Invited talks

Homomorphisms to the clone of projections

Manuel Bodirsky
École Polytechnique, Paris

Clones carry a natural topology under which composition is continuous, the topology of pointwise convergence. We study the following question: suppose that a clone has a homomorphism to the clone of projections, does it then also have a continuous homomorphism to the clone of projections? This is for instance true for all clones of term functions of locally finite algebras. The question is particularly interesting for polymorphism clones of omega-categorical structures Gamma, since then a continuous homomorphisms to the clone of projections implies that the constraint satisfaction problem (CSP) of Gamma is NP-hard. In this talk, we give an introduction to topological clones and their importance for the study of the complexity of CSPs.

Forbidden subalgebra theorems in semigroup theory

Michael Kinyon
University of Denver

The paradigm of forbidden subalgebra theorems is, of course, the usual characterization of distributive lattices in the variety of all lattices via forbidding the pentagon and diamond from occurring as sublattices. In this talk, I will first survey classical forbidden subalgebra theorems in semigroup theory, and then focus on some recent examples, including a characterization of adequate semigroups in the quasivariety of amiable semigroups, and a characterization of Clifford semigroups in the variety of inverse semigroups. No background in semigroup theory will be required.

This is joint work with João Araújo (Univ. Aberta, Lisbon).

The growth rate of solvable algebras

Emil W. Kiss
Loránd Eötvös University, Budapest
Coauthors: Keith A. Kearnes, Ágnes Szendrei

We investigate how the behavior of the function $d_A(n)$, which gives the size of a least size generating set for the n-th direct power of A, influences the structure of a finite algebra A. We concentrate on the case, when A is solvable (or even nilpotent, or abelian). In this case the function $d_A(n)$ is always at least linear in n, and it is an open problem whether this function is always linear or exponential for finite solvable algebras. We give an affirmative answer in special cases, and introduce several related growth-restricting structural conditions.

Finitely generated varieties with edge term

Peter Mayr
Johannes Kepler University, Linz
Coauthor: Erhard Aichinger

A variety is finitely generated if it is generated by a single finite algebra. From work of Oates and Powell (1964) it follows that every finitely generated variety of groups has only finitely many subvarieties and all of them are finitely generated as well. By Jónsson's Lemma, a corresponding result holds for congruence distributive varieties. We investigate whether this carries over to varieties with edge term in general.

The term $x^{1/2}yx^{1/2}$

Petr Vojtěchovský
University of Denver

The derived operation $x * y = x^{1/2}yx^{1/2}$ proved useful in non-commutative and/or non-associative algebras $(A, \cdot)$ where the square root $x^{1/2}$ is well-defined, for instance in C^*-algebras, Hilbert spaces and Moufang loops. In all these settings the algebra $(A, *)$ forms a so-called Bruck loop. Borrowing from Glauberman, Gudder, Molnár and Ungar, I will give examples and show connections of $(A, *)$ to foundations of quantum mechanics, special theory of relativity and Moufang loops of odd order.

In badly non-associative settings one has to decide how to interpret the product $x^{1/2}yx^{1/2}$ or its alter ego $(xy^2x)^{1/2}$. For instance, the term $(x^{-1}\backslash(y^2x))^{1/2}$

is used in automorphic loops, that is, loops where all inner mappings are automorphisms. Based on this term, we can prove many deep results on automorphic loops, including the Odd Order Theorem, despite the fact that we do not understand even small examples, say automorphic loops of order pq or p^3. Automated deduction comes into play in a crucial way.

This is joint work with P. Jedlička, K. Johnson, M. Kinyon, K. Kunen, G. Nagy and J. D. Phillips

Valued CSPs and weighted clones

Stanislav Živný
University of Oxford

Universal algebra and in particular clone theory has been behind the recent progress on the computational complexity of non-uniform constraint satisfaction problems (CSPs), which are decision problems ubiquitous in computer science, logic, and combinatorics. In this talk, I will present the theory of weighted clones developed together with Cohen, Cooper, Creed, and Jeavons [SICOMP 13]. Our motivation is to understand the computational complexity of optimisation problems called valued constraint satisfaction problems (VCSPs).

On key relations preserved by a weak near unanimity operation

Dmitriy Zhuk
Moscow State University

We introduce a notion of a key relation, which almost coincides with the notion of a critical relation introduced by Keith A. Kearnes and Ágnes Szendrei. We need only key relations to describe all clones on finite sets. We describe all key relations on 2 elements; these are relations that can be defined as a disjunction of linear equations. Then we show that in general key relations do not have such a nice characterization, but we can get a characterization of key relations preserved by a weak near-unanimity operation. Finally, we show that this characterization can be very helpful in studying of the lattice of clones.

Contributed talks

Some results on near-rings of skew polynomials over nil rings

Abdollah Alhevaz
Department of Pure Mathematics, Faculty of Mathematics and Computer Sciences, Amirkabir University of Technology (Tehran Polytechnic), P.O. Box: 15875-4413, Tehran, Iran &
Institute for Algebra, Johannes Kepler University Linz, 4040 Linz, Austria

Let R be any associative ring, α an endomorphism of R, and δ an α-derivation of R (i.e., δ is an additive operator on R with the property that $\delta(ab) = \delta(a)b + \alpha(a)\delta(b)$). Then the set of all skew polynomials over R (i.e., polynomials with usual addition and multiplication defined subject to the relation $xa = \alpha(a)x + \delta(a)$ for all $a \in R$) forms a (left) near-ring with respect to the usual addition and composition of skew polynomials. Our goal in this talk is to see to what extent we can generalize the results obtained about the usual polynomial ring $R[x]$ to the near-ring of skew polynomials. Because of the complexity of the coefficients that arise upon substitution in this structure, this generalization is fraught with difficulties. For these types of near-ring constructions, we discuss various annihilator properties and it is shown that, with relatively mild conditions on the base ring R and the endomorphism α and α-derivation δ, the problem that whether near-rings of skew polynomials over a base nil ring are nil, has a positive solution in a particular case. We are also interested determining which of the standard nilpotence properties on usual polynomial rings pass to the near-rings of skew polynomials.

The author would like to express his deep gratitude to Professor Erhard Aichinger for valuable comments and suggestions. This work were done during a visit by the author to the Institut für Algebra, Johannes Kepler Universität Linz; he is grateful for the generous hospitality and support he received.

Centraliser clones on finite domains

Mike Behrisch
Vienna University of Technology

We report on recent interest in clones described as polymorphism sets of graphs of functions, i.e. sets of operations described by commutation with a given set of functions. In 1987 it has been proven by Ross Willard and Stanley Burris that on finite domains there exists only a finite number of such clones, called *centraliser clones*. A complete description the lattice of centraliser clones has been achieved for two-element domains (Alexandr Vladimirovič Kuznecov, 1977) yielding 25 clones, and this result was confirmed to be true by Miki Hermann in 2008. Kuznecov's work was continued by Anna F. Daniľčenko, who proved in 1977 that there exist 2986 centraliser clones on a three-element carrier set. Beyond these results not much seems to be known about the structure of the lattice of centralisers on finite domains in general.

In the talk we will consider join-irreducible elements of this lattice (the key to Daniľčenko's proof), and focus in particular on minimal elements (atoms).

Maltsev constraints via LFP+Rank

Dejan Delić
Ryerson University, Toronto
Coauthor: Aklilu Habte

This talk is motivated by the attempt to characterize tractable constraint satisfaction problems with fixed finite relational templates using tools from descriptive complexity. Namely, we can consider the aforementioned CSPs as multisorted relational structures and, in those instances when the fixed template admits a particular polymorphism, attempt to define the class of tractable instances as a class of models of a sentence in some "nice" extension of the first-order logic. The content of this talk is based on the ongoing work with A. Habte (Ryerson University) which is concerned with verifying the tractability of CSPs with Maltsev templates, as well as those that can be handled using the Few Subpowers Algorithm, via LFP+Rank logic

Complexity of variety membership

Marcel Jackson
La Trobe University, Melbourne

A number of authors have given examples of finite algebras $\mathbf{A}$ for which (under reasonable complexity-theoretic assumptions) the following problem is not solvable in polynomial time:

INSTANCE: a finite algebra $\mathbf{B}$ of the same signature as $\mathbf{A}$.
QUESTION: does $\mathbf{B}$ lie in the variety generated by $\mathbf{A}$?

In this talk we will give a suite of fresh examples illuminating the boundaries of complexity for this problem, as well as touching on connections with a number of conjectures from universal algebra and finite model theory.

Linear Datalog and k-permutability = symmetric Datalog

Alexandr Kazda
Vanderbilt University, Nashville

Solving a Constraint Satisfaction Problem (CSP) using the full Datalog language is equivalent to local consistency checking. Therefore, by a result of L. Barto and M. Kozik, CSP($\mathbf{A}$) is solvable by a Datalog program iff the algebra of polymorphisms of $\mathbf{A}$ is congruence meet semidistributive. By restricting the shape of rules a Datalog program can use, one obtains first linear and then symmetric Datalog languages. Characterizing the CSPs solvable by these fragments of Datalog is an open problem. We show that if $\mathbf{A}$ is a finite relational structure whose algebra of polymorphisms is k-permutable for some k and CSP($\mathbf{A}$) can be solved using linear Datalog, then CSP($\mathbf{A}$) can be solved by symmetric Datalog (which is weaker). This supports the conjecture that CSP($\mathbf{A}$) is solvable by symmetric Datalog iff the algebra of polymorphisms of $\mathbf{A}$ is semidistributive and k-permutable for some k.

Term-equivalence of semigroups

Oleg Košik
University of Tartu
Coauthor: Peter Mayr

A *term operation* of an algebra $\mathbf{A}$ is any operation on A that can be constructed from basic operations of $\mathbf{A}$ and projection maps. Two algebras are called *term-equivalent* if they have the same universes and the same term operations. In this talk we will discus term-equivalence of semigroups.

Algebras and constant-factor approximable Min CSPs

Andrei Krokhin
Durham University, UK
Coauthors: V. Dalmau, R. Manokaran

In the Min CSP problem, one is given a CSP instance and the goal is to find a solution minimising the number of *un*satisfied constraints. This problem is obviously equivalent to Max CSP (maximising the number of satisfied constraints) when only optimal solutions are of interest, but the approximation properties of the two problems can be very different. For example, every Max CSP with a fixed finite constraint language admits a constant-factor approximation algoithm, but this is not the case for Min CSP, assuming P is not NP. In this talk, I will discuss algebraic properties of constraint languages such that the corresponding Min CSP admits a constant-factor approximation algorithm. If you don't come to my talk, your toes will turn green.

Finite characterizability of equational classes of threshold functions

Erkko Lehtonen
University of Lisbon
Coauthors: Miguel Couceiro, Karsten Schölzel

A Boolean function $f\colon \{0,1\}^n \to \{0,1\}$ is *threshold*, if there exists a hyperplane in $\mathbb{R}^n$ strictly separating the true points of f from the false points of f, considered as elements of $\mathbb{R}^n$. The class of threshold functions is closed under taking minors (i.e., permutation of arguments, identification of arguments, introduction of dummy arguments), and hence it is known to be characterizable by relational constraints. However, it was shown by Hellerstein that this class is not finitely characterizable. Motivated by the question whether the class of "majority games", i.e., self-dual monotone threshold functions, is finitely characterizable, we determine which clones C on $\{0,1\}$ have the property that the intersection of C with the class of threshold functions is finitely characterizable by relational constraints.

On regular ordered ternary semigroups

Nareupanat Lekkoksung
Khon Kaen University, Thailand
Coauthor: Prakit Jampachon

In this talk, we introduce fuzzy quasi-ideal and characterize regular ordered ternary semigroups. We characterize regular ordered ternary semigroups by means of their fuzzy generalized bi-ideals and fuzzy quasi-ideals, fuzzy generalized bi-ideals and fuzzy ideals, fuzzy bi-ideals and fuzzy ideals. And the last one, we characterize regular ordered ternary semigroups in terms of fuzzy generalized bi-ideals, fuzzy left ideals and fuzzy right ideals and fuzzy bi-ideals, fuzzy left ideals and fuzzy right ideals.

On optimal Mal'cev conditions for congruence meet-semidistributivity

Jelena Jovanović
University of Belgrade

In our search for optimal strong Mal'cev conditions describing congruence meet-semidistributivity of a locally finite variety we came to a single system Σ of term-equations that could be this optimal condition. We examined the algebras of polymorphisms of small digraphs and the results obtained support the conjecture that the system Σ is optimal. The computer search used both algorithms we wrote for this purpose and the model-builder Paradox.

Some observations on centralizing monoids and minimal clones

Hajime Machida
ICU, Tokyo

For a non-empty set A, denote by $\mathcal{O}_A$ and $\mathcal{O}_A^{(1)}$ the set of all multi-variable functions on A and the set of unary functions on A, respectively. For $F \subseteq \mathcal{O}_A$, the *centralizer* F^* of F is the set of functions in $\mathcal{O}_A$ which commute with all functions in F. A submonoid M of $\mathcal{O}_A^{(1)}$ is called a *centralizing monoid* if it is the unary part of some centralizer. When M is the unary part of F^* we call F a *witness* of M. A 'maximal' centralizing monoid is known to have a singleton witness consisting only of a minimal function. On a three-element set A, an interesting and amazing (to me), as well as amusing (at least, to me), relation was found between maximal centralizing monoids and majority functions generating minimal clones. Namely, (1) a centralizing monoid having a majority

minimal function as its witness is always maximal, and (2) any maximal centralizing monoid has a majority minimal function or a constant function as its witness. In this talk, we review the above result and consider the possibility of generalizing it from a three-element set to a larger finite set A.

This is a joint work with I. G. Rosenberg.

Commutators for near-rings

László Márki
Rényi Institute, HAS, Budapest
Coauthors: George Janelidze, Stefan Veldsman

We show that the Huq and the Smith commutators do not coincide for near-rings.

Quantified constraint satisfaction on semicomplete digraphs

Petar Marković
University of Novi Sad
Coauthors: Petar Šapić, Barnaby Martin

Quantified constraint satisfaction, denoted QCSP($\mathcal{A}$), where $\mathcal{A}$ is a fixed finite relational structure, is the problem which inputs a certain kind of sentence and asks whether this sentence is true in $\mathcal{A}$. The sentences which are valid inputs are in prenex form and the quantifier-free part is just a conjunction of atomic formulae. In their groundbreaking 2009 paper, Bórner, Bulatov, Chen, Jeavons and Krokhin connected the set of surjective compatible operations of $\mathcal{A}$ with the complexity of QCSP($\mathcal{A}$). Unfortunately, no nice delineation between the hard and easy case in terms of Mal'cev conditions is speculated, like in the case of usual Constraint satisfaction problem, but at least there are some known sufficient conditions for hardness. We managed to apply these conditions and (with some 50 pages' worth of extra work) prove a trichotomy result (Pspace-complete, NP-complete, or P) for QCSP($\mathcal{A}$), where $\mathcal{A}$ is a semicomplete digraph (loopless digraph such that for any pair of distinct vertices there is at least one directed edge between them).

The lattice of quasivarietes of modules over a Dedekind ring

Anna Mućka
Warsaw University of Technology
Coauthors: Katarzyna Matczak, Přemysl Jedlička

A ring $\mathcal{R}$ is *a Dedekind ring* if it is an integral domain and if every nonzero proper ideal of $\mathcal{R}$ is a finite product of prime ideals. Any ideal of a Dedekind ring has the unique factorization. Using this fact, we show that the lattice of quasivarieties of modules over a Dedekind ring $\mathcal{R}$ is isomorphic to the Biełkin lattice $L(\alpha)$, where α is the cardinality of the set of prime ideals of the ring $\mathcal{R}$.

Extension property and quasi-Maltsev term conditions

Anvar M. Nurakunov
Kyrgys Academy of Sciences

For a quasivariety $\mathcal{R}$, a congruence θ on $\mathbf{A}$ is called an $\mathcal{R}$-*congruence*, if $\mathbf{A}/\theta \in \mathcal{R}$. The set $\mathrm{Con}_{\mathcal{R}}\,\mathbf{A}$ of all $\mathcal{R}$-congruences on $\mathbf{A}$ is a complete lattice. For any set $X \subseteq A^2$ there is a least $\mathcal{R}$-congruence containing X; we denote it by $\theta_{\mathcal{R}}(X)$. A quasivariety $\mathcal{R}$ has an *Extension Property* if for any algebra $\mathbf{A} \in \mathcal{R}$ and every congruences $\alpha,\ \beta \in \mathrm{Con}\,\mathbf{A}$ we have $\theta_{\mathcal{R}}(\alpha \cap \beta) = \theta_{\mathcal{R}}(\alpha) \cap \theta_{\mathcal{R}}(\beta)$. A *strong quasi-Maltsev term condition* is a first-order sentence in the language of clones of the form '$\exists(\wedge$atomics $\to$ atomic)'. The main purpose of the talk is to show that Extension property can be charachterized by the set (possibly, infinite) of strong quasi-Maltsev term conditions.

On universal homogeneous polymorphisms and automatic homeomorphicity for clones

Christian Pech
Technical University Dresden & University of Novi Sad
Coauthors: Maja Pech

This talk conceptually continues the talk "Automatic homeomorphicity for topological monoids" by Maja Pech. Instead of topological monoids, we will consider topological clones. Recently, it has been shown by Bodirsky, Pinsker and Pongrácz, that the polymorphism clones of a number of countable homogeneous structures (including the Rado graph) have automatic homeomorphicity with respect to the class of polymorphism clones of countable structures. One of their main tools in proving automatic homeomorphicity are so called gate coverings. In out talk we are going to strengthen the notion of gate coverings

by introducing universal homogeneous polymorphisms. We will characterize all homogeneous structures that have universal homogeneous polymorphisms. Using these results we will show automatic homeomorphicity for the polymorphism clones of a number of homogeneous structures for which this property hitherto was not known to hold.

On automatic homeomorphicity for topological monoids

Maja Pech
TU Dresden/University of Novi Sad
Coauthors: Christian Pech

The endomorphism monoid of an infinite structure is naturally equipped with a topology (the topology of pointwise convergence). Let $\mathcal{M}$ be a class of topological monoids, and let $\mathbb{M}$ be a topological monoid. We say that $\mathbb{M}$ has automatic homeomorphicity with respect to $\mathcal{M}$ if every monoid-isomorphism of $\mathbb{M}$ to an element of $\mathcal{M}$ is already a homeomorphism. Automatic homeomorphicity has been studied intensively, when $\mathcal{M}$ is the class of automorphism groups of countable structures and $\mathbb{M} \in \mathcal{M}$. In this talk we report on recent results about automatic homeomorphicity with respect to the class $\mathcal{M}$ of endomorphism monoids of ω-categorical structures. For a number of ω-categorical structures we will show automatic homeomorphicity with respect to $\mathcal{M}$.

Entropicity and generalized entropic property in idempotent n-semigroups

Agata Pilitowska
Warsaw University of Technology
Coauthors: Erkko Lehtonen

An algebra (A, F) is entropic, if every pair of its fundamental operations commutes. A weaker version of the entropic law is the so-called generalized entropic property. On the level of varieties, the generalized entropic property may be characterized by the so-called subalgebra closure property: the variety V has the generalized entropic property iff for each algebra (A, F) in V its complex algebra of subalgebras is also a subalgebra of (A, F). In general, these two properties are not equivalent. But in some cases they are. During the talk we will present certain classes of idempotent n-semigroups for which entropicity and the generalized entropic property are equivalent. These results support the conjecture that every idempotent algebra with only one at least binary operation, with the generalized entropic property is entropic.

Clones on Ramsey structures

Michael Pinsker
Vienna University of Technology

A countable structure S is Ramsey iff for all finite substructures P and all colorings of the isomorphic copies of P in S there are arbitrarily large monochromatic substructures in S. We survey how clones on Ramsey structures relate to finite clones, and explain possible applications of this relationship: in particular, the future reduction of CSPs of a nice class of infinite structures to finite structures.

Uncountable critical point for congruence distributive varieties

Miroslav Ploščica
Slovak Academy of Sciences

For varieties $\mathcal{V}$ and $\mathcal{W}$ we define the critical point $\mathrm{Crit}(\mathcal{V};\mathcal{W})$ as the smallest cardinality of a semilattice isomorphic to $\mathrm{Con}_c A$ for $A \in \mathcal{V}$ but not for $A \in \mathcal{W}$. (Here $\mathrm{Con}_c A$ is the semilattice of all compact congruences of A.) We present a general method for constructing pairs of (congruence distributive) varieties with the critical point $\aleph_1$. We present several examples and discuss possible generalizations. Our method is based on lifting of semilattice diagrams by the Con functor.

On syntactic structures

Libor Polák
Masaryk University, Brno
Coauthor: Ondřej Klíma

The algebraic language theory contributes to the classification of regular languages and to the decision of memberships of a given language to various important classes of languages (star-free, piecewise testable, ...) The crucial notion is here the variety of languages (for each finite alphabet A one has a Boolean algebra $\mathcal{V}(A)$ of regular languages over A which is closed with respect to quotients, and for each morphism $f\colon B^* \to A^*$ and $L \in \mathcal{V}(A)$ one has also $f^{-1}(L) \in \mathcal{V}(B)$. We consider also varieties of finite deterministic automata and pseudovarieties of finite monoids. The links are via the minimal automaton of a language and via constructing the transformation monoid of an automaton. There are numerous significant classes of languages which are not varieties. So we need to weaken the closure properties in the definition. There are several variants and

one has to enrich automata and monoids with additional algebraic structures in the same time to get the correspondences: languages $\longleftrightarrow$ automata $\longleftrightarrow$ monoids as above.

Reconstructing the topology of the polymorphism clone of the random graph

András Pongrácz
École Polytechnique, Paris
Coauthors: Manuel Bodirsky, Michael Pinsker

We study polymorphism clones $\mathrm{Pol}(F)$ where F is a countable structure. The clone $\mathrm{Pol}(F)$ is not merely an algebraic object with n-ary compositions for operations, but also a topological space under the pointwise convergence topology. We say that $\mathrm{Pol}(F)$ has reconstruction if whenever it is isomorphic to a clone $\mathrm{Pol}(F')$, then there is also an isomorphism between $\mathrm{Pol}(F)$ and $\mathrm{Pol}(F')$ that is a homeomorphism. The analogous notion for automorphism groups has been studied intensively. We present some general techniques to obtain reconstruction for (monoids and) clones, and conclude that the polymorphism clone of the random graph has reconstruction.

Duality for some classes of convex sets

Anna Romanowska
Warsaw University of Technology
Coauthors: Anna Mućka

There is a well known self-duality for the category of finite-dimensional (real) vector spaces which can be adjusted to the category of corresponding affine spaces. This duality can be extended to the category of all affine spaces. However, it cannot be restricted to provide a duality for the category of convex subsets of (real) affine spaces. We will discuss the problem of duality for some classes of convex sets. We consider subsets of real affine spaces as abstract algebras, so-called barycentric algebras. Dualities for some classes of barycentric algebras will be described. In constructing some od these dualities, we use methods of dualizing so-called Plonka sums of algebras.

Poset Loops

Jonathan Smith
Iowa State University

Given a ring and a locally finite poset, a poset loop is obtained from a new and natural extended convolution product on the set of functions mapping intervals of the poset to elements of the ring. The talk will examine the interplay between properties of the ring, the poset, and the loop. The annihilation structure of the ring and extremal elements of the poset determine commutative and associative properties of elements of the loop. Nilpotence of the ring and height restrictions on the poset force the loop to become associative, or even commutative. Constraints on the appearance of nilpotent groups of class 2 as poset loops are given. The main result shows that the incidence loop of a poset of finite height is nilpotent, of nilpotence class bounded in terms of the height of the poset.

On the subpower membership problem for semigroups

Markus Steindl
Johannes Kepler University, Linz
Coauthor: Peter Mayr

Given a fixed, finite algebra $\mathbf{A}$, the *subpower membership problem for* $\mathbf{A}$ is the following decision problem:

Input: Tuples $a_1, \ldots, a_k, b \in A^n$
Question: Is b in the subalgebra of A^n generated by $a_1, \ldots, a_k$?

We investigate the complexity of this problem for particular semigroups.

Structural completeness for discriminator varieties

Michał Stronkowski
Warsaw University of Technology

The notion of structural completeness (SC for short) in logic has received considerable attention since many years. In the eighties Clifford Bergman noted that the algebraic counterpart of SC property for a quasivariety or a variety is to be generated as a quasivariety by its free algebras. It appears that there are varieties that are as "good" as SC varieties from the logical perspective (strictly from the proof theoretical perspective) but are not SC. Among them there are e.g. varieties of monadic algebras. That is why Wojciech Dzik introduced an adjusted notion: almost structural completeness (ASC for short). We claim that

ASC property is mare adequate notion than SC. We hope to present plenty of examples of ASC varieties which are not SC. From the work of Stanley Burris about unification in discriminator varieties it follows that all discriminator varieties are ASC. In the talk we will present a characterization of SC discriminator varieties. It will follow that there are many discriminator varieties which fail to be SC.

Category equivalences of clones

Worakrit Supaporn
University of Potsdam

All clones can be written in a form $\operatorname{Pol} Q$ for some set Q of relations and all maximal clones can be written in a form $\operatorname{Pol}\rho$ for some relation ρ. I. G. Rosenberg has classified all maximal clones on finite sets into 6 classes by 6 types of relations. One of them is a class of all $\operatorname{Pol}\rho$ where ρ is a nontrivial equivalence relation. In this talk, we present a characterization of the clones $\operatorname{Pol}\rho$ (ρ is a nontrivial equivalence relation) by category equivalence.

Operator properties of congruence permutable varieties with strongly definable principal congruences

Bože Tasić
Ryerson University, Toronto

In an attempt to describe the partially ordered monoid of operators generated by the operators H (homomorphic images), S (subalgebras), P_f (filtered products) for the variety $\mathcal{R}_c$ of commutative rings several results about congruence permutable varieties have been discovered.

Let us recall that the variety $\mathcal{R}_c$ is congruence permutable and for any $\mathbf{R} \in \mathcal{R}_c$, and $a, b, c_1, d_1, \dots c_k, d_k \in R$ we have

$$(a,b) \in \operatorname{Cg}((c_1,d_1),\dots,(c_k,d_k)) \leftrightarrow \exists e_1 \dots \exists e_k \ ((a-b) = \sum_{i=1}^{k} e_i(c_i - d_i)).$$

It will turn out that these two properties are the main reason why $\mathcal{R}_c$ satisfies $HP_f \leq SP_fH$.

We will actually prove that whenever a congruence permutable variety $\mathcal{V}$ has finitely generated congruences definable by a special type of formula we will have $HP_f(\mathcal{K}) \subseteq SP_fHS(\mathcal{K})$ for every class $\mathcal{K} \subseteq \mathcal{V}$.

On the complexity of testing for Jónsson terms

Matt Valeriote
McMaster University, Hamilton
Coauthor: Alex Kazda

We show that for $k > 0$ the problem of deciding if a given finite idempotent algebra has a sequence of Jónsson terms of length k can be solved by a polynomial time algorithm. This is in contrast to earlier results that show that in general testing for a sequence of Jónsson terms of length $k > 1$ is an EXP-TIME complete problem. Our techniques can be used to show that other similar special Maltsev conditions can be tested in polynomial time for finite idempotent algebras.

On the cardinality of the C-clone lattice

Edith Vargas-García
University of Leeds
Coauthor: Mike Behrisch

A *clausal relation* is the set of all tuples over a finite set D satisfying disjunctions of inequalities of the form $x \geq d$ and $x \leq d$, where x, d belong to $D = \{0, 1, \ldots, n-1\}$. The definition of clausal relations is based on the notion of *clausal constraints*, which first appeared in connection with constraint satisfaction problems, studied by Creignou, Hermann, Krokhin and Salzer (2008). In this talk I will present some results of Behrisch and Vargas on the description of clones determined by clausal relations, so-called *C-clones*. More precisely, I will focus on the open problem of determining the exact cardinality κ of the lattice of all C-clones.

Special elements of the lattice of epigroup varieties

Boris M. Vernikov
Ural Federal University
Coauthors: Shaprynskii V. Yu., Skokov D. V.

An *epigroup* is a semigroup in which some power of any element lies in a subgroup of the given semigroup. Epigroups can be treated as *unary semigroups*, that is semigroups equipped by an additional unary operation defined by the following way. If S is an epigroup and $x \in S$ then some power of x lies in a maximal subgroup of S. We denote this subgroup by G_x and the unit element of G_x by x^ω. It is well known that the element x^ω is well defined and $xx^\omega = x^\omega x \in G_x$. We denote the element inverse to xx^ω in G_x by $\overline{x}$. The map $x \longmapsto \overline{x}$ is just

the mentioned unary operation on an epigroup S. The element $\overline{x}$ is called *pseudoinverse* to x. So, we can consider varieties of epigroups as algebras with two operations, namely multiplication and pseudoinversion. An idea to examine epigroups in the framework of the theory of varieties was promoted by L. N. Shevrin in [1,2]. An overview of first results obtained here may be found in Section 2 of the survey [3].

An element x of a lattice $\langle L; \vee, \wedge \rangle$ is called *neutral* if, for all $y, z \in L$, the sublattice of L generated by x, y and z is distributive; *modular* if $(x \vee y) \wedge z = (x \wedge z) \vee y$ for all $y, z \in L$ with $y \leq z$; *upper-modular* if $(z \vee y) \wedge x = (z \wedge x) \vee y$ for all $y, z \in L$ with $y \leq x$. Elements of these and several other types (for instance, lower-modular, distributive or codistributive elements) are intensively examined with respect to the lattice of all semigroup varieties and certain its sublattices. Results obtained here are overviewed in [4].

In the present work we start with an investigation of special elements in the lattice **Epi** of all epigroup varieties. We completely determine all neutral elements of this lattice. It turns out that there are four neutral elements in **Epi** only: the trivial variety, the variety of all semilattices, the variety of semigroups with zero multiplication and the join of two previous varieties. Further, we find a strong necessary condition for modular elements of the lattice **Epi** that completely reduces the problem of description of corresponding varieties to nilvarieties satisfying identities of some special type. Finally, we completely classify commutative varieties that are modular elements of the lattice **Epi** and commutative varieties that are upper-modular elements of this lattice.

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Presentation for semigroups satisfying $x = x^k$, $k > 2$

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Bands are semilattices of rectangular bands. We want to find a similar presentation for semigroups satisfying $x = x^k$, $k > 2$. We will present an approach and meaningful examples. There is complete solution for $k = 3$.

The structure of polynomial operations associated with smooth digraphs

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With every digraph we associate an algebra whose fundamental operations are the polymorphisms of the digraph. In a 2012 paper M. Maroti and I proved that the digraph of endomorphisms of any finite connected reflexive digraph is connected, provided that the algebra associated with the digraph lies in a congruence join-semidistributive over modular variety. A digraph is smooth, if it has no sinks and no sources. Smooth digraphs of algebraic length 1 are a broad generalization of reflexive digraphs. In a 2009 paper, Barto et al. proved that every finite smooth digraph of algebraic length 1 whose associated algebra lies in a congruence meet-semidistributive over modular variety has a loop edge. In the talk I present our recent result that the digraph of unary polynomial operations of the algebra associated with a finite connected smooth digraph of algebraic length 1 is connected, provided that the algebra lies in a congruence join-semidistributive over modular variety. This generalizes our connectivity result mentioned above and implies the restricted version of Bartos' result in the congruence join-semidistributive over modular case. Some further consequences and related open questions will also be discussed in the talk.

The structure of medial quandles

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Quandles are binary algebras motivated by knot theory. In this talk we will describe the structure of *medial quandles*, i.e. quandles in which the identity $(xy)(zt) = (xz)(yt)$ holds. The most important examples are *affine quandles*, taking the operation $x * y = (1-k)(x)+k(y)$ over any abelian group $(A,+)$ with an automorphism k. They form a proper subclass of all medial quandles and they turn out to play a major role in the theory. Our main result states that each medial quandle Q is built from affine pieces using a heterogeneous affine structure. The decomposition to affine pieces is given by the action of a certain subgroup of the automorphism group of Q.