Local Poincaré inequalities on metric spaces with Ricci-curvature bounded from below

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Abstract

Recently Sturm, and independently Lott and Villani gave synthetic notions of Ricci-curvature bounds from below using the theory of optimal transportation. These notions were given in terms of geodesic convexity of functionals on the Wasserstein space $(\mathcal{P}_2(X), W_2)$.

The definitions by Sturm and by Lott and Villani have subtle differences. Both of them are stable under the measured Gromov-Hausdorff convergence, they extend the Riemannian definitions, and they imply many useful properties for the space such as the Brunn-Minkowski and Bishop-Gromov inequalities. Lott and Villani also proved a local $(1,1)$-Poincaré inequality in non-branching metric spaces. The assumption on non-branching was rather unfortunate as it is not stable under the measured Gromov-Hausdorff convergence.

In this presentation I will first talk about the different definitions of Ricci-curvature bounds in metric spaces. Ater that I will prove a local $(1,1)$-Poincaré inequality from the definitions by Lott and Villani without making the extra assumption on non-branching.