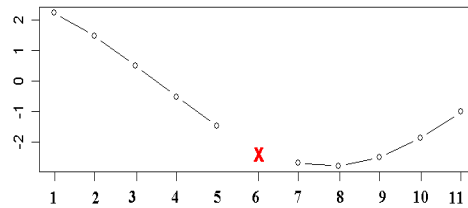


■ Missing Values

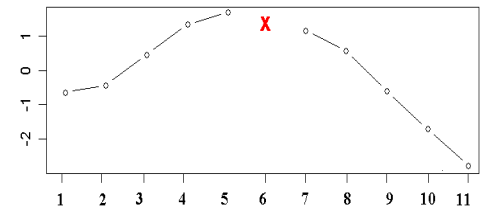
■ Horizontal



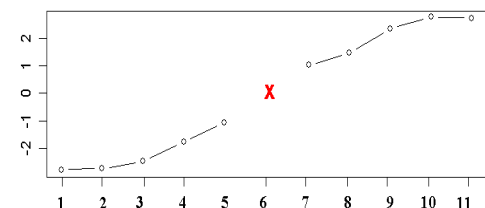
■ Vertical



■ Positive



■ Negative



(1) Fit Regression

$\hat{Z}_d, d = 1, \dots, 4.$

(2) Calculate weights

$$\text{slope}_d[i] = y_d[i + 1] - y_d[i], i = 1, \dots, 9.$$

$$w_d = \frac{1}{\text{var}(\text{slope}_d)}$$

(3) Impute values

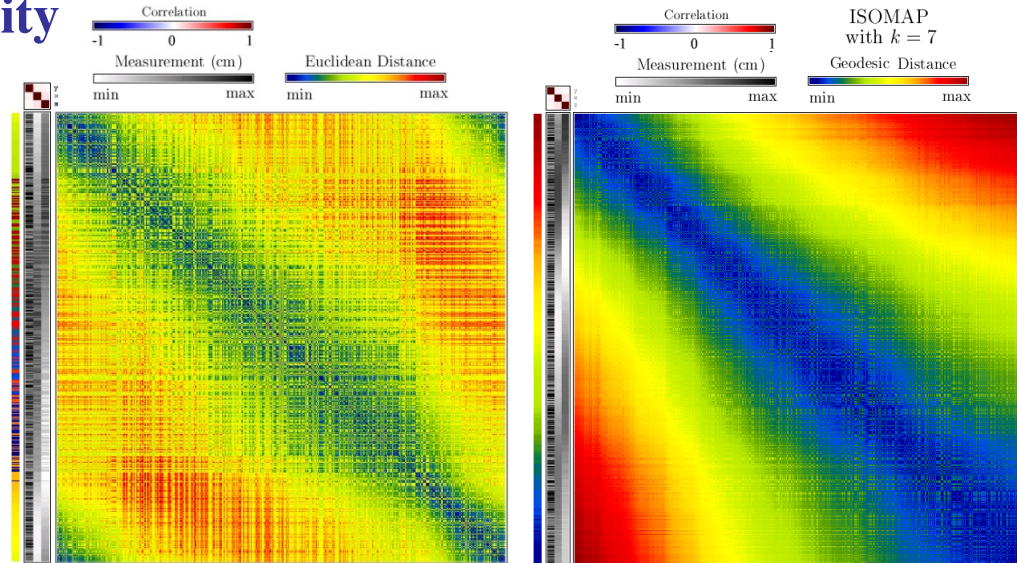
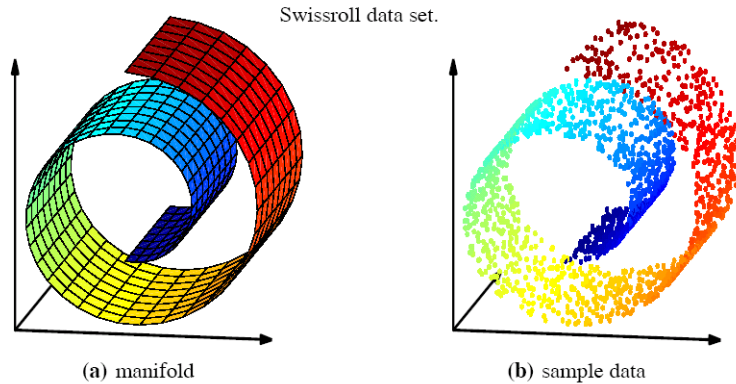
$$\text{ImputedValue} = \frac{\sum_{d=1}^4 w_d \hat{Z}_d}{\sum_{d=1}^4 w_d}$$



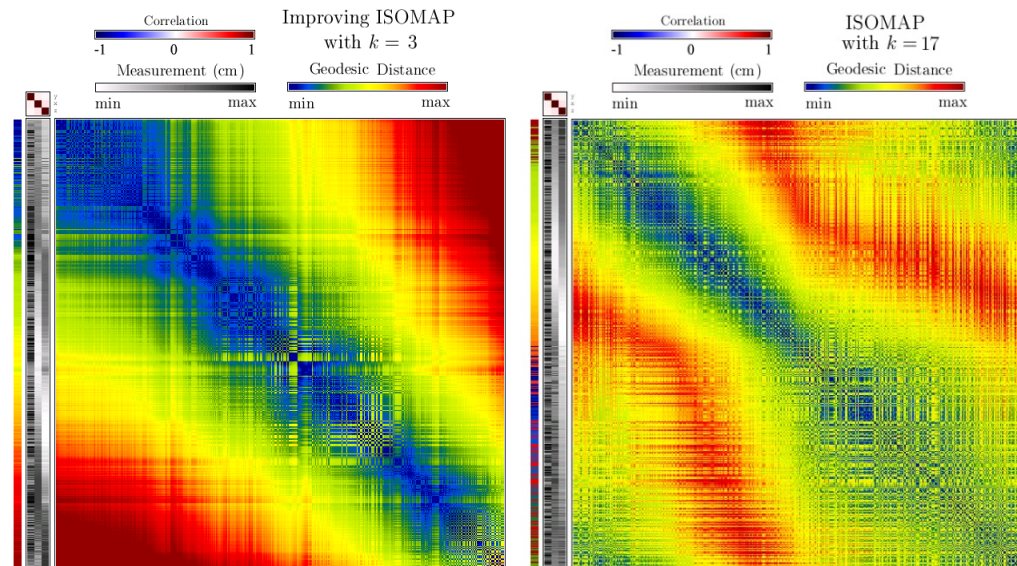
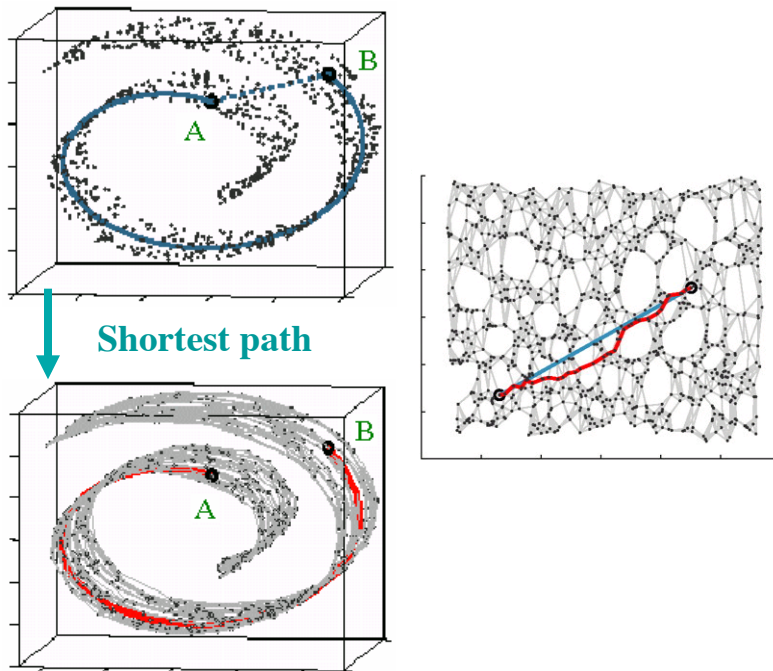
# Approaching Statistics & Statistical Approach

## 5. MV with nonlinear proximity measurement

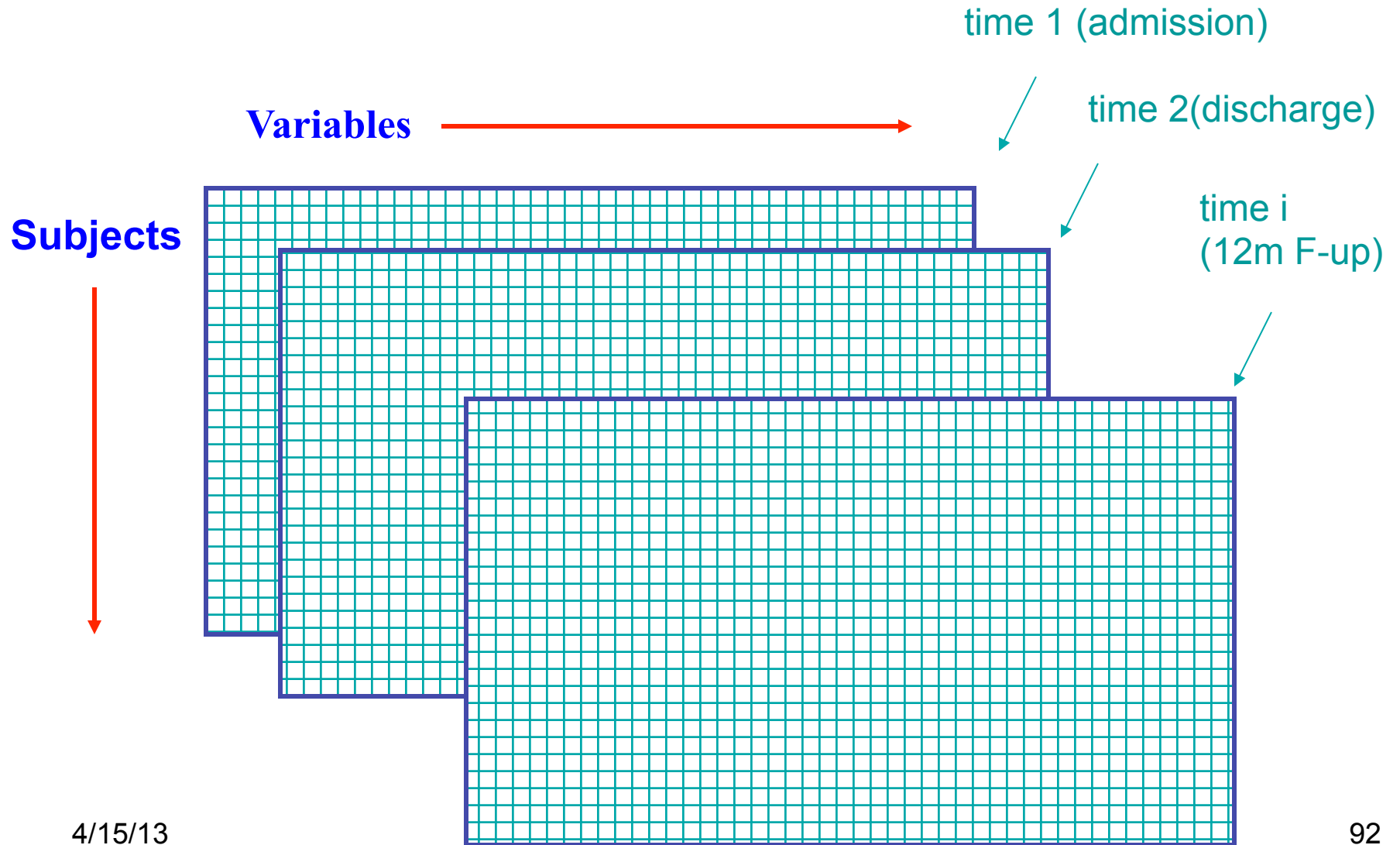
### Concept of Manifolds and Nonlinearity



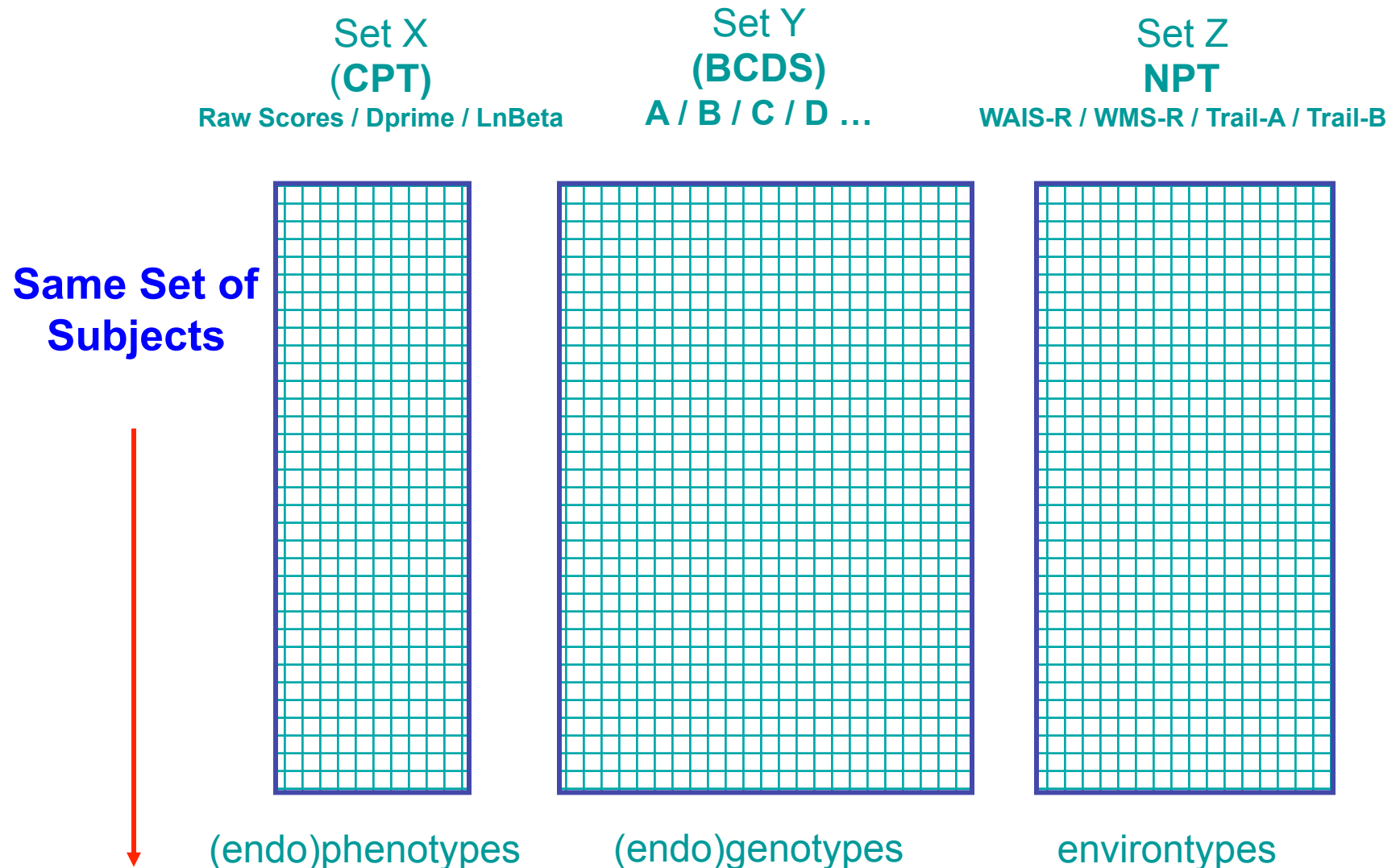
### Isometric Mapping (isomap)



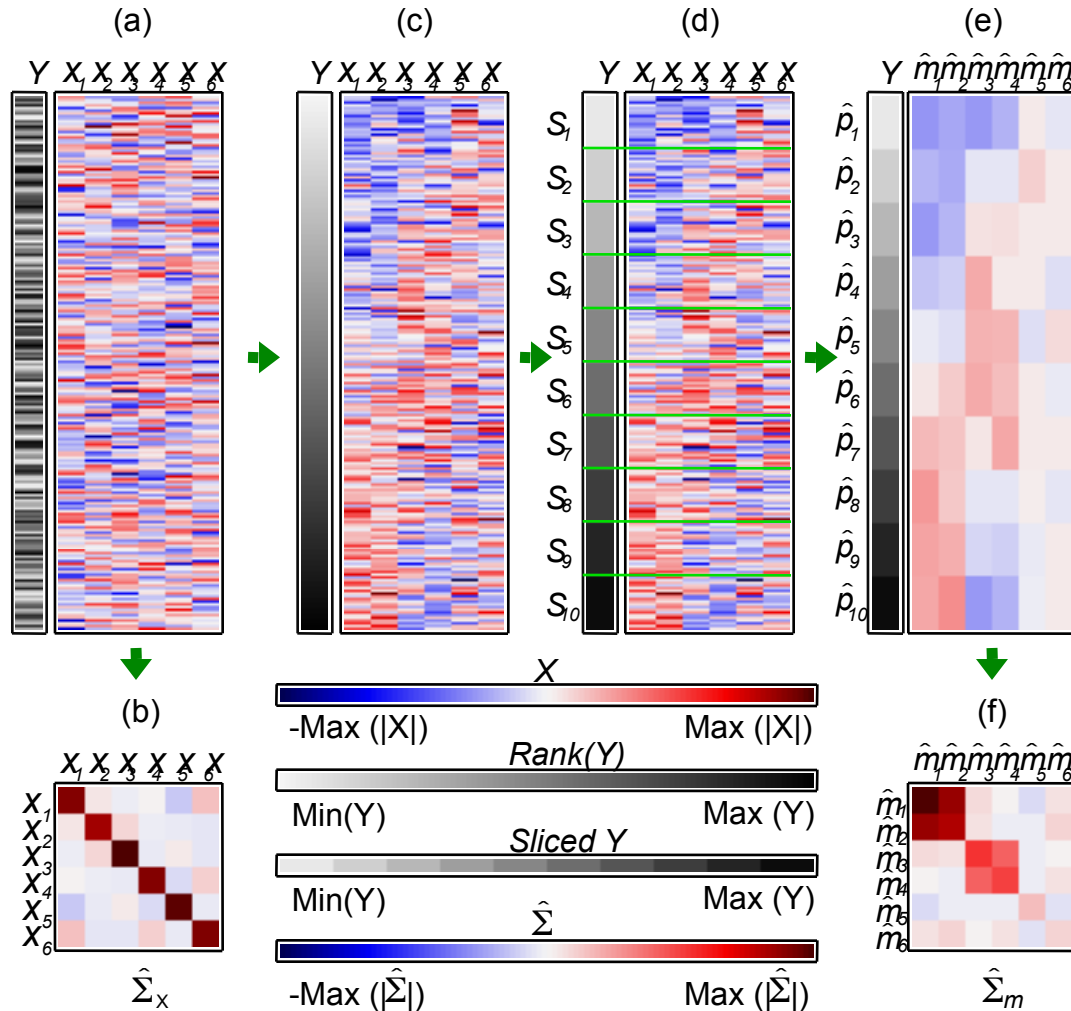
## 6. MV for longitudinal multivariate data



# 7. MV for multi-conditioned multivariate data



# 8. MV with dependent variable(s)



MV for a regression context with dependent variables is similar but not identical to MV with adjusting covariates.

Sliced inverse regression (SIR)  
Li (1991) is a natural starting point.

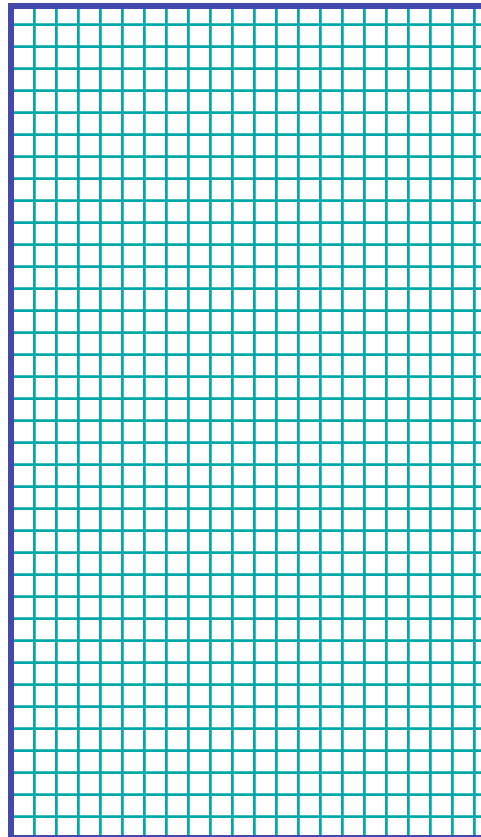
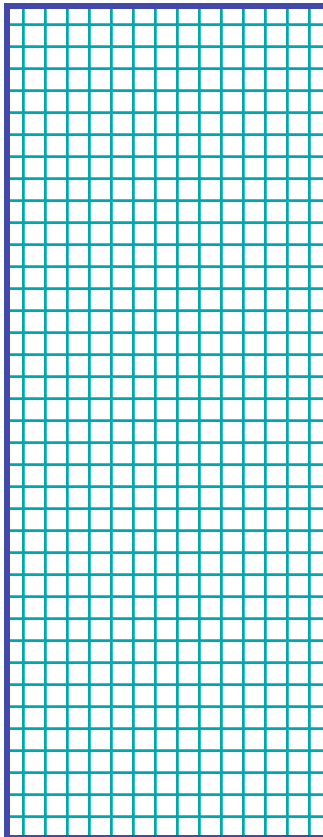
Figure 4. Matrix map of the raw data matrix  $(Y, x)$  with a PCA analysis and the SIR algorithm. (a). original (unsorted) matrix map; (b). sample covariance matrix of  $x$  in (a),  $\hat{\Sigma}_x$ ; (c). sorted (by rank of  $Y$ ) map; (d). sliced sorted map; (e) map for sliced mean matrix  $\hat{m}$ ; (f). sample covariance matrix of sliced mean matrix in (e),  $\hat{\Sigma}_m$ .



# 9. MV for **mixed** data

## Continuous    Categorical

Same Set of  
Subjects



1. Calculation of **proximity** matrices for variables and subjects

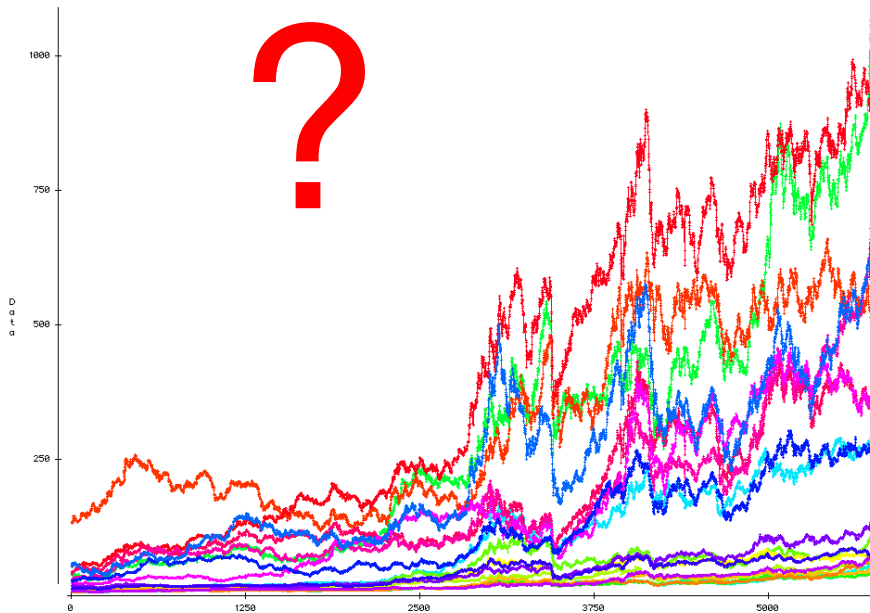
General similarity coefficients Gower (1971)

General weighted two-way dissimilarity coefficients introduced Cox and Cox (2000)

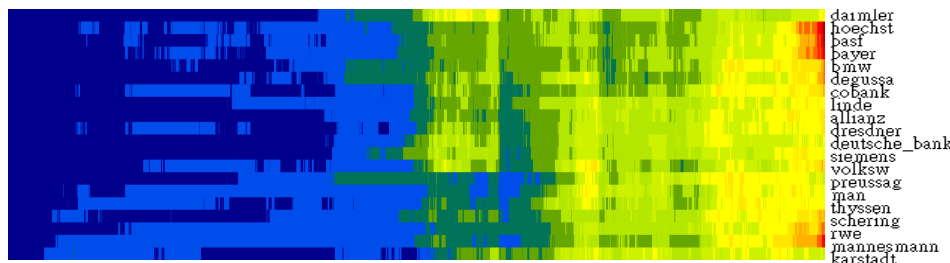
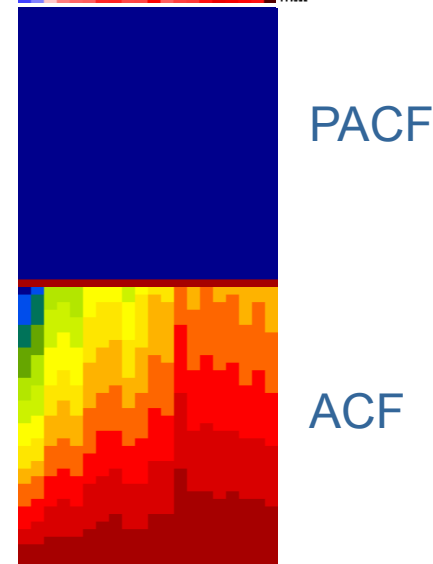
2. **Color coding** for a data matrix with mixed data is a more difficult task.

# Approaching Statistics & Statistical Approach

## 11. MV for multiple time series data



(Euclidean)

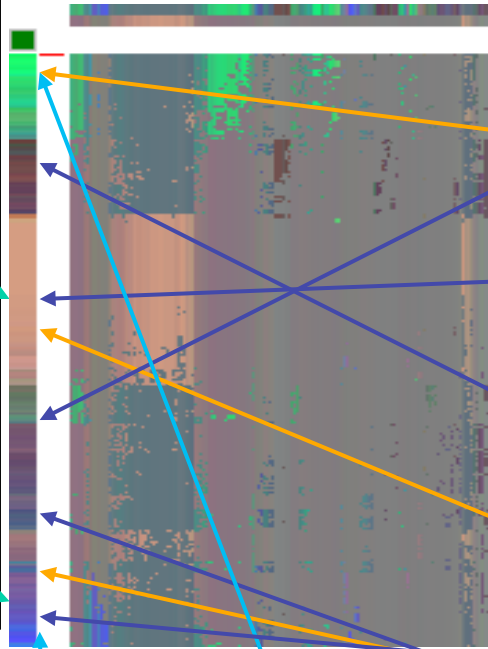
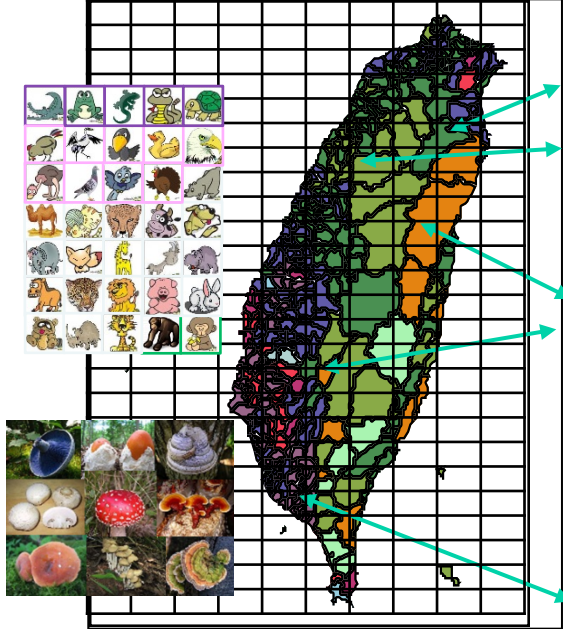


Normalized Data (Matrix condition) (Euclidean)

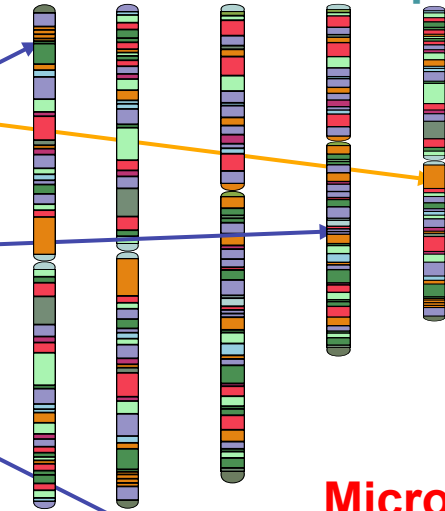
Data provided by Professor WOLFGANG HÄRDLE

# From physical maps to conceptual maps

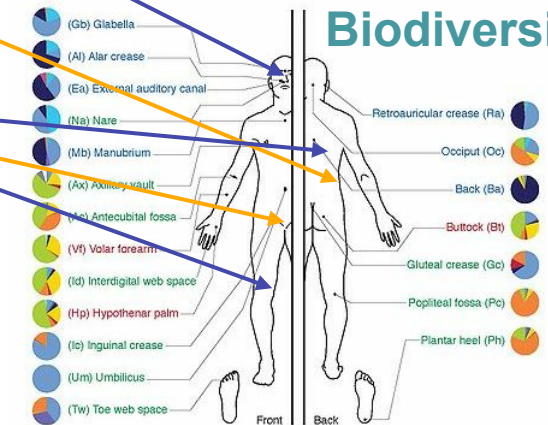
## Macro Biodiversity



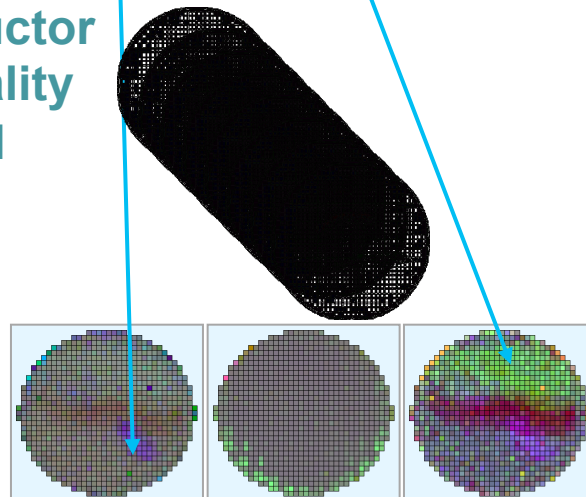
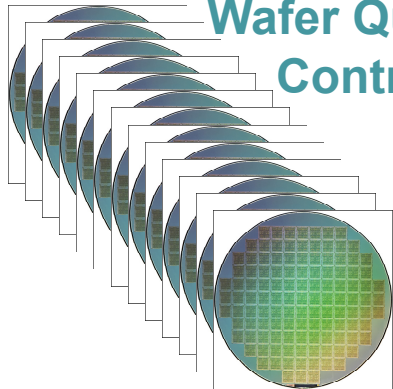
## Chromosome Map



## Micro Biodiversity



## Semiconductor Wafer Quality Control



# Approaching Statistics & Statistical Approach

## 12. MV for Color Blind people

Vischeck



<http://www.vischeck.com/examples/>

Types of color blind

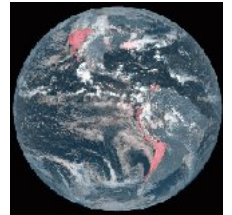
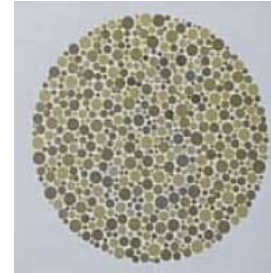
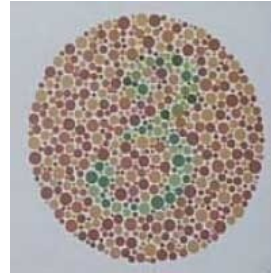
Monochromacy

Dichromacy

Protanopia and deuteranopia

Hereditary tritanopia

Anomalous Trichromacy



To act **passively** to prevent from using color systems that are difficult for color blind people to understand. **or**

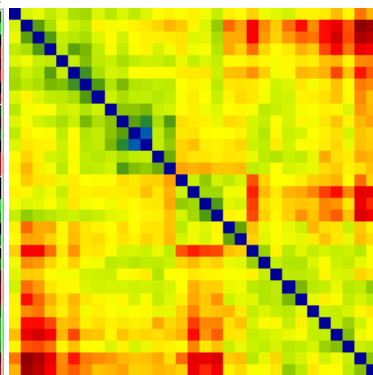
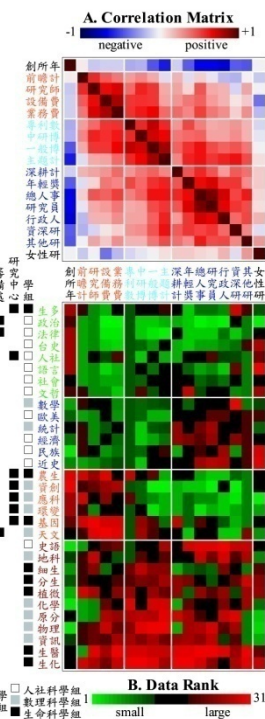
To work **actively** in assisting people with visual impairments to have better visualization of data/information.

“I believe there are more **mathematics/statistics blind people** than **color blind people**”

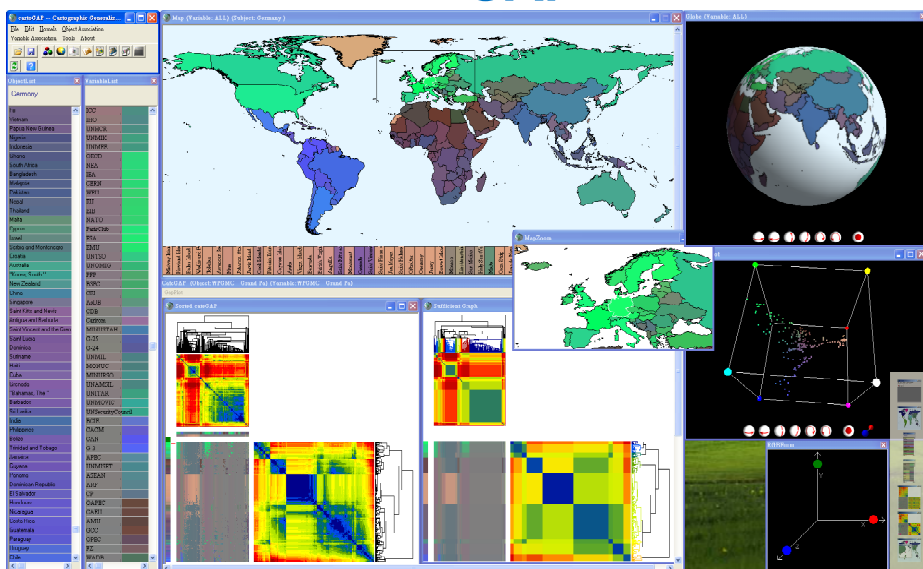


# Approaching Statistics & Statistical Approach

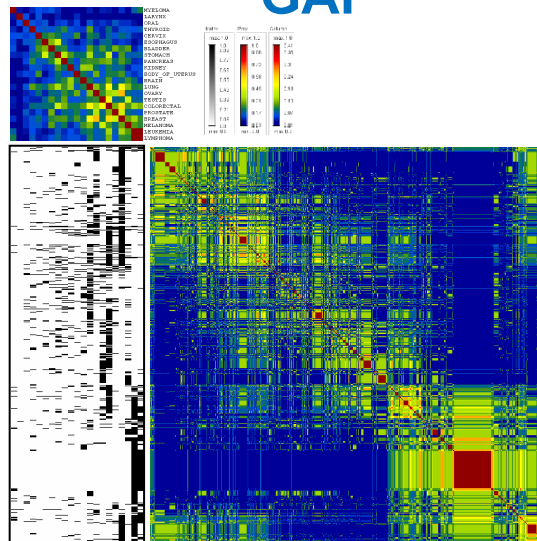
## Continuous GAP



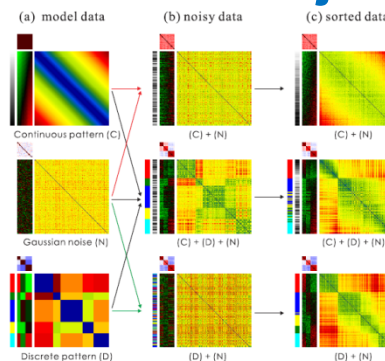
## Cartography GAP



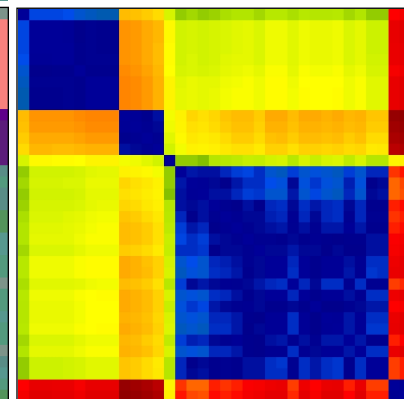
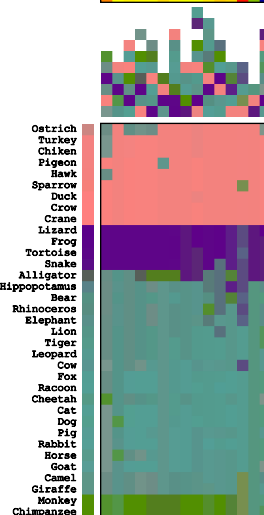
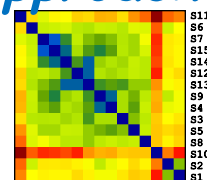
## Binary GAP



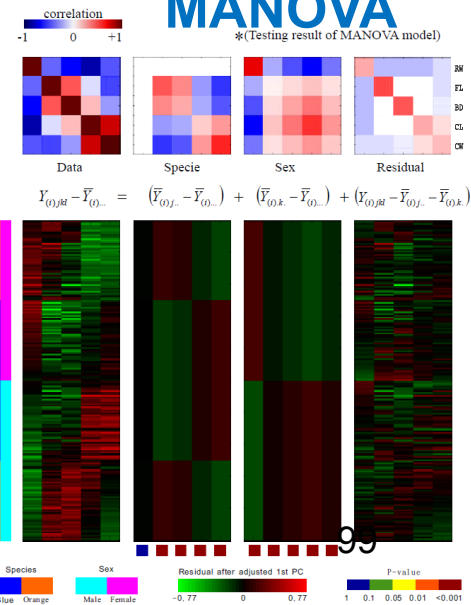
## GAP with Covariate-Adjust



## Categorical GAP



## GAP for MANOVA





*Thank You!*