

NONLINEAR GAGLIARDO-NIRENBERG INEQUALITIES,

NONLINEAR EIGENVALUE PROBLEMS AND ISOPERIMETRIC INEQUALITIES

Jan Peszek

University of Warsaw

joint works with Angieszka Kałamajska

University of Warsaw

Abstract

We present inequalities

$$(1) \int_{\mathbf{R}} |f'(x)|^p h(f(x)) dx \leq \left(\sqrt{p-1}\right)^p \int_{\mathbf{R}} \left(\sqrt{|f''(x)\mathcal{T}_h(f(x))|}\right)^p h(f(x)) dx,$$

$$(2) \int_{\mathbf{R}} M(|f'|h(f)) \leq C \int_{\mathbf{R}} M\left(\sqrt{|f''\mathcal{T}_h(f)|} \cdot h(f)\right) dx,$$

where f belongs to some subset of the Sobolev space $W_{loc}^{2,1}$. We assume, that $h(\cdot)$ is a given function and $\mathcal{T}_h(\cdot)$ is it's certain transform. Furthermore, M is a Young function satisfying some additional assumptions. Next, we present applications of inequality (1) to nonlinear eigenvalue problems of the form:

$$(3) \quad \begin{cases} f''(x) = g(x)\tau(f(x)) \text{ a.e. in } (a, b), \\ f \in \mathcal{R} \end{cases}$$

with function $g \in L^q(a, b)$ and a set \mathcal{R} describing boundary conditions.

Finally, we show how to apply inequality (2) to new capacity estimates and isoperimetric type inequalities that generalise some results from [3] chapters 8.2.1 and 8.3.

References

- [1] A. Kałamajska, Jan Peszek, *On some nonlinear extensions of the Gagliardo-Nirenberg inequality with applications to nonlinear eigenvalue problems*, preprint 2011, <http://arxiv.org/abs/1104.1967>.
- [2] A. Kałamajska, Jan Peszek, *On certain generalizations of the Gagliardo-Nirenberg inequality and their applications to capacity estimates and isoperimetric inequalities*, in preparation.
- [3] Mazy'a, V. G., *Sobolev Spaces*, Springer-Verlag 1985.
- [4] Opial, Z. *Sur une inégalité*, (French) Ann. Polon. Math. **8** (1960), 29–32.